Engineering 9867 Advanced Computing Concepts
Assignment #1

Due: Tuesday, March 12 at 0900

1. [10 points] Express the following in predicate logic, using the given predicate symbols and types.

   a) [3 points] There is a smallest integer.
   Predicates: $\leq$
   Types: **Integer**

   b) [3 points] The array `A[N]` is bitonic. (An array is said to be *bitonic* iff the elements are in non-decreasing order in some initial portion of the array, and in non-increasing order for the remainder. For example, $[1, 1, 2, 3, 4, 4, 3, 2, 1]$ is bitonic, but $[1, 1, 2, 3, 4, 3, 4, 2, 1]$ is not.)
   Predicates: $<, \leq, >, \geq$

   c) [4 points] The definition of "$\lim_{x \to a} f(x) = L$". (Hint: Quantify variables x, ϵ and δ over **Real** and relate $|f(x) - L|$ to ϵ and $|x - a|$ to δ.)
   Predicates: $<, \leq$
   Types: **Real**

2. [10 points] A *permutation* of an array is an array containing exactly the same values in another order, i.e.,
   $$permuation(a, b) \stackrel{\text{df}}{=} \begin{pmatrix} length(a) = length(b) \wedge \\ \forall i, (0 \leq i < length(a) \rightarrow \begin{pmatrix} card(\{j \mid 0 \leq j < length(b) \wedge a[i] = b[j]\}) = \\ card(\{j \mid 0 \leq j < length(a) \wedge a[i] = a[j]\}) \end{pmatrix} \end{pmatrix}$$
   Prove that the number of permuations of an array of length N is $N!$.

3. [15 points] In this question you are to reason about a C++ function `int gcd(int x, int y)` which returns the greatest common divisor of the natural numbers `x` and `y`.

   a) [5 points] Give the specification for this function. You may find it helpful to recall that any common divisor, d, of natural numbers x and y, will also be a divisor of the GCD of x and y. You may use the following predicate in your specification:
   $divisor(d, x) \stackrel{\text{df}}{=} (\exists q : \textbf{int}, 0 < q \wedge x = d \times q)$

   b) [10 points] Implement the function in C++ and add comments to your implementation to reason, as formally as possible, that it is correct. You may find it helpful to recall the property of natural numbers, that
   $\forall x, y : \textbf{int}, (0 \leq x \wedge 0 \leq y) \rightarrow gcd(x, y) = gcd(y, x\%y)$

4. [15 points] A *palindrome* is a string that is the same when read forward and backward. Some examples of palindromes are "ABBA", "radar" and "200202202002". In this question you are to reason about a C++ function `bool isPalindrome(const string& s)`, which returns `true` if `s` is a palindrome and `false` otherwise.

   a) [5 points] Give the specification for this function.

   b) [10 points] Implement the function in C++ and add comments to your implementation to reason, as formally as possible, that it is correct.