Effective property models for homogeneous two-phase flows

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A B S T R A C T

Using an analogy between thermal conductivity of porous media and viscosity in two-phase flow, new definitions for two-phase viscosity are proposed. These new definitions satisfy the following two conditions: namely (i) the two-phase viscosity is equal to the liquid viscosity at the mass quality = 0% and (ii) the two-phase viscosity is equal to the gas viscosity at the mass quality = 100%. These new definitions can be used to compute the two-phase frictional pressure gradient using the homogeneous modeling approach. These new models are assessed using published experimental data of two-phase frictional pressure gradient in circular pipes, minichannels and microchannels in the form of Fanning friction factor ($f_m$) versus Reynolds number ($Re_m$). The published data include different working fluids such as R-12, R-22, argon (R740), R717, R134a, R410A and propane (R290) at different diameters and different saturation temperatures. Models are assessed on the basis minimizing the root mean square error ($RMS$). It is shown that these new definitions of two-phase viscosity can be used to analyze the experimental data of two-phase frictional pressure gradient in circular pipes, minichannels and microchannels using simple friction models.

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1. Introduction

The homogeneous flow model provides the simplest technique for analyzing two-phase (or multiphase) flows. In the homogeneous model, both liquid and vapor phases move at the same velocity (slip ratio = 1). Consequently, the homogeneous model has also been called the zero slip model. The homogeneous model considers the two-phase flow as a single-phase flow having average fluid properties, which depend upon mixture quality. Thus, the frictional pressure drop is calculated by assuming a constant friction coefficient between the inlet and outlet sections of the pipe.

The void fraction based on the homogeneous model ($x_m$) can be expressed as follows:

$$x_m = \frac{1}{1 + \left(\frac{\rho_l}{\rho_g}\right) \left(\frac{x_l}{x_g}\right)}$$

(1)

When ($\rho_l/\rho_g$) is large, the void fraction based on the homogeneous model ($x_m$) increases very rapidly once the mass quality ($x$) increases even slightly above zero, as shown in Fig. 1. The prediction of the void fraction using the homogeneous model is reasonably accurate only for bubble and mist flows since the entrained phase travels at nearly the same velocity as the continuous phase. Also, when ($\rho_l/\rho_g$) approaches 1 (i.e. near the critical state), the void fraction based on the homogeneous model ($x_m$) approaches the mass quality ($x$) and the homogeneous model is applicable at this case.

For the homogeneous model, the density of two-phase gas-liquid flow ($\rho_m$) can be expressed as follows:

$$\rho_m = \left(\frac{x}{\rho_g} + \frac{1-x}{\rho_l}\right)^{-1}$$

(2)

Eq. (2) can be derived knowing that the density is equal to the reciprocal of the specific volume and using thermodynamics relationship for the specific volume:

$$v_m = (1-x)v_l + xv_g$$

(3)

Eq. (2) can also be obtained based on the volume averaged value as follows:

$$\rho_m = x_m\rho_g + (1-x_m)\rho_l = \left(\frac{x}{\rho_g} + \frac{1-x}{\rho_l}\right)^{-1}$$

(4)

Eq. (2) satisfies the following limiting conditions between ($\rho_m$) and mass quality ($x$):

$$x = 0, \quad \rho_m = \rho_l$$

$$x = 1, \quad \rho_m = \rho_g$$

(5)
The Reynolds number based on the homogeneous model ($R_{m}$) can be expressed as follows:

$$R_{m} = \frac{Gd}{\mu_{m}}$$  \hspace{1cm} (6)

In the homogeneous model, there are some common expressions for the viscosity of two-phase gas–liquid flow ($\mu_{m}$). The expressions available for the two-phase liquid–gas viscosity are mostly of an empirical nature as a function of mass quality ($x$). The liquid and gas are presumed to be uniformly mixed due to the homogeneous flow. The possible definitions for the viscosity of two-phase gas–liquid flow ($\mu_{ml}$) can be divided into two groups. In the first group, the form of the expression between ($\mu_{m}$) and mass quality ($x$) satisfies the following important limiting conditions:

$$x = 0, \quad \mu_{m} = \mu_{l}$$
$$x = 1, \quad \mu_{m} = \mu_{g}$$  \hspace{1cm} (7)

For example, McAdams et al. [1], introduced the definition of two-phase viscosity ($\mu_{m}$) based on the mass averaged value of reciprocals as follows:

$$\mu_{m} = \left( \frac{x}{\mu_{l}} + \frac{1-x}{\mu_{g}} \right)^{-1}$$  \hspace{1cm} (8)

They proposed their viscosity expression by analogy to the expression for the homogeneous density ($\rho_{m}$). Eq. (8) leads to the homogeneous Reynolds number ($R_{m}$) is equal to the sum of the liquid Reynolds number ($R_{l}$) and the gas Reynolds number ($R_{g}$).

Cicchetti et al. [2], introduced the definition of two-phase viscosity ($\mu_{m}$) based on the mass averaged value as follows:

$$\mu_{m} = x\mu_{l} + (1-x)\mu_{g}$$  \hspace{1cm} (9)

They used the above definition of $\mu_{m}$ in place of the definition proposed by McAdams et al. [1]. The only reason for doing this, in addition to simplicity, was a reasonable agreement with experimental data.

Dukler et al. [3], introduced the definition of two-phase viscosity ($\mu_{m}$) based on the mass averaged value of kinematic viscosity as follows:

$$\mu_{m} = \rho_{m} \left[ \frac{\nu_{l}}{\nu_{l}} + (1-x)\frac{\nu_{g}}{\nu_{l}} \right]$$  \hspace{1cm} (10)

or

$$v_{m} = x\nu_{l} + (1-x)\nu_{l}$$  \hspace{1cm} (11)

Beattie and Whalley [4] presented a simple two-phase pressure drop calculation method. They adapted a theoretically based flow pattern dependent calculation method to yield a simple predictive method in which flow pattern influences were in an implicit method and hence need not to be explicitly taken into account when using the method. For both bubble flow and annular flow, they proposed that the average two-phase viscosity ($\mu_{m}$) was replaced by a hybrid definition:

$$\mu_{m} = \mu_{i}(1-z_{1})(1 + 2.5\gamma_{m}) + \mu_{g}z_{1}$$

$$= \mu_{i} - 2.5\mu_{i}\left( \frac{x\rho_{l}}{x\rho_{l} + (1-x)\mu_{g}} \right)^{2} + \frac{x\rho_{l}(1.5\mu_{l} + \mu_{g})}{x\rho_{l} + (1-x)\mu_{g}}$$  \hspace{1cm} (12)

Lin et al. [5] introduced the definition of two-phase viscosity as follows:
\[
\mu_m = \frac{\mu_1 \mu_2}{\mu_1 + \sqrt{4(x-x_0)(\mu_1 - \mu_2)}}
\] (13)

In their study, the range of \( x \) appearing in the capillary tubes was \( 0 < x < 0.25 \). For the best fit to their experimental data, they took the value of the exponent in Eq. (13) as 1.4.

Fourar and Bories [6] presented an unusual expression of the two-phase viscosity as follows:

\[
\mu_m = \rho_m \left( \sqrt{\rho_v} + (1-x) \sqrt{\rho_l} \right)
\] (14)

Eq. (14) can be rewritten in terms of the kinematic viscosity as follows:

\[
v_m = \left( \sqrt{\rho_v} + (1-x) \sqrt{\rho_l} \right)^2
\] (15)

It should be noted that Fourar and Bories [6] definition of two-phase viscosity is similar to the Dukler et al. [3] definition of two-phase viscosity but with the addition of an extra term.

In the second group, the form of the expression between \( \mu_m \) and mass quality \( x \) does not satisfy the limiting conditions of Eq. (7).

For example, Davidson et al. [7], defined the viscosity of two-phase gas–liquid flow \( (\mu_m) \) as follows:

\[
\mu_m = \mu_l \left[ 1 + \frac{x}{\rho_l} \right]
\] (16)

The reason for this definition is that when Davidson et al., plotted the experimental two-phase friction factor \( (f_{p2}) \) of their high pressure steam-water pressure drop data against the Reynolds number for all-liquid flow \( (Re_2) \), they observed that there were large discrepancies from the single-phase friction factor at \( Re_2 < 2 \times 10^5 \). They found that considerably better agreement with the normal single-phase flow relationship represented by the Blasius Equation [8] was obtained if they plotted the experimental two-phase friction factor against the Reynolds number for all-liquid flow multiplied by the ratio of the inlet to outlet mean specific volumes. It should be noted that the above definition of the viscosity of two-phase gas–liquid flow \( (\mu_m) \), does not extrapolate to the gas viscosity \( (\mu_g) \) as the mass quality \( x \) approaches 1.

Owens [9] introduced a definition of two-phase viscosity based on the liquid viscosity, simply as \( \mu_m = \mu_l \). The rationale being that in most two-phase flows, the liquid is the dominant phase.

Garcia et al. [10], defined the Reynolds number of two-phase gas–liquid flow using the kinematic viscosity of liquid flow \( (\nu_l) \) instead of the kinematic viscosity of two-phase gas–liquid flow \( (\nu_m) \). They used this definition because the frictional resistance of the mixture was due mainly to the liquid. This was equivalent to defining \( \mu_m \) as

\[
\mu_m = \mu_l \left( \frac{\rho_m}{\rho_l} \right) = \frac{\mu_l \rho_g}{\rho_l + (1-x) \rho_g}
\] (17)

The main disadvantage for the various forms of \( \mu_m \) in the second group is that they are not accurate as the mass quality \( x \) approaches 1. A robust model should capture the physics of all limiting cases.

In the realm of two-phase flow viscosity models, Collier and Thome [11] mention that the definition of \( \mu_m \) proposed by McAdams et al. Eq. (8), is the most common definition of \( \mu_m \).

2. Proposed methodology

In the current work, many of the existing two-phase flow viscosity models along with several new definitions are examined. Heat transfer studies involving porous media and two component systems, have lead to many definitions for effective thermal conductivity of the homogeneous medium. Using a one dimensional transport analogy between thermal conductivity in porous media [12] and viscosity in two-phase flow, new definitions for two-phase viscosity will be introduced. These new definitions are generated by analogy as follows:

(i) \( \mu_m \) is analogous to \( \kappa_e \)
(ii) \( \mu_l \) is analogous to \( \kappa_1 \)
(iii) \( \mu_g \) is analogous to \( \kappa_2 \)
(iv) \( \mu_x \) is analogous to \( \mu_v \) (volume fraction of component 2).

These new definitions for two-phase viscosity are given in Table 1. As one can see the series and parallel combination rules are analogous to existing rules proposed by McAdams et al. [1], and Cicchitti et al. [2]. Definition 3 for two-phase viscosity is generated by analogy to the effective thermal conductivity using the Maxwell-Eucken 1 model [13]. Maxwell-Eucken 1 [13] is suitable

<table>
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<th>Table 1</th>
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<tr>
<td><strong>Analog between thermal conductivity in porous media and viscosity in two-phase flow</strong></td>
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<tr>
<td>System Property</td>
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<tr>
<td>Definition 1</td>
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<td>Definition 5</td>
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<tr>
<td>Definition 6</td>
</tr>
</tbody>
</table>

Comment on all definitions:

(i) \( x = 0, k_v = k_1 \)
(ii) \( v_2 = 1, k_v = k_2 \)

They satisfy the following conditions:

(i) \( x = 0, \mu_m = \mu_l \)
(ii) \( x = 1, \mu_m = \mu_g \)
for materials in which the thermal conductivity of the continuous phase is higher than the thermal conductivity of the dispersed phase ($k_{\text{cont}} > k_{\text{disp}}$) like foam or sponge. In this case, the heat flow essentially avoids the dispersed phase. In the case of momentum transport, this is akin to a bubbly flow, where the dominant phase is the liquid. Definition 4 for two-phase viscosity is generated by analogy to the effective thermal conductivity using the Maxwell-Eucken 2 model [13]. Maxwell-Eucken 2 [13] is suitable for materials in which the thermal conductivity of the continuous phase is lower than the thermal conductivity of the dispersed phase ($k_{\text{cont}} < k_{\text{disp}}$) like particulate materials surrounded by a lower conductivity phase. In this case, the heat flow involves the dispersed phase as much as possible. In the case of momentum transport, this is akin to droplet flow, where the dominant phase is the gas. Definition 5 for two-phase viscosity is generated by analogy to the effective thermal conductivity using the Effective Medium Theory (EMT [14,15]). The Effective Medium Theory (EMT [14,15]) is suitable for the structure that represents a heterogeneous material in which the two components are distributed randomly, with neither phase being necessarily continuous or dispersed. In the case of momentum transport, this averaging scheme seems reasonable given the unstable and random distribution of phases in a liquid/gas flow. Finally, Definition 6 for two-phase viscosity is based on the arithmetic mean of Maxwell-Eucken 1 [13] and Maxwell-Eucken 2 [13] models. This is proposed here as a simple alternative to the Effective Medium Theory.

Figs. 2 and 3 show the analogy between thermal conductivity in porous media [12] and viscosity in two-phase flow where Fig. 2 shows $k_e/k_1$ versus $v_2$ [12] while Fig. 3 shows $\mu_m/\mu$ versus $x$ for air–water system at atmospheric conditions.

These new definitions overcome the disadvantages of some definitions of two-phase viscosity such as Davidson et al.’s definition [7], Owens' definition [9] and Garcia et al.’s definition [10] that do not satisfy the condition at $x = 1$, $\mu_m = \mu_q$. These new definitions of two-phase viscosity can be used to compute the two-phase frictional pressure gradient using the homogeneous modelling approach.

Often, it is desirable to express the two-phase frictional pressure gradient, $(dp/dz)_m$, versus the total mass flux ($G$) in a dimensionless form like the Fanning friction factor ($f_m$) versus the Reynolds number ($Re_m$) as follows:

$$f_m = \frac{\rho_m(d\rho/\rho_0) d}{2G^2}$$

Eqs. (2) and (6) represent the two-phase density based on the homogeneous model ($\rho_m$) and Reynolds number based on the homogeneous model ($Re_m$).

To satisfy a good agreement between the experimental data and well-known friction factor models, assessment of the best definition of two-phase viscosity among the different definitions (old and new) is based on the definition that corresponds to the minimum the root mean square (RMS) error.

The fractional error ($e$) in applying the model to each available data point is defined as

$$e = \frac{|\text{Predicted} - \text{Available}|}{\text{Available}}$$

For groups of data, the root mean square error, $e_{\text{RMS}}$, is defined as

$$e_{\text{RMS}} = \left[ \frac{1}{N} \sum_{x=1}^{N} e_x^2 \right]^{1/2}$$

The Fanning friction factor ($f_m$) can be predicted using the Hagen–Poiseuille flow [16] for laminar–laminar flow and the Blasius equation [8] for turbulent–turbulent flow as follows:

$$f_m = \begin{cases} \frac{16}{Re_m} & \text{if } Re_m < 2300 \\ \frac{0.079}{k_{\text{disp}}} & \text{if } Re_m > 4000 \end{cases}$$

For the case of minichannels and microchannels, the friction factor is calculated using the Churchill model [17] that allows for prediction over the full range of laminar-transition-turbulent regions. The Fanning friction factor ($f_m$) can be predicted using the Churchill model [17] as follows:

$$f_m = 2 \left[ \frac{8}{Re_m} \right]^{12} \left( \frac{1}{(a_m + b_m)^{1/2}} \right)^{1/12}$$

$$a_m = \frac{2.457 \ln \left( \frac{1}{7(Re_m)^{0.3}} + (0.27\epsilon/d) \right)}{16}$$

$$b_m = \frac{37.530}{(Re_m)^{16}}$$
The Churchill model [17] is preferable since it encompasses all Reynolds numbers and includes roughness effects in the turbulent regime.

The total pressure drop is the sum of frictional, acceleration and gravitational pressure drops. The proposed correlation gives us the frictional term using the homogeneous model. The acceleration term can be calculated as follows:

\[ \Delta p_a = G^2 \left\{ \frac{(1 - \chi_0)^2}{\rho_1(1 - \chi_0)} + \frac{\chi_0^2}{\rho_2^{\alpha \chi_0^2}} \right\} \left\{ \frac{(1 - \chi_0)^2}{\rho_1(1 - \chi_0)} + \frac{\chi_0^2}{\rho_2^{\alpha \chi_0^2}} \right\} \]  

(25)

The acceleration term is negligible in adiabatic channels. The gravitational term can be calculated using the homogeneous model as follows:

\[ \left( \frac{dp}{dz} \right)_{grav.m} = \frac{g \sin \theta}{\rho_2^{\alpha \chi_0^2}} \]  

(26)

From Eq. (26), it is clear that the gravitational term equals zero in horizontal flow \((\theta = 0)\).

3. Results and discussion

Comparisons of the two-phase frictional pressure gradient versus mass flux from published experimental studies in circular pipes, minichannels and microchannels are undertaken, after expressing the data in dimensionless form as Fanning friction factor versus Reynolds number. The published data include different working fluids such as R-12, R-22, Argon (R740), R717, R134a, R410A and Propane (R290) at different diameters and different saturation temperatures.

3.1. Data for circular pipes

Figs. 4–9 show the Fanning friction factor \((f_m)\) versus Reynolds number \((Re_m)\) for data obtained on circular pipes, using the six
different definitions of two-phase viscosity shown in Table 1 on log–log scale. The sample of the published data includes Bandel’s data [18] for R12 flow at $x = 0.3$ and $T_s = 0^\circ C$ in a smooth horizontal pipe at $d = 14$ mm, Hashizume’s data [19] for R12 flow at $x = 0.5$ and $T_s = 39^\circ C$ and R22 flow at $x = 0.5$ and $T_s = 20^\circ C$, and Müller-Steinhagen’s data [20] for argon (R740) flow at $x = 0.3$ and reduced pressure of 0.188 in a smooth horizontal pipe at $d = 14$ mm. Eq.(18) represents the measured Fanning friction factor while Eq.(21) represents the predicted Fanning friction factor. From Table 2, $e_{RMS}^\%$ values based on measured Fanning friction factor and predicted Fanning friction factor using the six different definitions of two-phase viscosity for this sample of the published data, it can be seen that two-phase viscosity based on Effective Medium Theory (EMT [14,15]) gives the best agreement between the published data and the Blasius equation [8] with the root mean square error ($e_{RMS}$) of 19.99%.

![Fig. 9. $f_m$ versus $Re_m$ in circular pipes using definition 6 of two-phase viscosity.](image1)

![Fig. 10. Predicted frictional pressure gradient versus measured frictional pressure gradient in circular pipes.](image2)

![Fig. 11. $f_m$ versus $Re_m$ in minichannels and microchannels using definition 1 of two-phase viscosity.](image3)

![Fig. 12. $f_m$ versus $Re_m$ in minichannels and microchannels using definition 2 of two-phase viscosity.](image4)

![Fig. 13. $f_m$ versus $Re_m$ in minichannels and microchannels using definition 3 of two-phase viscosity.](image5)
The data in the current dimensionless form as $f_m$ versus $Re_m$ (Figs. 4–9) can be presented as predicted frictional pressure gradient versus measured frictional pressure gradient as shown in Fig. 10.

3.2. Data for minichannels and microchannels

Figs. 11–16 show the Fanning friction factor ($f_m$) versus Reynolds number ($Re_m$) in minichannels and microchannels using the six different definitions of two-phase viscosity shown in Table 1 on log–log scale. The sample of the published data includes Ungar and Cornwell’s data [21] for R717 flow at $T_s = 74 \, {\text{°C}}$ (165.2 °C) in a smooth horizontal tube at $d = 0.1017$ in. (2.583 mm), Tran et al.’s data [22] for R134a flow at $p_s = 365$ kPa and $x = 0.73$ in a smooth horizontal pipe at $d = 2.46$ mm, Cavallini et al.’s data [23] for R410A flow at $T_s = 40$ °C and $x = 0.74$ in smooth multi-port minichannels at hydraulic diameter of 1.4 mm, and Field and Hrnjak’s data [24] for R1233zd(E) flow at $p_s = 430$ kPa and $x = 0.74$ in a smooth horizontal pipe at $d = 2.46$ mm.

Table 3 $\epsilon_{RMS}$ values based on measured fanning friction factor and predicted fanning friction factor in minichannels and microchannels using different definitions of two-phase viscosity

<table>
<thead>
<tr>
<th>Definition</th>
<th>$\epsilon_{RMS}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McAdams et al. [1]</td>
<td>20.76</td>
</tr>
<tr>
<td>Cicchitti et al. [2]</td>
<td>31.06</td>
</tr>
<tr>
<td>Maxwell-Eucken 1 [13]</td>
<td>24.78</td>
</tr>
<tr>
<td>Effective Medium Theory (EMT [14,15])</td>
<td>23.60</td>
</tr>
<tr>
<td>Arithmetic Mean of Maxwell-Eucken 1 and 2 [13]</td>
<td>17.98</td>
</tr>
</tbody>
</table>

The data in the current dimensionless form as $f_m$ versus $Re_m$ (Figs. 4–9) can be presented as predicted frictional pressure gradient versus measured frictional pressure gradient as shown in Fig. 10.

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Hrnjak data [24] for propane (R290) flow at reduced pressure of 0.23 and $G = 330 \text{ kg/m}^2\text{ s}$ in a smooth horizontal pipe at hydraulic diameter of 0.148 mm. Eq. (18) defines the measured Fanning friction factor while Eqs. (22)–(24) represent the predicted Fanning friction factor. Table 3 presents $\epsilon_{\text{RMS}}$ values based on measured Fanning friction factor and predicted Fanning friction factor using the six different definitions of two-phase viscosity for this sample of the published data. It can be seen that two-phase viscosity based on Maxwell-Eucken 2 model [13] gives the best agreement between the published data and the Churchill model [17] with the root mean square error ($\epsilon_{\text{RMS}}$) of 16.47%. It can be seen from Fig. 14 and Table 3, that the definition of effective viscosity based on the Maxwell-Eucken 2 model [13] appears to be more appropriate for defining two-phase flow viscosity in microchannels, and microchannels. On the basis of the data considered, a nominal 5–6% gain in accuracy can be achieved using the homogeneous flow modeling approach. When one considers the nature of the Maxwell-Eucken 2 definition, whereby the dominant phase is the lower viscosity phase, i.e. the gas, and considering the context of Fig. 1, it is clear that this definition is most appropriate for liquid/gas mixtures which have very high density ratios. Thus, even for small mixture qualities, a significant portion of the flow volume is occupied by gas, making the Maxwell-Eucken 2 definition most appropriate.

The data in the current dimensionless form as $fm$ versus $Re_m$ (Figs. 11–16) can be presented as predicted frictional pressure gradient versus measured frictional pressure gradient as shown in Fig. 17.

4. Summary and conclusions

Using the analogy between thermal conductivity in porous media and viscosity in two-phase flow, new definitions for two-phase viscosity are given in Table 1 (Definitions 3, 4, 5 and 6, respectively). These new definitions for two-phase viscosity satisfy the following two conditions: namely (i) $\mu_m = \mu_g$ at $x = 0$ and (ii) $\mu_m = \mu_e$ at $x = 1$. These new definitions of two-phase viscosity can be used to compute the two-phase frictional pressure gradient using a homogeneous modelling approach. Expressing two-phase frictional pressure gradient in dimensionless form as Fanning friction factor versus Reynolds number (Figs. 11–16) can be presented as predicted frictional pressure gradient versus measured frictional pressure gradient as shown in Fig. 17.

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