ABSTRACT

Theoretical and empirical models for the gas void fraction ($\alpha$) are reviewed. Simple rules are developed for obtaining rational bounds for the void fraction in two-phase flow. The lower bound is based on the separate cylinders formulation for turbulent-turbulent flow that uses the Blasius equation to predict the Fanning friction factor. The upper bound is based on the Butterworth relationship that represents well the Lockhart-Martinelli correlation. These two bounds are reversed in the case of liquid fraction ($1-\alpha$). The bounds models are verified using published experimental data of void fraction versus mass quality at constant mass flow rate. The published data include different working fluids such as R-12 and R-22 at different pipe diameters, different pressures, and different mass flow rates. It is shown that the published data can be well bounded for a wide range of mass qualities, pipe diameters, pressures and mass flow rates. Further comparisons are made using the published experimental data of void fraction ($\alpha$) and liquid fraction ($1-\alpha$) versus the Lockhart-Martinelli parameter ($X$), for different working fluids such as R-12, R-22 and air-water mixtures.

NOMENCLATURE

$A$ pipe cross-sectional area, m$^2$
$C_A$ Armand coefficient
$C_o$ Zuber-Findlay coefficient
$c$ constant in Butterworth’s expression, Eq. (35)
$d$ pipe diameter, m
$E_1$ parameter, Eq. (28)
$E_2$ parameter, Eq. (29)
$e$ fraction of the liquid entrained as droplets, Eq. (33)
$Fr$ Froude number $= G^2/gd\rho_l^2$
$f$ Fanning friction factor
$G$ mass flux, kg/m.s$^3$
$k$ index, Eq. (15)

$m$ mass flow rate, kg/s
$n$ index, Blasius index
$p$ pressure, Pa
$q$ index, Eq. (35)
$Re$ Reynolds number $= Gd/\mu$
$r$ index, Eq. (35)
$S$ slip ratio
$s$ index, Eq. (35)
$T$ temperature, °C
$U$ superfacial velocity of flow, m/s
$u$ average velocity of flow, m/s
$We$ Weber number $= G^2d/\rho l s$
$x$ mass quality
$Y$ parameter, Eq. (14)
$y$ distance from the pipe wall, m
$Z$ parameter, Eq. (19)
$z$ parameter, Eq. (27)

Greek Symbols

$\alpha$ void fraction
$\beta$ volumetric quality
$\rho$ density, kg/m$^3$
$\mu$ dynamic viscosity, kg/m.s
$\sigma$ surface tension, N/m

Subscripts

$av$ average
$g$ gas
$l$ liquid
$m$ homogeneous mixture
$max$ maximum
$s$ saturation
$tt$ turbulent-turbulent
INTRODUCTION

The void fraction ($\alpha$) is an important variable in any two-phase flow system. For example, in forced convection boiling in a vertical tube, the determination of pressure change due to gravity is a major problem because the density of the two-phase mixture changes with height. As a result, the determination of pressure change due to gravity requires the estimation of the void fraction. Also, the pressure change due to acceleration is dependent on the void fraction.

The void fraction ($\alpha$) is defined as the ratio of the pipe cross-sectional area occupied by the gas phase to the pipe cross-sectional area.

$$\alpha = \frac{A_g}{A} = \frac{A_g}{A_l + A_g}$$  \hspace{1cm} (1)

In order to characterize a two-phase flow, the slip ratio ($S$) is frequently used instead of void fraction. The slip ratio ($S$) is defined as the ratio of the average velocity of gas phase flow ($u_g$) to the average velocity of liquid phase flow ($u_l$). The void fraction ($\alpha$) can be related to the slip ratio ($S$) as follows:

$$S = \frac{u_g}{u_l} = \frac{G x / A \alpha \rho_g}{G (1-x)/A (1-\alpha) \rho_l} = \frac{\rho_l (1-\alpha)}{\rho_g (1-x) \alpha}$$  \hspace{1cm} (2)

$$S = \frac{u_g}{u_l} = \frac{Q_g / A \alpha}{Q_l / A (1-\alpha)} = \frac{Q_g (1-\alpha)}{Q_l \alpha}$$  \hspace{1cm} (3)

Equations (2) and (3) can be rewritten in the form:

$$\alpha = \frac{l}{1 + S \left( \frac{1-x}{x} \right) \left( \frac{\rho_g}{\rho_l} \right)}$$  \hspace{1cm} (4)

$$\alpha = \frac{Q_g}{S Q_l + Q_g}$$  \hspace{1cm} (5)

The research on pressure drop and void fraction in two-phase flow began in the 1940’s. Since then, pressure drop and void fraction data have been collected for horizontal, vertical, and inclined gas-liquid systems. From the pressure drop and void fraction data, many attempts have been made to develop general procedures for predicting these quantities.

The homogeneous flow model provides the simplest technique for analyzing two-phase (or multiphase) flows. In the homogeneous model, both liquid and vapor phases move at the same velocity (slip ratio = 1). Consequently, the homogeneous model has also been called the zero slip model. The homogeneous model considers the two-phase flow as a single-phase flow having average fluid properties depending on quality. The void fraction based on the homogeneous model ($\alpha_m$) can be expressed, by putting $S = 1$ in Eq. (4), as follows:

$$\alpha_m = \frac{l}{1 + \left( \frac{1-x}{x} \right) \left( \frac{\rho_g}{\rho_l} \right)}$$  \hspace{1cm} (6)

When ($\rho_l/\rho_g$) is large, the void fraction based on the homogeneous model ($\alpha_m$) increases very rapidly once the mass quality ($x$) increases even slightly above zero. The prediction of the void fraction using the homogeneous model is reasonably accurate only for bubble and mist flows since the entrained phase travels at nearly the same velocity as the continuous phase. Also, when ($\rho_l/\rho_g$) approaches 1 (i.e. near the critical state), the void fraction based on the homogeneous model ($\alpha_m$) approaches the mass quality ($x$) and the homogeneous model is applicable at this case.

The separated model is another example of the existing void fraction models. In the separated model, two-phase flow is considered to be divided into liquid and vapor streams. Hence, the separated model has been referred to as the slip flow model. The separated model was originated from the classical work of Lockhart and Martinelli [1] that was followed by Martinelli and Nelson [2]. The separated model is used by both analytical and semi-empirical methods. In the analytical theories, some quantities like the momentum or the kinetic energy is minimized to obtain the slip ratio ($S$). The momentum flux model and the Zivi model [3] are two examples of this technique, where the slip ratio ($S$) equals ($\rho_l/\rho_g$)$^{1/2}$ and ($\rho_l/\rho_g$)$^{1/3}$.

The drift flux model is the third example of the existing void fraction models. The drift flux model is a type of separated flow model. In the drift flux model, attention is focused on the relative motion rather than on the motion of the individual phases. The drift flux model was developed by Wallis [4]. The advantage of the drift flux model is that it takes into account the effect of mass flux on the void fraction. The drift flux model has widespread application to bubble flow and plug flow. The drift flux model is not particularly suitable to a flow such as annular flow that has two characteristic velocities in one phase: the liquid film velocity and the liquid drop velocity. Despite this, the drift flux model has been used for annular flows, but with no particular success. The Rouhani and Axelsson [5] model is an instance for this type of model.

The prediction of void fraction in two-phase flow using models for specific flow regimes is a fourth example of the existing void fraction models. The Taitel and Dukler [6] model is an example for this type of model.

The prediction of void fraction in two-phase flow can also be achieved by empirical correlations. Correlating the experimental data in terms of chosen variables is a convenient way of obtaining design equations with a minimum of analytical work. There are a considerable number of empirical correlations for the prediction of void fraction. Although the empirical correlations require a minimum of knowledge of the system characteristics, they are limited by the range of data available for correlation construction. The empirical correlations are usually presented in terms of the slip ratio ($S$).

The volumetric quality ($\beta$) is defined as the ratio of the volumetric flow rate of gas phase ($Q_g$) to the total volumetric flow rate ($Q$).
\[
\beta = \frac{Q_v}{Q} = \frac{Q_v}{Q + Q_g}
\] (7)

From Eqs. (5), and (7), it is obvious that the volumetric quality \((\beta)\) is equivalent to the void fraction based on the homogeneous model \((S = 1)\).

The void fraction can be calculated using one of the void fraction correlations, which are usually correlations of the slip ratio \((S)\). But, in certain situations such as subcooled boiling processes, measurement of the void fraction is necessary since the void fraction cannot be calculated from an energy balance because of the non-equilibrium nature of the flow. There are several experimental ways of measuring void fraction. Measurement of void fraction can be performed using direct or indirect methods. One of the well-known direct methods is quick-closing valves. In this technique, simultaneously operated quick-closing valves are used to measure the void fraction. This technique can be used only on systems with negligible phase changes and give only the average void fraction for the section between the two valves. Other direct methods include photography using visible light or X-ray and sampling probe. Indirect void fraction measurements are based on ability of two-phase mixture to absorb particles such as alpha, beta, and neutrons. Other indirect methods are based on ability of two-phase mixture to absorb radiation from microwaves, infrared, visible light, ultraviolet, gamma, and X-rays.

In the present study, simple rules are presented for obtaining rational bounds for void fraction in two-phase flow. The advantage of using bounds is in prediction of lower and upper limits that the experimental data fall between. The lower bound is based on Carey correlation [7] for turbulent-turbulent flow that uses the separate-cylinders model [8] and the Blasius equation [9] to represent the Fanning friction factor while the upper bound is based on the Butterworth relationship [10] that represents well the Lockhart-Martinelli correlation [1]. These two bounds are reversed in the case of liquid fraction \((1-\alpha)\). The models are verified using published experimental data of void fraction versus mass quality at constant mass flow rate. The published data include different working fluids such as R-12, and R-22 at different pipe diameters, different pressures, and different mass flow rates. Further, the model is verified using published experimental data of void fraction \((\alpha)\) and liquid fraction \((1-\alpha)\) versus the Lockhart-Martinelli parameter \((X)\) for different working fluids such as R-12, R-22 and air-water mixtures.

**LITERATURE REVIEW**

Armand [11] correlated data for the void fraction during air-water flow in a horizontal pipe of 26 mm diameter at 1 bar by plotting \(\alpha\) versus \(\beta\). He observed that up to \(\alpha\) of about 0.72 \((\beta = 0.9)\), the relationship between \(\alpha\) and \(\beta\) was linear and could be represented by the following equation:

\[
\alpha = C_\alpha \beta
\] (8)

where \(C_\alpha = 0.833\).

For \(\beta > 0.9\), Massena [12] suggested the following approximate equation:

\[
\alpha = \left[C_\alpha + (1-C_\alpha) \frac{x}{x}\right] \beta
\] (9)

Martinelli and Nelson [2] presented an empirical correlation for the void fraction in a graphic manner on semi-log plot. In their study, they assumed that the flow regime would always be ‘turbulent-turbulent’ since any normal forced circulation boiler design for all practical purposes would involve this flow mechanism only. On the horizontal axis, the independent parameter was the mass quality \((x)\). On the vertical axis, the dependent parameter was the void fraction \((\alpha)\). On the grid, they plotted a family of curves for pressures from atmospheric pressure to the critical pressure. They derived their correlation in the following method: at the critical pressure, when \(p_1 = p_g\) and \(\mu_1 = \mu_g\), the relationships that \(\alpha = x\) and \(X_n = (1-x)/\alpha\) were valid. So, the relationship between the void fraction and \(X_n\) at critical pressure could be calculated easily. Also, the relationship between the void fraction and \(X_n\) could be known from the curve at atmospheric pressure. Using these two known curves of void fraction versus \(X_n\) for both critical and atmospheric pressures, curves at intermediate pressures were interpolated. They replaced \(X_n\) by \(x\), with pressure as a parameter. However, this method was based upon a meager amount of data. So, further experimental verification was required, particularly at the higher pressures, before this method could be considered valid.

Lockhart and Martinelli [1] presented data for the simultaneous flow of air and liquids including benzene, kerosene, water, and different types of oils in pipes varying in diameter from 0.0586 in. to 1.017 in. They developed their method for pressures from atmospheric to 50 psi. There were four types of isothermal two-phase, two-component flow. In the first type, flow of both the liquid and the gas were turbulent. In the second type, flow of the liquid was laminar and flow of the gas was turbulent. In the third type, flow of the liquid was turbulent and flow of the gas was laminar. In the fourth type, flow of both the liquid and the gas were laminar. They correlated the pressure drop resulting from these different flow mechanisms by means of the Lockhart-Martinelli parameter \((X)\). The Lockhart-Martinelli parameter \((X)\) was defined as:

\[
X^2 = \frac{(dp/dz)_{f,ji}}{(dp/dz)_{f,g}}
\] (10)

In addition, they expressed the two-phase frictional pressure gradient in terms of factors, which multiplied single-phase gradients. These multipliers were given by:

\[
\phi^2_f = \frac{(dp/dz)_{f,ji}}{(dp/dz)_{f,g}}
\] (11)

\[
\phi^2_g = \frac{(dp/dz)_{f,gi}}{(dp/dz)_{f,g}}
\] (12)

They correlated the percent of pipe filled with liquid under any flow conditions for all four-flow types by means of the Lockhart-Martinelli parameter \((X)\).
Baker [13] gave an empirical equation to allow for the effect of mass flux on the void fraction in vertical upward flow. His equation was:

\[
x = \frac{\alpha^2(Y^{1/3} - 1) + \alpha}{Y - \alpha (Y - Y^{1/3})}
\]  

(13)

\[
Y = 0.021 \left( \frac{\rho_l}{\rho_g} \right) 0.686
\]  

(14)

Equations (13) and (14) were valid over the range \( 7.5 < Y < 300 \) and \( G < 950 \text{ kg/m}^2\text{s} \).

Bankoff [14] presented a variable density single-fluid model for two-phase flow. This model was an extension of the homogeneous model with correction for two-dimensional effects. In his model, he proposed that the mixture flowed as a suspension of bubbles in the liquid, where radial gradients existed in the concentration of bubbles. The bubble concentration had a maximum value at the center of the pipe, decreased monotonically in a radial direction, and reached zero at the pipe wall. He assumed that the gas and liquid had the same velocity at any radial position. The relative velocity of the bubbles with respect to the surrounding liquid was considered to be negligible compared to the stream velocity. The average velocity of the gaseous phase was greater than that of the liquid phase only because the gas concentration was in the regions of higher velocity. The mixture might be considered to be a single fluid whose density was a function of radial position because the slippage at any point was considered to be negligible. He assumed a power law distribution for both the velocity and the void fraction.

\[
u = u_{max} \left( \frac{2y}{d} \right)^{\frac{1}{3}}
\]  

(15)

\[
\alpha = \alpha_{max} \left( \frac{2y}{d} \right)^{n}
\]  

(16)

Integration and manipulation of the above two equations lead directly to the result that the average void fraction \( \alpha_{av} \), was related to the average volumetric quality \( \beta_{av} \), through the following relationship

\[
\alpha_{av} = C_A \beta_{av}
\]  

(17)

where \( C_A \) was a function of \( k \) and \( n \) as follows:

\[
C_A = \frac{2(k + n + kn)(k + n + 2kn)}{(n + 1)(2n + 1)(k + 1)(2k + 1)}
\]  

(18)

The symmetry of the above equation in \( k \) and \( n \) was noteworthy. For \( k = 2-7 \) and \( n = 0.1-5 \), \( C_A \) had an effective range of 0.5-1. Bankoff compared the prediction of average void fraction versus mass quality in steam-water flow with the Martinelli-Nelson correlation [2] using a constant average value of 0.89 for \( C_A \). He found that the agreement the Martinelli-

Nelson correlation was good over a range of pressures from 100 to 2 500 psia and a range of average void fractions from 0 to 0.85.

Hughmark [15] developed a correlation for the void fraction. He related \( C_A \) to a parameter \( Z \) in a non-linear function in his correlation. He defined \( Z \) as follows:

\[
Z = \left[ \frac{d G}{(1 - \alpha) \mu_l + \alpha \mu_g} \right]^{1/6} \left[ \frac{U^2}{g d} \right]^{2/6} \left[ I - \beta \right]^{1/4}
\]  

(19)

Table 1 gives values of \( C_A \) as a function of \( Z \).

<table>
<thead>
<tr>
<th>( C_A )</th>
<th>1.3</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z )</td>
<td>0.185</td>
<td>0.225</td>
<td>0.325</td>
<td>0.49</td>
<td>0.605</td>
<td>0.675</td>
<td>0.72</td>
</tr>
<tr>
<td>( C_A )</td>
<td>8</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>40</td>
<td>70</td>
<td>130</td>
</tr>
<tr>
<td>( Z )</td>
<td>0.767</td>
<td>0.78</td>
<td>0.808</td>
<td>0.83</td>
<td>0.88</td>
<td>0.93</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Hewitt et al. [16] linearized Lockhart-Martinelli’s data because Lockhart-Martinelli’s empirical correlation for void fraction was presented between liquid void fraction, \( 1 - \alpha \), and Lockhart-Martinelli parameter \( X \) on log-log scale. They fitted Lockhart-Martinelli’s empirical correlation for void fraction using a six-term equation. Their equation was:

\[
\ln(X) - 1.482 + 4.915 \ln X - 5.955(\ln X)^2
\]

\[
+ 2.675(\ln X)^3 + 6.399(\ln X)^4 - 8.768(\ln X)^5
\]  

(20)

Zivi [3] developed a correlation to determine the void fraction using the concept of minimum entropy production. His correlation was

\[
\alpha = \frac{1}{1 + \left( \frac{1 - x}{x} \right) \left( \frac{\rho_g}{\rho_l} \right)^{\frac{2}{3}}}
\]  

(21)

Comparing Eqs. (4), and (21), he found that the slip ratio \( S \) in an idealized two-phase flow with zero wall friction and zero entrainment was equal to \( (\rho/\rho_g)^{1/3} \). Therefore, the slip ratio \( S \) was dependent only on the phase density ratio. Also, the slip ratio \( S \) should decrease as the pressure increased. This agreed with experimental data [17]. This model proved particular successful in predicting pressure drop [18] and heat transfer [19] during condensation. Also, this model was used to calculate the acceleration pressure drop during two-phase flow [20,21].

Thom [22] correlated steam-water void fraction data in terms of slip factor, which was a unique function of pressure and mass quality. His equation was given by:

\[
\alpha = \frac{S x}{I + x (S - I)}
\]  

(22)

Baroczy [23] presented a generalized correlation in a
graphical manner for the liquid fraction in two-phase flow. He proposed his correlation for use with all fluids, including liquid metals. The correlation was based on isothermal, two-phase, two-component liquid fraction data for liquid mercury-nitrogen, and water-air. His correlation was for liquid void fraction data as a function of the Lockhart-Martinelli parameter ($X$) and the property index, $(\mu_\ell/\mu_g)^{0.2}/(\rho_g/\rho_\ell)$. The property index had the advantage of not requiring knowledge of the critical pressure and temperature in order to establish the property ratios at the critical point, where they had a value of 1. By similar reasoning, it could be used to establish the analogous condition for two-phase, two-component flow, that was, equal viscosity and density for each phase. Thus, the physical properties of single and two-component, two-phase fluids could be described on a common basis. He found that there was a good agreement between the liquid fraction predicted by this correlation and the Martinelli-Nelson correlation for steam, experimental data for steam, and experimental data for Santowax R, an organic coolant. He also showed the prediction of liquid fraction by this method for sodium, potassium, rubidium, and mercury. He demonstrated the application of this method to boiling mercury, for a range of temperatures and exit mass qualities.

Turner [8] developed the separate-cylinders model of two-phase flow. He assumed that the two phases were to flow independently of each other in two horizontal separate parallel circular cylinders and that the areas of the cross sections of these cylinders added up to the actual pipe. The frictional pressure gradient in each of the imagined cylinders was the same as in the actual flow and was calculated from single-phase flow theory. The separate-cylinders model formulation resembled Lockhart and Martinelli correlation [1] but had the advantage that it could be pursued to an analytical conclusion whereas Martinelli was content with a correlation. The results of his analysis gave:

$$\alpha = \frac{1}{1 + X^{2.7/n}}$$  \hspace{1cm} (23)

The values of $n$ were dependent on whether the liquid and gas phases were laminar or turbulent flow. The different values of $n$ are given in Part I. However, it should be noted that this model was not a particularly good representation of experimental data because no function of the form of Eq. (23) would predict Lockhart-Martinelli's data. For all values of $n$, Eq. (23) would predict $\alpha = 0.5$ at $X = 1$ instead of the correct value of 0.77. Also, it was noted that Lockhart-Martinelli's empirical correlation for void fraction could be represented well by the equation

$$\alpha = \left( \frac{1}{1 + X^{0.8}} \right)^{1/2.65}$$  \hspace{1cm} (24)

which is easier to use than Eq. (20).

Zuber et al. [24] studied the effect of mass flux on the void fraction for the evaporation of R22 in a vertical 1 cm heated pipe. For non-equilibrium nature of the flow at 11.5 bar, the effects of mass flux on the void fraction were clearly visible at low void fractions. For $x > 0$ at 11.5 bar, the void fraction increased with increased mass flux at a given value of $x$. They also showed on the same graph that the void fractions predicted from the Martinelli-Nelson correlation for the steam-water system at the same value of $(\rho_v/\rho_\ell)$ might be expected to correspond to mass fluxes near 1 000 kg/m.s$^2$. In addition, the void fractions predicted from the homogeneous model were higher than the void fractions predicted from the Martinelli-Nelson correlation and might be expected to correspond to mass fluxes > 2 000 kg/m.s$^2$.

Rouhani and Axelsson [5] divided the complex problem of void calculation in the different regions of flow boiling in two parts. In the first part, they included only the description of the mechanisms and the calculation of the rates of heat transfer for vapour and liquid. They assumed that heat was removed by vapour generation, heating of the liquid that replaced the detached bubbles, and in some parts, by single phase heat transfer. By considering the rate of vapour condensation in liquid, they obtained an equation for the differential changes in the true steam mass quality throughout the boiling regions. Integration of this equation gave the vapour weight fraction at any position. In the second part of the problem, they were concerned with the determination of the void fractions corresponding to the calculated steam mass qualities. For this purpose, they used the derivations of Zuber and Findlay [25]. This model was a type of drift flux model, and yielded the following equation for vertical flows:

$$\alpha = \frac{x}{\rho_g} \left[ \left( 1 + 0.2(1-x) \left( \frac{gd\rho_l^2}{G^2} \right)^{0.25} \right) \left( \frac{x + l - x}{\rho_g \rho_l} \right) \right]^{-1}$$  \hspace{1cm} (25)

Equation (26) provided a method for calculating void fractions including the effects of mass flux and surface tension. Rouhani and Axelsson [5] compared this model with data from different geometries including small rectangular channels and large rod bundles. The data covered pressures from 19 to 138 bars, heat fluxes from 18 to 120 W/cm$^2$ with many different subcoolings and mass fluxes. They found that the agreement between the model and data was generally very good.

Premoli et al. [26] presented an empirical correlation of void fraction. This correlation is usually known as the CISE correlation. They covered a wide range of data in their correlation and took into account the effect of mass flux on the void fraction. They expressed their correlation in terms of slip ratio ($S$). Equation (4) is used with the slip ratio ($S$) defined as:

$$S = I + E_1 \left( \frac{z}{1 + z} \right)^{0.5}$$  \hspace{1cm} (26)

$$z = \frac{\beta}{I - \beta}$$  \hspace{1cm} (27)

$$E_1 = 1.578 Re^{-0.16} \left( \frac{\rho_l}{\rho_g} \right)^{0.22}$$  \hspace{1cm} (28)
\[ E_z = 0.0273 \text{We}^{-0.31} \left( \frac{\rho_l}{\rho_s} \right)^{0.08} \]  \hspace{1cm} (29)

\[ Re = \frac{Gd}{\mu_l} \]  \hspace{1cm} (30)

\[ We = \frac{G^2 d}{\rho_s \sigma} \]  \hspace{1cm} (31)

The CISE correlation was the most accurate generally applicable correlation to calculate the mean density (\( \overline{\rho} \)). The mean density (\( \overline{\rho} \)) was defined as:

\[ \overline{\rho} = (1-\alpha) \rho_l + \alpha \rho_s \]  \hspace{1cm} (32)

The standard deviation of the mean density calculated by the CISE correlation was approximately 40%, although it was a little less in the case of steam-water flow.

Smith [27] assumed a separated flow consisting of a liquid phase and a gas phase with a fraction \( e \) of the liquid entrained as droplets. He obtained the following expression in terms of slip ratio (\( S \)) assuming that the momentum fluxes of the two separated phases were equal:

\[ S = e + (1-e) \left( \frac{\rho_l / \rho_s + e (1/x - 1)}{1 + e (1/x - 1)} \right)^{0.5} \]  \hspace{1cm} (33)

Equation (33) is in conjunction with Eq. (4). A value of \( e = 0.4 \) gave the best fit to the data.

Chisholm [28] presented a particularly simple correlation of void fraction, Eq. (4), in terms of slip ratio (\( S \)):

\[ S = \left[ 1 - x \left( 1 - \frac{\rho_l}{\rho_s} \right) \right]^{0.5} \]  \hspace{1cm} (34)

It was clear that as \( (\rho_l/\rho_s) \) approached 1 (i.e. approaching the critical state), the slip ratio (\( S \)) approached 1 as the flow became homogeneous in character. Although the Chisholm correlation is very simple, but it provides a reasonably accurate result.

Butterworth [10] presented a short note about a comparison of some void fraction relationships for co-current gas-liquid flow. His comparison was among six different models and correlations. The models and correlations considered were: the homogeneous model, Zivi model, separate-cylinders model, Eq. (23) with \( n = 2.5 \), Lockhart-Martinelli correlation, Thom correlation, and Baroczy correlation respectively. He suggested that the following equation could be used as the basis for a new void fraction correlation.

\[ \alpha = \frac{1}{1 + c \left( \frac{1-x}{x} \right)^{\left( \frac{\rho_s}{\rho_l} \right) \left( \frac{\mu_l}{\mu_g} \right)}} \]  \hspace{1cm} (35)

The values of \( c, q, r, \) and \( s \) for the different models and correlations were given in the Table 2.

**Table 2. Values of \( c, q, r, \) and \( s \) for Different Models and Correlations**

<table>
<thead>
<tr>
<th>Model or Correlation</th>
<th>( c )</th>
<th>( q )</th>
<th>( r )</th>
<th>( s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous Model</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Zivi [3]</td>
<td>1</td>
<td>1</td>
<td>0.67</td>
<td>0</td>
</tr>
<tr>
<td>Separate-Cylinders [8]</td>
<td>0.72</td>
<td>0.40</td>
<td>0.08</td>
<td>0</td>
</tr>
<tr>
<td>Lockhart-Martinelli [1]</td>
<td>0.28</td>
<td>0.64</td>
<td>0.36</td>
<td>0.07</td>
</tr>
<tr>
<td>Thom [22]</td>
<td>1</td>
<td>1</td>
<td>0.89</td>
<td>0.18</td>
</tr>
<tr>
<td>Baroczy [23]</td>
<td>0.74</td>
<td>0.65</td>
<td>0.13</td>
<td>0</td>
</tr>
</tbody>
</table>

The values of \( c, q, r, \) and \( s \) could be determined by fitting Eq. (35) to experimental data. However, two of these constants might be removed on physical grounds. For example, at the critical state, when \( \rho_l = \rho_s \) and \( \mu_l = \mu_g \), the relationship that \( \alpha = x \) was valid. So, both \( c \) and \( q \) had a value of 1.

Taitel and Dukler [6] presented a theoretical model for determining flow regime transitions in two-phase gas-liquid flow. Their model predicted the relationship between the following variables at which the flow regime transitions took place: gas and liquid mass flow rates, properties of the fluids, pipe diameter, and angle of inclination of the pipe to the horizontal. The regimes considered in their model were intermittent (slug and plug), stratified smooth, stratified wavy, dispersed bubble, and annular-annular dispersed liquid flow. They based the mechanisms for transition on physical concepts. These mechanisms were fully predictive in that no flow regime transition was used in their development. They presented a generalized flow regime map based on this theory. Two-phase flow patterns could be predicted using this mechanistic approach rather than correlated using experimental data. Also, they developed a method to predict the liquid height in stratified flows for their flow pattern map. This flow pattern map could be used to obtain the void fraction.

Nabizadeh [29] modified Zuber and Findlay equation [25]

\[ \alpha = \frac{x \rho_s - \rho_l}{\rho_s} \left[ C_a \left( \frac{x}{\rho_s} + \frac{1-x}{\rho_l} \right) + 1.18 \frac{G \sigma (\rho_l - \rho_s)^{1/25}}{G \rho_l^{1/2}} \right]^{-1} \]  \hspace{1cm} (36)

Using a large number of his own measurements, Nabizadeh developed an empirical correlation for the \( C_a \) factor in the equation of Zuber and Findlay [25] of the form

\[ C_a = \left[ 1 + \frac{1-x}{x} \frac{\rho_s}{\rho_l} \right]^{1/2} \left[ 1 + \frac{1-x}{x} \frac{\rho_s}{\rho_l} \right]^{1/2} \]  \hspace{1cm} (37)

\[ Fr = \frac{G^2}{g \rho_l^{1/2}} \]  \hspace{1cm} (38)

\[ n = \sqrt{0.6 \frac{\rho_l - \rho_s}{\rho_l}} \]  \hspace{1cm} (39)

Applying the extended and improved Zuber-Findlay's
equation, he obtained good agreement between correlated and measured data for water, R12, and R113.

Using a force balance under steady-state conditions, Hart et al. [30] derived for small liquid void fraction \((1-\alpha) \leq 0.06\) the following equation for the liquid void fraction in the stratified, wavy and annular flow regimes:

\[
\frac{1-\alpha}{\alpha} = \frac{U_i}{U_s} \left\{ 1 + \left[ 10.4 \frac{\rho_1}{\rho_s} \left( \frac{\rho_s}{\rho_1} \right)^{0.5} \right] \right\}^{1/2} \tag{40}
\]

They obtained good agreement between the experimentally determined values of the liquid void fraction and the values calculated with Eq. (43) for the air-water and four different air-water+ethylene glycol systems.

Steiner [31] modified the Rouhani-Axelsson [5] void fraction model for application to horizontal flows. His equation was:

\[
\alpha = \frac{x}{\rho_s} \left( \frac{1 + 0.12(1-x)}{\rho_1} \left( \frac{x}{\rho_1} + \frac{1-x}{\rho_1} \right) \right)^4 \right\}^{1/4} + \frac{1.18(1-x)}{Gp_g^{0.5} \sqrt{\sigma (\rho_1 - \rho_g)}} \tag{41}
\]

He did not provide a comparison of Eq. (41) to void fraction data but only noted that he found it to work for R-12 and R-22. Kattan et al. [32] used Steiner expression in their two-phase flow heat transfer model. Also, Steiner expression was used in the new condensation heat transfer model and flow pattern map of Thome and coworkers [33,34].

A comparison of several two-phase flow void fraction models and correlations is presented in Fig. 1. It is clear from the figure that no two models provide the same result. Since all models were developed in conjunction with experimental data, which are prone to measurement error, it is reasonable to expect that any prediction is also subject to similar error. The use of bounds can alleviate some of the subjectivity of the published models by providing definite limits on two-phase void fraction values.

### PROPOSED METHODOLOGY

In this section, we develop rational bounds for two phase void fraction. These bounds may be used to determine the maximum and minimum values that may reasonably be expected in a two phase flow. Further, by averaging these limiting results an acceptable prediction for the void fraction is obtained, which is then bracketed by the bounding values. This approach is very useful when conducting new experiments, since it provides a reasonable envelope for the data to fall within.

As mentioned in Part I, several issues may be considered when developing bounds. For example, one could consider extreme conditions such as laminar and turbulent flow, variability in properties, type of modelling approach such as homogeneous or separated flow, and so on.

As in part I of this paper, the method is applied for the case of turbulent flow. In practice, both \(Re_l\) and \(Re_g\) are most often greater than 2000 unless microchannel flows of interest. The general methodology for choosing the upper and lower bounds for a turbulent/turbulent flow will now be developed based on the same concepts and rationale as used in Part I, and comparisons made with published data.

### The Lower Bound

Carey [7] used the separate cylinders analysis [8] and introduced the Blasius equation [9] to represent the Fanning friction factor. He obtained the following equations:

\[
\phi_l^2 = \frac{l}{\alpha^{5-n/2}} \tag{42}
\]

\[
\phi_g^2 = \left[ l + X^{4(5-n)/2} \right]^{5-n/2} \tag{43}
\]

Combining Eqs. (42) and (43), Carey [32] expressed \(\alpha\) for turbulent-turbulent flow \((n = 0.25)\) as:

\[
\alpha = \frac{l}{l + X^{16/19}} \tag{44}
\]

For turbulent-turbulent flow, Lockhart-Martinelli parameter \((X)\) can be expressed as [21]:

\[
X = \left( \frac{1-x}{x} \right)^{0.875} \left( \frac{\rho_s}{\rho_l} \right)^{0.5} \left( \frac{\mu_l}{\mu_g} \right)^{0.125} \tag{45}
\]

From Eqs. (44) and (45), we obtain

\[
\alpha = \frac{l}{l + \left[ \left( \frac{1-x}{x} \right)^{0.875} \left( \frac{\rho_s}{\rho_l} \right)^{0.5} \left( \frac{\mu_l}{\mu_g} \right)^{0.125} \right]^{16/19}} \tag{46}
\]

### The Upper Bound

Butterworth [10] represented well the Lockhart-Martinelli correlation by the relation:

![Figure 1 Comparison of Selected Two-Phase Flow Void Fraction Models and Correlations](image-url)
\[ \alpha = \frac{1}{1 + 0.28X^{0.77}} \]  

(47)

From Eqs. (45) and (47), we obtain

\[ \alpha = \frac{1}{1 + 0.28 \left( \frac{1 - x}{x} \right)^{0.875} \left( \frac{\rho_s}{\rho_t} \right)^{0.5} \left( \frac{\mu_s}{\mu_t} \right)^{0.125}} \]  

(48)

Since \((1-\alpha)\) represents the liquid fraction, the lower and upper bounds are reversed in the case of liquid fraction data.

**Mean Model**

A simple model may be developed by averaging the two bounds. This is defined as follows:

\[ \alpha_{av} = \frac{0.5}{1 + X^{16.79}} + \frac{0.5}{1 + 0.28X^{0.77}} \]  

(49)

or

\[ \alpha_{av} = \frac{0.5}{1 + \left( \frac{1 - x}{x} \right)^{0.875} \left( \frac{\rho_s}{\rho_t} \right)^{0.5} \left( \frac{\mu_s}{\mu_t} \right)^{0.125} X^{16.79}} + \frac{0.5}{1 + 0.28 \left( \frac{1 - x}{x} \right)^{0.875} \left( \frac{\rho_s}{\rho_t} \right)^{0.5} \left( \frac{\mu_s}{\mu_t} \right)^{0.125}} \]  

(50)

In the next section, both bounds and the average of the bounds models are compared with published data.

**RESULTS AND DISCUSSION**

Examples of two-phase void fraction versus mass quality at constant mass flow rate from published experimental studies are presented to show features of the bounds. The published data include different working fluids such as R-12, and R-22 at different pipe diameters, different pressures, and different mass flow rates. Also, the model is verified using published experimental data of void fraction \((\alpha)\) and liquid fraction \((1-\alpha)\) versus the Lockhart-Martinelli parameter \((X)\) for different working fluids such as R-12, R-22, and air-water mixtures in turbulent-turbulent flow.

**Comparison of the Present Model with Data**

Figures 2-4 show the void fraction versus mass quality. Equation (46) represents the lower bound and Eq. (48) represents the upper bound while Eq. (50) represents the average. Figure 2 compares the present model with Hashizume’s data [35] for R 12 flow at \(T_i = 50^\circC\) and \(m = 25, 35, 50, 70\) and 100 kg/hr respectively in a smooth horizontal pipe at \(d = 10\) mm. Figure 3 compares the present model with Hashizume’s data [35] for R 22 flow at \(T_i = 39^\circC\) and \(m = 25, 35, 50, 70\) and 100 kg/hr respectively in a smooth horizontal pipe at \(d = 10\) mm while Fig. 4 compares the present model with for Wojtan et al’s data [36] R 410A flow at \(T_i = 5^\circC\) and \(G = 70, 150, 200\) and 300 kg/m\(^2\).s respectively in a smooth horizontal pipe at \(d = 5/8\) in. (15.875 mm). In Figs. 2-4, the mean model predicts the published data of \(\alpha\) with the root mean square (RMS) error of 9.04%, 7.39% and 26.86% respectively. In Fig. 4, if the two lower points at \(G = 70\) kg/m\(^2\).s are not included due to a non-representative acquisition time period and the maximum errors \(\pm2\%\) in energy balance [36], RMS will be 10.6% instead of 26.86%.
simple expressions are developed for obtaining bounds for void fraction \((\alpha)\) in two-phase flow. The lower bound is based on Carey correlation for turbulent-turbulent flow. The upper bound is based on the Butterworth relation [10] for liquid fraction \((1-\alpha)\). These two bounds are reversed in the case of liquid fraction \((1-\alpha)\). The model is verified using published experimental data of void fraction versus mass quality at constant mass flow rate. The published data of \((1-\alpha)\) with the root mean square (RMS) error of 9.21% and 5.78% respectively.
data include different working fluids such as R-12 and R-22 at different pipe diameters, different pressures, and different mass flow rates. It can be seen that the published data can be well bounded for a wide range of mass qualities, pipe diameters, pressures and mass flow rates. Also, the models are verified using published experimental data of void fraction ($\alpha$) and liquid fraction ($1-\alpha$) versus the Lockhart-Martinelli parameter ($X$) for different working fluids such as R-12, R-22 and air-water mixtures. The following conclusions can be drawn based upon the present study.

First, the present model is very successful in bounding the two-phase void fraction well for different working fluids over a wide range of mass qualities, pipe diameters, pressures and mass flow rates.

Second, the present model is very successful in bounding two-phase void fraction ($\alpha$) and liquid void fraction ($1-\alpha$) versus Lockhart-Martinelli parameter ($X$) well for different working fluids.

Third, the mean model provides a simple and acceptable means of predicting void fraction.

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