Active Modeling: A Method for Creating and Simulating Variable-Complexity Models

D. Geoff Rideout
Assistant Professor
e-mail: g.rideout@mun.ca

Kazi T. Haq
Graduate Student
e-mail: b28kth@mun.ca

Faculty of Engineering and Applied Science,
Memorial University,
St. John’s, NL, A1B 3X5, Canada

1 Introduction

Simulation-based design of dynamic systems requires mathematical models that are complex enough to predict system response accurately. However, models that are needlessly complex may inhibit understanding of interactions among system elements, decrease computation efficiency, and create difficult parameter estimation problems. There is typically a trade-off between model accuracy and computation time [1]. More complex models (with correct parameters) yield more accurate results, but take longer to create and analyze [2].

For certain systems such as vehicles, the inputs and parameters may change during the course of a long simulation event or “maneuver.” A typical truck mobility study might have a mixture of on- and off-road road profiles and varying road inclination (input changes). The analyst may wish to simulate the effect of sudden tire failure or the effect of decreasing mass due to payload offloading or fuel consumption (parameter changes). Selecting one model of constant complexity will require a compromise. The most conservative choice would be a model of maximal complexity that can capture all dynamics encountered during the maneuver but would run unnecessarily slowly during long steady-state periods under mild inputs.

Previous “proper modeling” techniques have shown the feasibility of predicting required model complexity by measuring and comparing energy-related outputs of lumped parameter dynamic elements. Prior research has also investigated methods for switching elements off or on. In this paper, a modified switching element removes the dynamic output of a model element to the rest of the system when the moving average of its absolute power falls below a user-defined threshold. When the element is “off,” the input from the system to the element is still passed through the transformer so that an estimate of element power and importance can continue to be calculated and the element switched back “on” if necessary. The bond graph formalism is used to facilitate implementation. Switch configurations are defined for both causally weak and causally strong energy storage and dissipative elements. The method is applicable to linear or nonlinear systems that can be modeled with lumped parameter elements. The approach is demonstrated for quarter- and half-car vehicle models subject to a road profile of varying frequency. The appropriate model complexity at all stages is determined and implemented continuously without prior knowledge of input or parameter changes.

[DOI: 10.1115/1.4002472]

2 Background and Literature Review

Considerable prior research has been directed toward using energy and power-based metrics to guide model reduction. Bond graphs have been the preferred modeling formalism and are used in this paper. In bond graphs, energy conservation is assured, power flow paths can be identified, and power flow associated with elements and their connections can be readily determined. In the bond graph graphical user interface, bond graph graphical user interfaces such as 208M [3] can rederive system equations automatically when an element is added or eliminated. The reader is referred to Karnopp et al. [4] for a thorough exposition of bond graphs.

2.1 Previous Proper Modeling and Switching Research.

Rosenberg and Zhou [5] correlated rms power flow through a bond with the contribution of the associated element to the overall system dynamics. Louca et al. [6] calculated and ranked “activity,” defined as the integral of absolute value of power flow through an I, C, or R element. The most active elements, whose cumulative activity accounted for a certain aggregate percentage of system power flow, were retained and the remaining elements were eliminated. The preceding methods generate reduced models that predict outputs of dynamically dominant elements. Rideout et al. [7] compared activity of bonds at each 1- and 0-junction to partition bond graphs into “driving” and “driven” submodels that could be then subject to model reduction methods or simulated in parallel.

Kypuros and Longoria [8] demonstrated that changes in input frequency can change the required elements of a model. Activity analysis was used to derive reduced models for a linear half-car subject to a road input for which frequency was varied at predetermined times. Separate activity analyses over the known input time intervals gave four different reduced models, which were
then simulated in sequence. The models were simulated in MATLAB [9], and the required time steps for the full and sequentially run reduced models were compared for three variable-step integrators. Sequential execution of proper models gave significant reduction in computation steps compared with running the full model through all four stages. The work in Ref. [8] focused on showing the potential improvement in numerical effort from variable-complexity modeling, and on identifying when model complexity changes should occur. The problem of automating the complexity changes, rather than changing the model manually at predetermined times, was not addressed.

This paper addresses the variable-complexity implementation issue by adding and removing the effects of individual bond graph elements based on changes in their energetic contributions. To that end, this modeling theory must be joined with an appropriate method of switching. Switched systems, or systems with discontinuities, can be more difficult to model, simulate, and design [10].

A variable structure system (VSS) formulation was proposed by Kypuros and Longoria to break up a switched system into a set of continuous substructures. Individual bond graphs were derived, and then synthesized into a single VSS model with switching functions. Elements with switching behavior must be determined a priori, and the approach becomes laborious when there are many substructures.

The preferred switching method will facilitate inclusion into commercial software such as Simulink, by not requiring model causality or parameter changes when switching occurs. The switches will be placed “in-line” with external or internal junction structure bonds in order to turn off individual components or groups of components, respectively. A critical requirement of the switch is that even if an element is off, it must still be possible to predict the power that would be flowing through the element if it were still on. For a causally weak C element at a 1-junction, for example, the flow from the rest of the system must be delivered to the element. The constitutive law would then be able to calculate effort, from which power could be estimated. The relative contribution of the element to the system dynamics could continue to be calculated so that if necessary it could be reinstated. While it is not difficult to eliminate elements when their energetic contribution becomes negligible, it is not straightforward to monitor how important an element would be if it had not been taken out of the model [11].

The switching method must also tolerate a large number of switches (possibly one per element) without requiring an excessive number of separate bond graphs or sets of state equations. The goal is to assess a system with no prior knowledge or bias, so that the resulting tool will be of use to a modeler without considerable experience or domain expertise. The increasingly interdisciplinary models encouraged in industry and academia make it likely that a single person will not have mature intuition about all aspects of a model, and thus would be advisedly reluctant to eliminate elements in a nonsystematic way.

Bond graph switching research has primarily focused on modeling systems with inherently discontinuous dynamics such as power electronics or mechanical systems with collisions. Asher [12] modeled power electronics with composite switching elements consisting of zero flow or effort generators combined with causality generating \( R \) elements. Switching requires changing causality and equations of the generator. Ducruix et al. [13] modeled power electronic switching with an \( R \) element representing the switch resistance when the component is on, along with an \( MTF \) with logic statements that can impose zero flow onto connected elements when the component is switched off. The combination of \( MTF \) and \( R \) element was also used by Garcia et al. [14]. Stromberg et al. [15] used ideal zero flow or effort sources and switch causality as a system changes modes. They argued that causality reassignment can be done automatically; however, this would require symbolic derivation of new system equations with each switching event. Mosterman and Biswas [16] developed a hybrid discrete-continuous modeling method with controlled junctions in which the control logic is implemented as a state transition graph or table. The switching is not executed strictly by bond graph elements. Demir and Poyraz [17] created software to model systems switched with ideal effort or flow sources. The number of bond graphs required is 2 \( n \), making the method cumbersome when more than a few switches are present. Umakar and Umanand [18] and Junco et al. [19] modeled ideal switching and maintained causality invariance through switched power junctions (SPJs). SPJs choose among multiple causal effort (0-junction) or flow (1-junction) inputs at mutually exclusive times. SPJs maintain invariance causality and a constant state vector, show all possible system modes on one graph, and can represent switching at the bond graph level in commercial software. However, SPJs would not allow monitoring of power flow to elements in nonselected bond graph branches. Dauphin-Tanguy and Rombaut [20] argued for constant causality and noted numerical issues that can arise from using stiffness-inducing \( R \) elements to generate nonideal switch behavior. Numerical stiffness is important in the current context as it can offset computational gains from model reduction. Karnopp [21] proposed an approach that could reduce numerical stiffness by limiting the rate of flow or effort equalizations during switch transients, but is not intended to deliver power variables from the system to the “switched-off” element.

Monitoring “off-element” activity in this paper is done with nonpower conserving transformations. The non-power conserving transformations are based on standard bond graph elements, unlike those of Paynter and Longoria [22], which use a wave-scattering approach and nonstandard bond graphs.

Section 3 describes an energetic element contribution metric and a switching method suitable for variable-complexity modeling.

3 Energetic Metric and Switching Method

3.1 Energetic Metrics. An input or parameter change that could necessitate a model complexity change will modify the relative power flow through elements. “Activity” is defined in Ref. [6] as a power-based metric for assessing the contribution of an individual element to the overall system dynamics over a particular time interval. Activity is the integral of absolute value of power flow:

\[ A = \int_{t_i}^{t_f} |P| \, dt \]  

Activity index was defined as each element’s fraction of total system activity:

\[ A_i = \frac{A_i}{\sum_j A_j} \]  

In the model order reduction algorithm (MORA) proposed in Ref. [6], elements in a model are ranked from highest to lowest activity. The cumulative activity index of element \( i \) allows the modeler to determine the percentage of total system aggregate power flow that is accounted for by that element and all others with greater activity. A threshold corresponding to a high cumulative activity index gives a reduced model, which predicts dominant system dynamics accurately.

While activity is a useful metric in that it is positively valued and monotonically increasing, it can be slow to respond to sudden changes in dynamic response due to input changes. If, for example, activity is calculated over a 120 s time interval, then a transient at 110 s may not contribute significantly to the integrated absolute value of power from 0 s to 120 s. A required model change at 110 s would therefore not be predicted.

To increase responsiveness of the metric, the moving average of absolute power (MAP) was chosen as in Ref. [8]. Switching times
based on MAP do not occur exactly at the instants when input frequency content changes [8]. Prior knowledge of input changes is thus not sufficient for defining when model complexity changes should occur. MAP of element $i$ is defined by

$$\text{MAP}_i = \frac{1}{T_o} \int_{t-T_o}^{t} |P_i| dt$$

(3)

where $T_o$ is the averaging time interval. Modifying $T_o$ changes the responsiveness of the metric.

To calculate the relative contribution of element $i$ to overall system dynamics, a moving average power index (MAPI) similar to the activity index is defined by

$$\text{MAPI}_i = \frac{\text{MAP}_i}{\sum_j \text{MAP}_j}$$

(4)

where the denominator of Eq. (4) is the sum of the MAP for all system elements.

Model complexity switching will therefore occur when the MAPI of an element goes above or below a user-defined threshold value. When an active element goes below the threshold, its effect on the system will be eliminated, and when an inactive element rises above the threshold, it will begin sending an output quantity into the system equations.

### 3.2 Bond Graph-Based Switching

#### 3.2.1 Switching Approach and Illustrative Example

An element whose MAPI falls below a threshold, thereby causing it to be switched off, may become important later on in a simulation maneuver. A significant challenge is to estimate how important the eliminated element would be if it was reinstated, and to restore its contribution to system dynamics when its metric value would have crossed back over the threshold. To monitor this, the constitutive law of an “eliminated” element must be retained and its power calculated, while the remainder of the system simulation proceeds. For example, consider a generalized compliance (bond graph $C$ element) such as a spring, connected to a bond graph 1-junction representing the relative velocity of its end points. Figure 1 shows such an element. In the constitutive law of the $C$ element, the flow $f_1$ from the rest of the system is the input, and the effort $e_1$ is the output from the $C$ element.

$$e_1 = \Phi_C \left( \int f_1 dt \right)$$

(5)

where $\Phi_C$ is a nonlinear capacitance function.

The power bond in Fig. 1 carries both effort and flow information between the system and element. Eliminating the $C$ element requires setting the effort $e_1$ to the system equal to zero. In order to monitor the hypothetical contribution of the $C$ element after its elimination, its power must be calculated, requiring that the system flow $f_1$ continue to be passed to the element so that the constitutive law can generate a value of $e_1$. Therefore, the bond must not be entirely broken but must be converted into a one-way interaction conduit. A nonpower conserving transformer is introduced, as shown in Fig. 2 along with its constitutive law. System flow $f_1$ flows through the transformer at all times, allowing continuous computation of $C$ element power $P_C$ from which moving average of absolute value of power can be calculated.

$$P_C = f_{ib} \cdot \Phi_C \left( \int f_{id} dt \right)$$

(6)

To implement automated switching, a sensor is placed inline with the bond to calculate MAP, as shown in Fig. 3. The MAP signal is sent to a summation block, which sums all system MAP values to determine the denominator of Eq. (4). The total system MAP is then sent back to the sensor to calculate the respective element’s MAPI (MAP index). A MAPI value below the threshold sets an internal $U$ variable value to 0, whereas above the threshold sets $U$ equal to 1. The $U$ value is then outputted from the MAPI sensor to modulate the transformer. See Fig. 4.

To reduce the severity of the discontinuity (which can impact the stability of implicit numerical integrators) when switching the transformer modulus between 0 and 1, the modulus is varied linearly between 0 and 1 (or vice versa) over a finite time span.

#### 3.2.2 Effect of Causality on Switching

The illustrative example of Figs. 1–4 shows an element with a weak causal connection to the remainder of the system. An element has weak causal connection to a junction if it provides causal effort to a 1-junction or flow to a 0-junction. Switching of weakly causal elements is easier due to the fact that setting the output of the switched element to zero does not affect the common junction variable. Had the $C$ element above been causally strong, prescribing the flow to the 1-junction, then setting its output to zero would create zero

---

**Fig. 1** Bond graph element interaction with system

**Fig. 2** Nonpower conserving transformer to partially disconnect element

**Fig. 3** MAPI sensor to calculate MAPI for $C$ element

**Fig. 4** Switching via MAPI sensor output to MTF
flow to all other connected elements. The result would be not only the removal of the \( C \) element but also of the kinematic node corresponding to its velocity.

Whether or not this outcome is correct depends on the physical interpretation of “removing the element from the system.” As described in Ref. [23], an “inactive” \( C \) element at a 1-junction is assumed to be inactive due to low effort, in which case its stiffness is set to zero and its effort on the system is removed. The 1-junction flow is assumed to be significant, and any inactive elements bonded to it are assumed to be inactive because they contribute negligible effort. If a \( C \) element such as a spring is in integral causality and connected to a 0-junction, then inactivity is assumed to be due to the negligible flow and the removal of the element implies a rigid connection.

A common occurrence in mechanical systems is for generalized inertias (\( I \) elements) to be bonded to 1-junctions representing their velocity. Suppose a small mass is inactive because it has low inertia. In switching off the element, the goal is to remove the inertial force rather than to set the flow equal to zero. Causally strong small masses are treated as in Fig. 5. A parasitic \( I \) element [24] \( I_{\text{par}} \) (or a parasitic \( R \) element) is added to the 1-junction as a causality generator, and the original \( I \) element is bonded to a 0-junction along with a parasitic damper \( R_{\text{par}} \). The 0-junction and damper allow the integral causality of the original \( I \) element to be retained, while creating an effort output to the rest of the system which can be set to zero if the small mass is off.

An alternative configuration would place the original \( I \) element in derivative causality. While the proposed method adds a parasitic state variable and damper (thereby increasing system order and/or numerical stiffness), it avoids derivative causality and thus allows an explicit integrator to be used.

Table 1 summarizes switch elements for causally weak elements. Causally weak elements require only a nonpower conserving \( MTF \). Switches for causally strong elements (with causality generator elements) are shown in Table 2. The values of the parasitic element parameters are indicated as “low” or “high.”

### 3.2.3 Implications of Low-Valued Common Junction Variable

Consider an \( I \) element with low \( MAPI \), bonded to a 1-junction. There are two scenarios in which an \( I \) element can be inactive, as shown in Fig. 6. In the typical scenario, a small mass in a mechanical system contributes little effort, but the mass may have significant velocity. Analogies exist for electrical, hydraulic systems, etc.

If, however, the 1-junction flow is low, then all bonds connected to the junction may be inactive and elimination of junction structure as well as energetic elements may be required. A large mass, suspended from a moving base by a spring and damper, will have very low velocity for sufficiently high-frequency base motion and could be inactive. Simply setting the mass to zero will obviously create model prediction errors.

As mentioned in Section 3.2.2, Louca and Stein [23] assumed that element inactivity arises from the former scenario, in the left of Fig. 6. To ensure that inactivity of the mass does not arise from low velocity, a second test must be added in addition to the mod-
elwide ranking of MAP. Suppose a causally strong inertia is connected to a 1-junction with flow $f$. To test if $f$ is non-negligible in the model, the relative moving average power of bonds with flow $f$ must be calculated at each junction containing a bond with flow $f$. MAP of the element bond is compared with the highest MAP at the junction. If

$$\frac{\text{MAP}}{\text{MAP}_{\text{max,junction}}} < \varepsilon$$  \hspace{1cm} (7)

for each junction at which $f$ is an input, then $f$ can be removed. In the arrangement in Fig. 7, low I-element activity will result in zero effort out as in the tables above. However, if the MAP of the flow $f$ is also relatively low at all affected junctions, a zero flow can prescribed to the 1-junction by a second switch. The activation of the switch “MTF internal” will supersede the activation of the switch “MTF external” in dictating the output of the I-element branch to the rest of the system. In the figure, $U$ is 0 if the element’s MAP goes below the threshold. This creates a zero effort out of bond 1 to the rest of the system, representing the case where the mass is small and has low effort. Meanwhile, if the relative MAP of bond 5 is low compared with bonds 6 and 7, then the flow $f$ is a negligible input to the left branch of the bond graph and $V_1$ is zero. If $V_1$ and $V_2$ are both zero, then the flow $f$ is a negligible system quantity and the flow $f_4$ is set to zero. Flow, rather than effort, is then treated as the low power variable that creates mass inactivity.

### 4 Vehicle Dynamics Case Studies

#### 4.1 Quarter Car

Figure 8 shows a typical quarter-car vehicle model. The road input $u(t)$ excites the unsprung and sprung masses $m_u$ and $m_s$ through the tire stiffness $k_t$ and damping $b_t$. The suspension stiffness and damping are represented by the $k_s$ and $b_s$ elements, respectively. Parameters are given in Table 3.

The sinusoidal road input velocity $\frac{du(t)}{dt}$ is as follows: amplitude is 1.5 m/s at a frequency of 100 rad/s for $t = 0$–2 s; 10 rad/s from 2 s to 4 s, and 100 rad/s thereafter. The high-frequency portions of the input are designed to lower the activity of the sprung mass in this demonstration. During the low-frequency portion of the input, the sprung mass should be excited.

Two bond graphs are generated. The first has a sign convention (half-arrow directions), which renders the sprung mass causally weak, suggesting a switch configuration from Table 2. The second bond graph has a causally strong attachment of the sprung mass to the $z$-1-junction, requiring a switch element from Table 3 and an internal bond switch as in Fig. 7.

#### 4.1.1 Results With Causally Weak Sprung Mass

In the bond graph of Fig. 9, half-arrow directions are chosen so that $m_s$ is bonded to a 0-junction representing the combined suspension spring and damper force. Switching MTFs and MAP sensors are shown. Modulating signals to and from a summation element that calculates total system MAP are omitted for clarity. A parasitic $R$ element is chosen as a causality generator for the $z_s$ 1-junction at which the unsprung mass has a causally strong attachment. The system was simulated for 15 s using a Runge–Kutta–Fehlberg

---

**Table 3** Quarter car parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprung mass $m_s$</td>
<td>267 kg</td>
</tr>
<tr>
<td>Suspension stiffness $k_s$</td>
<td>18,742 N/m</td>
</tr>
<tr>
<td>Suspension damping $b_s$</td>
<td>700 N s/m</td>
</tr>
<tr>
<td>Unsprung mass $m_u$</td>
<td>36.6 kg</td>
</tr>
<tr>
<td>Tire stiffness $k_t$</td>
<td>193,915 N/m</td>
</tr>
<tr>
<td>Tire damping $b_t$</td>
<td>200 N s/m</td>
</tr>
</tbody>
</table>
integrator with absolute and relative tolerances of $1 \times 10^{-6}$. MAPI threshold was 0.03. Figure 10 shows the transitioning $U$ values indicating when elements were switched off or on. Some numerical simulation-related noise in the $U$ signal is present; however, the simulation remained stable despite the switching. Figure 11 plots the state (momentum) of the sprung mass. Output from the switched model is plotted on top of output from a constant-complexity full model to show that the MAPI metric and switching method preserve the model’s ability to predict the system response.

The sprung mass becomes inactive during the first high-frequency stage, along with the suspension spring, as shown in Fig. 10. The sprung mass has low velocity, and the spring has low force. The suspension damper effectively filters out road motion. The unsprung mass oscillations atop the tire spring and damper remain significant. During the second, low-frequency stage, sprung mass oscillation is insignificant and the mass and suspension elements must be switched on. The tire damping becomes insignificant.

Figure 11 shows good agreement between the switched model and the full model except for the period immediately after the reinstatement of the sprung mass. Releasing the mass from rest, as opposed to the mass transitioning from a period of very low velocity as in the full model, will cause slight changes in the initial condition at the beginning of the second input stage. Given the magnitude of the mass parameter, these slight changes appear to have caused overshoot of the mass during the first cycle of low-frequency motion of the switched model.

4.1.2 Results With Causally Strong Sprung Mass. Figure 12 is a bond graph of the quarter-car in which the 1-junction $z_s$ is not a flow-through junction and cannot be eliminated. The sprung mass now generates the flow input to the 1-junction. The causally strong switch element from Table 3 is added, along with sensors and junction structure to monitor the contribution of the flow $z_s$ at the $Fs$ 0-junction.

Figure 13 shows the same elements switching on and off at approximately the same times as for the model of Sec. 4.1. The presence of the extra switch creates more numerical noise overall. The sprung mass effort remains significant throughout the simulation as expected, with some numerical noise appearing in the switch modulus early in the second input stage, as shown in the lowest plot of Fig. 13. Figure 14 shows good agreement between the full and switched models.

4.1.3 Eigenvalue Analysis of Strong-Causality Switching Method. The causally strong mass and internal junction switch of Fig. 7 gives four possible scenarios. The mass may be active or inactive, and one or both of the mass effort and flow may be approximately zero. To assess the soundness of the strong causali-

![Fig. 10](image1.png)  
Fig. 10  $U$ values indicating switch time and status of elements

![Fig. 11](image2.png)  
Fig. 11  Comparison of switched and nonswitched model outputs

![Fig. 12](image3.png)  
Fig. 12  Quarter car bond graph with causally strong sprung mass
itivity switching method, consider the effect on eigenvalues of having one or both switches off. Figure 15 shows the quarter-car with only the causally strong sprung mass switch arrangement present. Modulus $A$ is zero if the effort is negligible, and modulus $B$ is zero if the flow is negligible. The combined effect of the switches should preserve eigenvalues of the remaining model elements regardless of whether the causally strong mass becomes inactive due to low inertial force or low velocity.

An eigenvalue analysis using the parameters of Table 3 is conducted using the system equations:

$$
\begin{align*}
\begin{bmatrix}
\dot{p}_1 \\
\dot{p}_7 \\
\dot{p}_{12} \\
\dot{q}_{10} \\
\dot{q}_{15}
\end{bmatrix} &= 
\begin{bmatrix}
\frac{-R_{hi}}{M_t} & \frac{R_{hi}}{I_{par}} & 0 & 0 & 0 \\
\frac{AR_{hi}}{M_t} - \frac{AR_{hi}}{I_{par}} - \frac{Bb_S}{I_{par}} & b_S \frac{M_s}{M_{us}} - k_s & -k_s & 0 & 0 \\
0 & b_S \frac{M_s}{M_{us}} - b_t & k_s - k_t & 0 & 0 \\
0 & 0 & \frac{B}{I_{par}} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{M_{as}} & 0 \\
0 & 0 & 0 & 0 & \frac{1}{M_{as}}
\end{bmatrix}
\begin{bmatrix}
p_1 \\
p_7 \\
p_{12} \\
q_{10} \\
q_{15}
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
$$

(8)

where the parasitic parameter values are $I_{par}=0.001 \text{ kg}$ and $R_{hi} = 1 \times 10^7 \text{ Ns/m}$, and $p_1$ is the sprung mass momentum, $p_7$ is the parasitic mass momentum, $p_{12}$ is the unsprung mass momentum, $q_{10}$ is the sprung suspension displacement, $q_{15}$ is the tire spring displacement, $A$ is the modulus of sprung mass switch (0=off), and $B$ is the modulus of internal bond switch (0=off).

Table 4 summarizes eigenvalues. When all switches are on, there are low- and high-frequency damped oscillations corresponding to ride and wheel hop frequency, along with a stiff overdamped parasitic eigenvalue.

In scenario 2, the sprung mass would be effectively set to zero, leaving the unsprung mass to oscillate atop only the tire spring and damper. The removal of the effect of suspension damping on $m_{us}$ manifests itself in the smaller real part of the eigenvalues. Fast overdamped parasitic modes remain. Scenario 3 sees the sprung mass set to zero and the velocity node fixed to ground. The separation of $I_{par}$, $R_{hi}$, and $M_t$ creates rigid body and parasitic modes. The unsprung mass oscillates between tire and suspension springs and dampers, giving eigenvalues very close to those of the original wheel hop mode. Scenario 4 is similar in that the sprung mass is a ground node. The switching modes thus correspond to the expected system configurations in Fig. 16.

### 4.1.4 Energy Analysis of “Off” Elements

The energy of elements that have been switched off can be estimated using the power variable that is passed through the switch $MTF$ and the conjugate power variable computed with the element constitutive law. In the quarter-car case study, both the sprung mass and suspension spring are switched off during the first input stage. Figure 17 compares estimated energy from the switched model and energy from an unswitched model. Energy levels are comparable, indicating that the off element contribution to system power flow is being predicted effectively and the element reinstated at an appropriate juncture.

#### 4.2 Half-Car

A half-car model similar to Ref. [8] is shown in Fig. 18. Parameters are given in Appendix A. Road input was $1.5 \cos(\omega t)$ where frequency $\omega$ was 10 rad/s for the first 3.14 s (stage 1), 20 rad/s for the next 3.14 s (stage 2), 100 rad/s for the next 1.26 s (stage 3), and 10 rad/s for the final 3.28 s (stage 4).

Figure 19 is a bond graph showing the system elements and switches. Solid circles represent elements that calculate $MTF$ moduli (U values) for element switching. Tire damping was neglected, as it had no significant effect on any of the four system responses. This resulted in tire springs becoming causally strong. Sprung masses were also causally strong. It was assumed that the large body mass and rotational inertia would only become inactive due to low flow. The flow of the respective 1-junctions $u_{ms}$ and $\omega$ was therefore evaluated relative to other flows at the $F_{rs}$ and $F_{fs}$ 0-junctions to predict if these elements should be switched.

### Table 4 Eigenvalue analysis for switched sprung mass scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Sprung mass</th>
<th>Sprung mass</th>
<th>Sprung mass</th>
<th>Sprung mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>“on” internal bond “on”</td>
<td>“off” internal bond “on”</td>
<td>“off” internal bond “off”</td>
<td>“on” internal bond “off”</td>
</tr>
<tr>
<td>1. Sprung mass</td>
<td>$-1 e^9$</td>
<td>$-3475$</td>
<td>$-3475$</td>
<td>$-1 e^9$</td>
</tr>
<tr>
<td>2. Sprung mass</td>
<td>$-1 + 7.95i$</td>
<td>$-6999$</td>
<td>$-12 + 75.2i$</td>
<td>$-12 + 75.2i$</td>
</tr>
<tr>
<td>3. Sprung mass</td>
<td>$-1 - 7.95i$</td>
<td>$-26.8$</td>
<td>$-12 - 75.2i$</td>
<td>$-12 - 75.2i$</td>
</tr>
<tr>
<td>4. Sprung mass</td>
<td>$-12.5 + 74.9i$</td>
<td>$-2.73 + 72.7i$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>5. Sprung mass</td>
<td>$-12.5 - 74.9i$</td>
<td>$-2.73 - 72.7i$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>
off (light circles in Fig. 19). For example, the rotational inertia $I_c$ will switch off if relative MAP from $F_{rs}p3$ at the $F_{rs}$ 0-junction, and $F_{fs}p3$ at the $F_{fs}$ 0-junction, are both below the threshold. The resulting MTF moduli are termed $V$ values in the results that follow.

To reduce computational overhead, the external signals to and from MAP summation blocks (as in Figs. 3 and 4) are replaced by global variable declarations in the $maxMAP$, $maxMAPr$, $maxMAPf$, and $Ms_{Ic\_Switch}$ blocks of Fig. 19.

The method is not restricted to linear systems, and this is demonstrated by giving the rear suspension damper a bilinear force-velocity relationship (higher damping coefficient in rebound than in jounce) and by giving the front damper a constitutive law reflective of a hydraulic piston and valve. The bilinear rear damper requires switching in the element’s equation model, and the front damper has an absolute value operator. The front damper force is given by

$$F_d = \frac{A_p \rho v^2}{2C_dA^2}$$

where $A_p$ is the piston area, $\rho$ is the hydraulic fluid density, $v$ is the damper velocity ($v_{fs}$ junc-junction in Fig. 19), $C_d$ is a discharge coefficient, and $A$ is the valve orifice area. See Appendix B for a schematic of the damper.

Figure 20 plots $U$ and $V$ values, with 1 indicating an element on and 0 indicating off. The threshold for both element and internal bond $MAP$ indices was 0.05, and moving averages were calculated over a 0.5 s time span. In the first two input stages, the proper model complexity remains constant with the exception of the unsprung masses. The model structure changes significantly in the third stage, where suspension elements switch off and tire dynamics dominate the system power flow. During this stage, the body rotational inertia switches off briefly as it has negligible velocity. Figure 21 compares the full and variable-complexity model predictions of front and rear unsprung mass displacements $y_{usf}$ and $y_{usr}$, sprung mass vertical displacement $y_{ms}$, and pitch angle $\theta$. The variable-complexity model predictions are typically in agreement with the full model. In the middle of stage 3, the $\theta$ prediction shows a spurious transient, and in stage 4 there is some overprediction and offset. The sprung mass displacement
overshoots exiting stage 3, but the prediction accuracy recovers in stage 4. Future work will examine the possibility of keeping element outputs at a constant static offset value in some cases when switching them off, instead of setting those outputs to zero. The accuracy of this automated variable-complexity model is comparable to the model formed by “gluing” separate reduced models together in Ref. [8]. In that work, stage 3 also presented the greatest errors, suggesting that a lower threshold might be appropriate. Tightening the threshold moves the accuracy versus complexity trade-off in the direction of greater accuracy at the expense of fewer model reduction opportunities. Computation speed issues are discussed in Sec. 5.

5 Discussion

The proposed active modeling method is applicable to lumped parameter system models comprised of elements for which power variables can be calculated. As shown in the case study, nonlinearity of elements (including switched constitutive laws) does not affect the implementation, since the MTF-based switch that turns the element “on/off” is external to the element and unaffected by the nonlinearity. The MTF switch can still pass flow or effort to the element as a causal input when it is off, for calculation and monitoring of MAP.

A fundamental limitation of the proposed active modeling method is that “eliminated” elements remain in the bond graph. The input to such elements is still manipulated by the element’s constitutive law to predict the power that would flow through that
element if it were still connected. Switch logic introduces additional computational overhead. Table 5 summarizes model size, computation time, and steps for the following models, using 20SIM’s Vode-Adams stiff system integrator with tolerances $1 \times 10^{-6}$ and maximum step size 0.01 s.

- constant-complexity half-car model with no MAP calculation or switches
- constant-complexity half-car with MAP calculation (switches disabled)
- variable-complexity switched half-car

The discontinuities inherent in switching, and the inclusion of parasitic elements when switching causally strong bonds, create the possibility of numerical instability. Numerical issues in stiff and discontinuous systems are widely recognized, for example, in Refs. [12,18,21]. As mentioned at the end of Sec. 3.2.1, transformer moduli are varied between 0 and 1 linearly over a finite time span rather than instantaneously. Instantaneous variation creates slow or unstable simulations with the implicit Vode-Adams, backward differentiation formula (BDF), and modified BDF methods. The significance of numerical issues as a function of variable-complexity model size, number of switches, and modulus blending functions remains an open research question.

The method herein proposed, despite not being able to reduce simulation time in its current form, does provide a method for predicting required model complexity on a continual basis. The analyst is provided with a prediction of not only the proper model at a given time but also how the system would actually perform if an adaptive proper model was used. MAP-based active modeling can automatically suggest a sequence of reduced models, which the user can then construct and simulate. In Ref. [8], running four reduced half-car models sequentially was more efficient than running the full model. Even small reductions in simulation time for an individual maneuver can accumulate if a model must be called multiple times in an optimization exercise. The active modeling method in this paper automates the generation of such a model sequence and its output. In Ref. [8], the time window over which each reduced model was valid did not correspond exactly to intervals of constant input frequency. The current work determines these time windows along with the appropriate extent of model reduction in those windows. Generating such a sequence of reduced models and then running them without switches appears to give the best opportunity for computation time reduction.

The computational expense of calculating and comparing MAP at each time step motivates future work into minimizing the frequency with which this is done. For a model operating in steady state under a periodic input, calculations of MAP to confirm adequate model complexity need only be done sporadically. An algorithm to trigger MAP calculation (and tune the frequency of calculation) only upon a change in a parameter value or some characteristic of the input would increase the efficiency of the method. For time-invariant models, the input sources would require constant monitoring, but the model elements would not. Another approach to improving efficiency would utilize an initial trial simulation to identify a target group of elements that switch most often and dictate the variations in complexity. Monitoring and switching this subset of model elements would reduce computational overhead. Such a trial run may suggest that cycling between high- and low-complexity versions of the original model (two models only as opposed to a spectrum of possible complexities) at appropriate intervals would give sufficient accuracy. Using two set models, and knowing when to move from one to the other, would allow simulation to proceed without switches in the models themselves.

### Table 5 Summary of half-car computation times

<table>
<thead>
<tr>
<th>Model description</th>
<th>Model size (No. of states, equations, and variables)</th>
<th>Computation time (s) and calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant-complexity, no MAP</td>
<td>15/53/33</td>
<td>1.371/24574</td>
</tr>
<tr>
<td>Constant-complexity, MAP calculation</td>
<td>37/316/160</td>
<td>2.840/51179</td>
</tr>
<tr>
<td>Variable complexity</td>
<td>37/316/160</td>
<td>3.927/79903</td>
</tr>
</tbody>
</table>
6 Summary and Conclusions

A method of “active modeling” has been developed to continuously monitor the contribution of an element to overall system dynamics using a moving average of absolute value of power, and to eliminate the effect of the element from a model when the metric falls below a threshold. Bond graph-based switches can remove elements regardless of causality. The use of nonpower conserving modulated transformers allows the metric to be calculated for elements that have been removed from the model, to estimate their importance if they were switched back on. Such elements are switched on if the metric rises above the threshold. Model complexity is therefore automatically modified as required. The method was demonstrated for quarter- and half-car vehicle models subject to road profiles of varying frequency. Sprung mass and suspension stiffness were removed from the model during high-frequency input stages where the system acted as a low-pass filter between road input and sprung mass response. These elements became active during lower-frequency input stages with greater sprung mass oscillation. Both causally weak and strong elements were switched effectively.

The variable-complexity model simulation was executed in unmodified commercial software. While the goal of active modeling is ultimately to reduce overall simulation time, the computational overhead of the metric calculation and monitoring offset any gains due to using a reduced model. Future work will increase the efficiency of active modeling by minimizing the number of switches and model complexity checks required. In its present form, active modeling automates the determination of time windows over which a fixed-complexity model is valid, and the required elements in the model for each time window. Output of a variable-complexity model can be generated, or the analyst can build and simulate a sequence of fixed proper models, already validated against the full model, to realize simulation accuracy and computational gains.

Acknowledgment

The authors would like to acknowledge the Natural Sciences and Engineering Research Council of Canada (NSERC) for funding this work under its Discovery Grant program.

Appendix A

The parameter values for the half-car model are as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translational velocity</td>
<td>V</td>
<td>72 km/h</td>
</tr>
<tr>
<td>Wheelbase</td>
<td>L</td>
<td>2.7 m</td>
</tr>
<tr>
<td>Distance between c.g. and rear axle</td>
<td>L_r</td>
<td>1.6 m</td>
</tr>
<tr>
<td>Distance between c.g. and front axle</td>
<td>L_f</td>
<td>1.1 m</td>
</tr>
<tr>
<td>Sprung mass</td>
<td>m_s</td>
<td>1700 kg</td>
</tr>
<tr>
<td>Rotational inertia</td>
<td>l_c</td>
<td>1700 kg m²</td>
</tr>
<tr>
<td>Front unsprung inertia</td>
<td>m_fus</td>
<td>100 kg</td>
</tr>
<tr>
<td>Rear unsprung inertia</td>
<td>m_rus</td>
<td>80 kg</td>
</tr>
<tr>
<td>Front suspension stiffness</td>
<td>k_fs</td>
<td>30 kN/m</td>
</tr>
<tr>
<td>Rear suspension stiffness</td>
<td>k_rs</td>
<td>20 kN/m</td>
</tr>
<tr>
<td>Front tire stiffness</td>
<td>k_ft</td>
<td>200 kN/m</td>
</tr>
<tr>
<td>Rear tire stiffness</td>
<td>k Rt</td>
<td>200 kN/m</td>
</tr>
<tr>
<td>Front suspension damper piston area</td>
<td>A_p</td>
<td>0.00196 m²</td>
</tr>
<tr>
<td>Front suspension damper valve orifice area</td>
<td>A</td>
<td>8.315 × 10⁻⁵ m²</td>
</tr>
<tr>
<td>Front suspension damper discharge coefficient</td>
<td>C_d</td>
<td>0.5</td>
</tr>
<tr>
<td>Front suspension damper fluid density</td>
<td>ρ</td>
<td>1000 kg/m³</td>
</tr>
<tr>
<td>Rear suspension damping, jounce</td>
<td>b_j</td>
<td>750 N/s</td>
</tr>
<tr>
<td>Rear suspension damping, rebound</td>
<td>b_r</td>
<td>1000 N/s</td>
</tr>
<tr>
<td>Front tire damping</td>
<td>b_f</td>
<td>0 N/s</td>
</tr>
<tr>
<td>Rear tire damping</td>
<td>b_r</td>
<td>0 N/s</td>
</tr>
</tbody>
</table>

Appendix B: Nonlinear Front Damper Schematic and Model

Figure 22 shows a simplified illustrative damper model in which piston motion (created by suspension deflection) directs fluid through an orifice.

From Ref. [4], a generic pressure-flow relation for an orifice is used along with piston area to create a nonlinear force-velocity constitutive law as follows:

\[ P_1 - P_2 = \frac{\rho Q |Q|}{2C_d A_p A^2} \]

\[ P_1 - P_2 = \frac{C_d}{A_p} Q = A_p v \]

\[ F_d = \frac{A_p \rho |v| v}{2C_d A^2} \]

References


