Bond Graph Dynamic Modeling and Stabilization of a Quad-Rotor Helicopter

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Keywords: Bond graph in aerospace, quad-rotor helicopter dynamics, stabilization of quad-rotor helicopters.

Abstract
Four-rotor/quad-rotor helicopters are emerging as a popular unmanned aerial vehicle configuration because of their simple construction, easy maintenance and high payload capacity. A quadrotor is an under-actuated mechanical system with six degrees of freedom and four lift-generating propellers arranged in cross configuration. Maneuvers are executed by varying the speed of the propellers, which causes moments that affect attitude control; and by varying thrust, which affects altitude control. This makes stabilization challenging.

This work presents a dynamic model of such a vehicle using bond graphs. Both an open loop unstable model and a closed loop stable model using different controllers are demonstrated in this paper. The Newton-Euler formalism with body fixed coordinates is used to model the dynamics of the platform. Rotor drag torque is assumed proportional to thrust of the rotor.

The graphical nature and explicit power flow paths inherent in the bond graph formalism facilitated model construction and troubleshooting. Existing commercial bond graph software allowed simultaneous modeling and control implementation.

The model results for different maneuvers and combinations of propeller thrust agree with existing theoretical results. Open loop simulation shows an uncontrolled revolving effect with increasing linear speed which results in instability of the system. The closed loop PID-controlled result nicely demonstrates the stabilization of the system from an initial roll, pitch, yaw and altitude to the desired steady state configuration.

1. INTRODUCTION

Recently, quad-rotors have been considered the best platform for experiments and applications in the field of unmanned aerial vehicle (UAV) because of their capacity of hovering and slow flight as well as reduced mechanical complexity with higher safety and higher payload. Research is mostly motivated due to their size and autonomous flight, which has practical implications for surveillance, targeting and disaster search in partially collapsed buildings [1]. Mesicopter [2] was an ambitious project which explored ways to fabricate centimeter-sized vehicles. The OS4 project [3] was started in March 2003, with an aim to develop devices for searching and monitoring hostile indoor environments. The X4 Flyer project of Australian National University [4] aims at developing a quad-rotor for indoor and outdoor applications. However, control of a quad rotor helicopter is very complex and virtually impossible without modern computer based control systems due to its under actuated nature with 6 degrees of freedom and only 4 actuating motors. Different researchers have modeled and simulated the system in different ways. This paper introduces bond-graph modeling to the field of UAV’s. The complex dynamics of quadrotor helicopters motivated the use of bond graphs for modeling their dynamics. Other available models are mostly code based differential equations which have less provision for modification and model expansion. This paper mostly emphasizes the theory behind quadrotor dynamic modeling and maneuver control. The control system of the vehicle is based on classical control methodology. The bond graph that is produced here follows the Newton-Euler formalism which has been widely used for modeling this kind of helicopter [5-8].

2. QUADROTOR HELICOPTER

![Figure 1. Quadrotor Helicopter Schematic](image-url)
The quad-rotor helicopter is an aircraft whose lift is generated by four rotors. Control of such a craft is accomplished by varying the speeds of the four rotors relative to each other. Quad-rotors have four fixed propellers in cross configuration (Figure 1).

(a) Climb/Descend  
(b) Yaw  
(c) Roll  
(d) Pitch

Figure 2. Theoretical maneuvers of a quadrotor

The front and the rear rotors rotate counterclockwise while the other two rotate clockwise, so that the gyroscopic effects and aerodynamic torque are canceled in trimmed flight. The rotors generate thrust forces \( T \) perpendicular to the plane of the rotors and moments \( M \) about \( x \) and \( y \) axes; moment about \( z \)-axis is obtained using counter torques of the rotor. Increasing or decreasing the speed of the four propellers simultaneously permits climbing and descending (figure 2a). Vertical rotation (yaw) is achieved by creating an angular speed difference between the two pairs of rotors which in turn creates reactive torques (figure 2b). Rotation about the longitudinal axis (pitch) and lateral axis (roll), and consequently horizontal motions, are achieved by tilting the vehicle. This is possible by changing the propeller speed of one pair of rotors (figure 2c, 2d)

3. DEVELOPMENT OF SYSTEM MODEL

The derivation of the system equations is now shown along with the evolution of the bond graph for the quadrotor vehicle. To derive the equations we assume the airframe is rigid, all the propellers are in the same horizontal plane and the quadrotor structure is symmetric. Figure 3 shows the coordinate system of a quadrotor helicopter where body fixed frame \( O=\{x, y, z\} \) is assumed to be at the centre of gravity and aligned with the principal axes of the body. The \( z \)-axis is considered to be positive upward. Due to the double symmetry of the quadrotor, it is logical to set the origin \( O \) at the intersection of the axes of the vehicle. Choosing the directions of the axes according to the common conventions in the literature results is a set of axes which is shown in figure 3. The body fixed frame \( O=\{x, y, z\} \) is considered to be moving with respect to Earth frame/Inertial frame \( E=\{X, Y, Z\} \).

The equation of motion of such a rigid body, under external forces \( F_{\text{external}} \) and \( M_{\text{external}} \) applied at the center of mass and expressed in the body frame, are developed in the Newton-Euler formalism [9-10]. With respect to these body fixed coordinates, the rotational inertia properties remain invariant and the products of inertia are all zero.

\[
\begin{align*}
F_{x,\text{external}} & = m\dot{v}_x - mv_y \omega_z + mv_z \omega_y \\
F_{y,\text{external}} & = m\dot{v}_y - mv_z \omega_x + mv_x \omega_z \\
F_{z,\text{external}} & = m\dot{v}_z - mv_x \omega_y + mv_y \omega_x \\
M_{x,\text{internal}} & = I_{xx} \dot{\omega}_x - (I_{yx} - I_{zx}) \omega_y \omega_z \\
M_{y,\text{internal}} & = I_{yy} \dot{\omega}_y - (I_{yz} - I_{xy}) \omega_x \omega_z \\
M_{z,\text{internal}} & = I_{zz} \dot{\omega}_z - (I_{zx} - I_{xy}) \omega_x \omega_y \\
\end{align*}
\]

The bond graph representation [11] of this Newton-Euler equation is shown in Figure 4 where the upper triangle relates the energy flow in translational dynamics and the lower triangle defines rotational dynamics. The three 1-junctions both in the upper and lower triangle provide the nodes for external forces and moments respectively.
3.1. Modeling of translational dynamics

The translational dynamics of such a helicopter can be rewritten from Equation (1) as,

\[
\begin{bmatrix}
    m & 0 & 0 \\
    0 & m & 0 \\
    0 & 0 & m \\
\end{bmatrix}
\begin{bmatrix}
    \dot{v}_x \\
    \dot{v}_y \\
    \dot{v}_z \\
\end{bmatrix}
= \begin{bmatrix}
    F_x \\
    F_y \\
    F_z \\
\end{bmatrix} - \begin{bmatrix}
    m \ddot{v}_x - v_y \\
    m \ddot{v}_y - v_x \\
    m \ddot{v}_z - v_x \\
\end{bmatrix} \begin{bmatrix}
    \omega_x \\
    \omega_y \\
    \omega_z \\
\end{bmatrix} \tag{2}
\]

The external force contribution on the quadrotor body is mainly the rotor thrust and the drag torque. The drag force is defined as [12] \( F_{\text{drag}} = (1/2) \rho A v^2 \), where \( \rho \) is the density of medium (air), \( A \) is the frontal area of the body and \( v \) is the velocity. Considering all the constants involved in this equation we rewrite \( F_{\text{drag}} = C_D v^2 \) where \( C_D \) is the combination of the constant terms and is defined as Drag Constant in this paper. Therefore,

\[
\begin{bmatrix}
    F_x \\
    F_y \\
    F_z \\
\end{bmatrix} = \begin{bmatrix}
    -C_D x^2 \\
    -C_D y^2 \\
    \sum_{i=1}^4 T_i - C_D z^2 - R \cdot mg \\
\end{bmatrix} \tag{3}
\]

Here, \( C_D \) has components in all three directions and \( T_i \) is the thrust mainly generated by the four rotors which are spinning with a velocity of \( \omega \) and is given by [12]

\[
T_i = C_T \rho \Omega_i^2
\]

\( R \in SO(3) \) is the rotation matrix representing ZYX Euler angles of roll(\( \phi \)), pitch(\( \theta \)) and yaw(\( \psi \)).

Due to the discontinuity of the time variation of angles (\( \phi, \theta, \psi \)) the body angular rates (\( \omega_x, \omega_y, \omega_z \)) need to be transformed by Euler angles using the matrix below [13],

\[
\begin{bmatrix}
    \omega_x \\
    \omega_y \\
    \omega_z \\
\end{bmatrix} = \begin{bmatrix}
    1 & 0 & -\sin \theta \\
    0 & \cos \phi & \sin \phi \cos \theta \\
    0 & -\sin \phi & \cos \phi \cos \theta \\
\end{bmatrix} \begin{bmatrix}
    \phi \\
    \theta \\
    \psi \\
\end{bmatrix} \tag{5}
\]

These transformation angles though suffer from singularity at \( \theta = \pm \pi/2 \) known as Gimbal lock but this does not affect UAV’s in regular flight maneuvers. In the bond graph the translational efforts arise from one gravitational effort source and four rotor effort sources, where gravitational effort is translated from Earth reference to body reference using Euler rotation angles. The upper side of figure 5 illustrates the translational dynamics of the quadrotor. As the quadrotor moves at low speed we can consider drag constant \( C_D \) as negligible and for simulation, rotor thrust is considered as a source of effort.

3.2. Modeling of Rotational Dynamics

Rotational dynamics of a quadrotor consist of three angular motions, namely roll pitch and yaw. Roll and pitch rotation are produced due to moments generated by the thrust difference of the rotors at \( y \)-axis and \( z \)-axis respectively, which are basically acting at a certain perpend-
-icular distance from the center of gravity. However, yaw is produced due to reactive torque in the rotors. From Equation 1 we can redefine that,

\[
\begin{bmatrix}
I_x & 0 & 0 \\
0 & I_y & 0 \\
0 & 0 & I_z
\end{bmatrix}
\begin{bmatrix}
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z
\end{bmatrix}
= \begin{bmatrix}
M_{xx} & 0 & -\omega_z \\
0 & M_{yy} & -\omega_x \\
-\omega_y & M_{zz} & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z
\end{bmatrix}
\ldots ...(6)
\]

Where,

\[
\begin{bmatrix}
M_{xx} \\
M_{yy} \\
M_{zz}
\end{bmatrix}
= C_T \begin{bmatrix}
0 & -l & 0 \\
l & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\omega_1^2 \\
\omega_2^2 \\
\omega_3^2
\end{bmatrix}
= \begin{bmatrix}
0 & -l & 0 \\
l & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
T_x \\
T_y \\
T_z
\end{bmatrix}
\ldots ...(7)
\]

In the above equation, \(l\) is the distance between centre of gravity and the centre of the rotors and \(\lambda\) is the proportionality constant between thrust \(T_i\) and motor torque \(\tau_i\). In this case for the simplicity of simulation we consider \(\lambda\) as 1. However, in practice motor reactive torque is a function of velocity according to [12]

\[\tau_i = C_R \omega_i^2 + I_i \ddot{\omega}_i\]

where \(C_R\) is the reactive torque constant due to drag terms and \(I_i\) is the rotational inertia of the rotor. In Figure 5 contributions of the rotor efforts are given to the three angular velocity ports using a skew matrix containing distances between the rotors and the center of gravity, which creates moments along their respective axis. However the
yaw occurs due to reactive torque of rotors which for simplicity of calculation is considered as proportional to thrust difference and is provided straight from the rotor to the z-axis rotation port. This gives a complete open loop model for the quadrotor vehicle. Obviously, only the dominant effects are modeled neglecting aerodynamic forces and moments exerted by drag and the dynamics of motor and propeller which can be extended easily by adding more external forces to their respective directional ports. The double lines used in figure 5 are vector bonds proposed in [14].

4. CONTROL MODELING

Stabilization is very important for an under-actuated system like a quadrotor, as it is inherently unstable due to its six degrees of freedom and four actuators. A control system is modeled for the quadrotor using four PID (Proportional-Integral-Derivative) controllers, where the input parameters are unstable attitudes (i.e., roll, pitch, yaw and height) and the controlling devices are the actuators or the rotors of the quadrotor. By controlling the thrust from the rotor one can have a stabilized system as well as a desired attitude and altitude. Table 1 shows the combinations of rotor excitations for any kind of positive maneuvers. For example, if we want to increase the pitch (CCW of $\theta$) we have to increase thrust at the back rotor as well as decrease thrust in the front rotor. The bond graph in Figure 6 shows the combination of the four PID controllers that stabilizes the quadrotor. The controller measures the error between the desired and actual attitude and actuates the rotors accordingly. Figure 7 shows the complete closed loop model utilizing this controller into the open loop system. However, as the attitude governs the position vector of the system, this controller can be further extended to move to any desired coordinate by adding position controller into the closed loop model. P, PI, PID controller were tested. The PID controller stabilizes the system faster and more accurately than the other two. The gains were defined using trial and error.

5. SIMULATION RESULTS

Initially to explore the performance of the model, open loop simulation was performed. The simulation were performed using 20SIM software, which converted the bond graph in to a simulation model and performed numerical integration using Backward Differentiation method with tolerance of 1e-005. The simulation demonstrates the flight maneuver which satisfies the theoretical trajectories that the quadrotor is supposed to perform at certain combinations of the rotor thrust. Figure 8 shows the freefall test. Providing no thrust to the quadrotor causes it to fall down due to gravity at a rate of 9.8 m s$^{-1}$. The parameters considered for simulation are realistic quadrotor parameters from [15] where the mass of the vehicle $m = 0.5$ kg. For lifting the helicopter it is obvious that there should be minimum combination thrust of 4.9 N provided by all the four rotors.

Table 1. Combinations of actuation for controlling altitude and attitude

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Front Rotor, F</th>
<th>Back Rotor, B</th>
<th>Right rotor, R</th>
<th>Left rotor, L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lift</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Roll</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Pitch</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Yaw</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 6. Block diagram of quadrotor craft controller
Figure 7. Closed loop bond graph of quadrotor craft

Figure 8. Open loop free fall simulation for zero thrust

(a) Rotation angles
(b) Linear velocities
Figure 9. Open loop roll motion due to \( (T_1, T_2, T_3, T_4) = (1.0, 1.5, 1.0, 0.5) \text{ N} \)

Figure 10. Open loop pitch motion due to \( (T_1, T_2, T_3, T_4) = (0.5, 1.0, 1.5, 1.0) \text{ N} \)

Figure 11. Open loop yaw motion due to \( (T_1, T_2, T_3, T_4) = (1.0, 1.5, 1.0, 1.5) \text{ N} \)

Figure 9 demonstrates the roll motion when subjected to an equal thrust at front and back rotor and unequal thrust at right and left rotor. In this case, front and back rotor was given a thrust of 1 N while right and left rotors were given thrusts of 1.5 N and 0.5 N respectively. Figure 9(a, b) shows a revolving nature of the body around its x-axis with no movement in the direction of the x-axis except the angular roll motion. However, the revolving effect is increasing due to inertia of the body according to Newton’s law of rotation which can be seen in Figure 9(c). Pitch
motion is very similar to roll motion due to the body symmetry as well as the mechanism of the pitch and roll motion. Figure 10 shows effects very similar to roll, but the actuating controls have been interchanged to create pitch motion.

The yaw motion was achieved by providing front and back rotor thrusts of 1 N and 1.5 N to the right and left rotors. As the right and left rotors rotate clockwise with higher magnitude of thrust compared to the front and back rotors, the reactive torque is dominated by right and left rotors rotational direction. Thus the motion of the body is in the counterclockwise direction which creates the positive yaw as seen in Figure 11. However in this case there is no motion in any other direction but ‘z’, which means the helicopter is moving upward with yaw motion. The craft moves up as the total thrust of rotors has overcome the gravitational weight.

![Figure 12. Simulated closed loop response of quadrotor](image)

All the above responses show that the system is highly unstable and coupled. The closed loop controlled model stabilizes the craft as expected. The response of the controller is shown below. As the linear motion along x and y is dominated by pitch and roll respectively, if angular motions are stabilized, the systems linear motion along x and y are also stabilized. The only remaining required control is for altitude. Thus control of roll, pitch, yaw and height creates a complete stabilization the system. Figure 12 shows how the body is stabilized from an intermediate condition of (roll, pitch, yaw, height = 0.6 rad, 0.8 rad, 0.5 rad, 5 m) to a stabilized condition of no angular motion, no lateral and longitudinal motion, and a desired height of 2 m. The closed loop can also create a desired orientation of the body from a stable position which can be further used for controlling the position vector of the helicopter.

6. SUMMARY & CONCLUSION

This paper presents the application of bond graph modeling to quadrotor unmanned aerial vehicles (UAV). A primary contribution of this paper is to demonstrate that bond graphs facilitate modeling of the complex and coupled dynamics of the system, as well as extension or simplification of the system model or its controller. As the quadrotor is a mechatronic system, involvement of different energy domains is a must. That is why selection of the modeling language is a significant decision if future extension is expected. The bond graph model presented herein is anticipated to be of value and interest to the UAV design community. Future model extensions include higher-fidelity rotor dynamics and actuator modeling. Possible extensions to the controller might involve control of its position or co-ordinates in space that may lead the model to be even more accurate and practical.

7. REFERENCES


APPENDIX: Model and Controller Parameters

<table>
<thead>
<tr>
<th>Model Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thrust Co-efficient, $C_T$</td>
<td>1</td>
<td>N s$^2$</td>
</tr>
<tr>
<td>Drag Co-efficient, $C_D$</td>
<td>1</td>
<td>N m s$^2$</td>
</tr>
<tr>
<td>CoG to rotor distance, $l$</td>
<td>0.2</td>
<td>m</td>
</tr>
<tr>
<td>Inertial moment on x and y, $I_{xx}=I_{yy}$</td>
<td>0.0226</td>
<td>kg m$^2$</td>
</tr>
<tr>
<td>Inertial moment on z, $I_{zz}$</td>
<td>0.0227</td>
<td>kg m$^2$</td>
</tr>
<tr>
<td>Quadrotor mass, $m$</td>
<td>0.5</td>
<td>kg</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PID Controller Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain constant for roll and pitch, $K_p$, $K_i$, $K_d$</td>
<td>0.90, 0.30, 0.20</td>
<td>-</td>
</tr>
<tr>
<td>Gain constant for yaw and height, $K_p$, $K_i$, $K_d$</td>
<td>0.06, 0.30, 0.02</td>
<td>-</td>
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</table>


