Control Volume Analysis
Control Volume Analysis

• Consider the control volume in more detail for both mass and energy
  – open and closed systems
  – steady and transient analysis

• Control Volume
  – encloses the system or region of interest
  – can have multiple inlets/exits or none at all if it is a closed system (as we have seen)
  – is important much like the free body diagram
**Control Volume Analysis**

- **Simple Two Port System (Mass Balance)**

- **Rate Balance**

\[
\frac{dm_{CV}}{dt} = \dot{m}_i - \dot{m}_e
\]
Conservation of Mass

• Extended to multiple inlet/exit control volumes, we may write the balance as:

\[
\frac{dm_{CV}}{dt} = \sum_{\text{inlets}} m_i - \sum_{\text{exits}} m_e
\]

• If there are no inlets/exits, the system is closed and we have:

\[
\frac{dm_{CV}}{dt} = 0 \quad \text{or} \quad m_{CV} = \text{Constant}
\]
Conservation of Mass

• If the system is under steady state then the left hand side of the mass balance becomes:

\[ \frac{dm_{CV}}{dt} = 0 \]

or

\[ \sum_{inlets} \dot{m}_i = \sum_{exits} \dot{m}_e \]

• We will examine many problems involving steady and unsteady flow
Conservation of Mass

- Evaluating Mass Flow Rate:
  - To determine the mass flow rate entering/exiting a control volume, we must consider a simple incremental area $dA$ with matter flowing across it with velocity $V$ over an interval $\Delta t$:
Conservation of Mass

• The volume of mass crossing \(dA\) in the interval \(\Delta t\) is the volume contained in the cylinder:
  – \(d\text{Volume} = V_n \Delta t \, dA\) \((V_n\) is normal to \(dA\))
  – \(d\text{Mass} = \rho \text{Volume} = \rho V_n \Delta t \, dA\)

• Thus the mass flow rate through \(dA\) is:

\[
\dot{m} = \frac{dm}{\Delta t} = \rho V_n dA
\]
Conservation of Mass

• Finally we integrate over the area area $A$, to get the mass flow rate through this region:

$$\dot{m} = \int_A \rho V_n \, dA$$

• If we have a stream with normal uniform velocity passing through an area $A$, the mass flow rate for this one-dimensional flow is:

$$\dot{m} = \rho \bar{V} A$$
Conservation of Mass

- The **Conservation of Mass** equation for a multiport control volume can then be taken as:
  
  \[
  \frac{dm_{CV}}{dt} = \sum_{\text{inlets}} (\rho \bar{V}A)_i - \sum_{\text{exits}} (\rho \bar{V}A)_e
  \]

- This will be the starting point for all future analyses. We will then make assumptions as to steady/unsteady or open/closed.
Example - 1

• Steam enters a turbine through a pipe with a 15 cm diameter. The inlet steam velocity is 90 m/s, and it has a pressure of 20 MPa and temperature of 600°C. The exit pipe has a diameter of 60 cm, and the exit pressure is 300 kPa and temperature of 150°C. Assuming steady state flow, calculate the mass flow rate through the turbine and the exit velocity.
Example - 2

• An air compressor supplies air to a rigid tank that has a volume of 4 m$^3$. Initially the pressure and temperature of the air in the tank are 101 kPa and 35°C. The supply pipe is 7 cm in diameter and the feed velocity is steady at 12 m/s. The pressure and temperature of the air in the inlet pipe are 600 kPa and 35°C. Find:
  – Time rate of change of mass inside the tank
  – Mass of air added to the tank if the compressor stops when pressure and temperature are 400 kPa and 55°C
  – Time the compressor is running to fill tank
Conservation of Energy: First Law of Thermodynamics
Introduction

• The First Law of Thermodynamics states that energy must be conserved, i.e. it cannot be created or destroyed.

• The energy balance for a control volume follows a similar approach to that for Conservation of Mass, but has additional considerations.

• As before we will consider open and closed systems and steady/transient flows
Conservation of Energy

- Energy Balances

Energy transfers can occur by heat and work.

Control volume

Dashed line defines the control volume boundary.

\[ \dot{m}_i \]

\[ u_i + \frac{V_i^2}{2} + g z_i \]

Inlet $i$

\[ \dot{Q} \]

Exit $e$

\[ u_e + \frac{V_e^2}{2} + g z_e \]

Faculty of Engineering and Applied Science
Memorial University of Newfoundland
St. John’s, Newfoundland, Canada
Conservation of Energy

• Rate Balance

\[
\frac{dE_{CV}}{dt} = \dot{Q} - \dot{W} + \dot{m}_i \left( u_i + \frac{V_i^2}{2} + g z_i \right) - \dot{m}_e \left( u_e + \frac{V_e^2}{2} + g z_e \right)
\]

• Referring to the figure we see that in equation form this becomes:
Conservation of Energy

• If the control volume contains multiple inlets/ exits then we may write:

\[
\frac{dE_{CV}}{dt} = \dot{Q} - \dot{W} + \sum_{\text{inlets}} \dot{m}_i \left( u_i + \frac{V_i^2}{2} + gz_i \right) - \sum_{\text{exits}} \dot{m}_e \left( u_e + \frac{V_e^2}{2} + gz_e \right)
\]

• We must now consider what the work term really represents. At this point it is the net work transfer.

• It is more useful to account for this in more detail as there are different types of work.
Conservation of Energy

• Since work is always done on or by a control volume when matter flows across inlets/exits, it is convenient to separate work into two contributions:
  – work associated with fluid pressure
  – work associated with rotating shafts, boundary displacement, etc

• The former is referred to frequently as “flow work”
Conservation of Energy

• Work due to fluid flow can be considered if consider that pressure times area is a force, \( p \times A \), and force times velocity, \( F \times V \), is power (or work), thus:

\[
\begin{align*}
\text{time rate of energy} & = pAV \\
\text{transfer by work due to flow} & = pAV
\end{align*}
\]

• We will define the work term as:

\[
\dot{W} = \dot{W}_{CV} + \sum_{\text{exits}} p_e A_e V_e - \sum_{\text{inlets}} p_i A_i V_i
\]
Conservation of Energy

• Redefining the above in terms of mass flow rate and density (or specific volume) leads to:

\[
\dot{W} = \dot{W}_{CV} + \sum_{\text{exits}} \frac{p_e \dot{m}_e}{\rho_e} - \sum_{\text{inlets}} \frac{p_i \dot{m}_i}{\rho_i}
\]

\[
\dot{W} = \dot{W}_{CV} + \sum_{\text{exits}} p_e v_e \dot{m}_e - \sum_{\text{inlets}} p_i v_i \dot{m}_i
\]

• Combining with the energy balance gives:

\[
\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W}_{CV} + \sum_{\text{inlets}} \dot{m}_i \left( u_i + p_i v_i + \frac{V_i^2}{2} + gz_i \right) - \sum_{\text{exits}} \dot{m}_e \left( u_e + p_e v_e + \frac{V_e^2}{2} + gz_e \right)
\]
Conservation of Energy

- Conservation of energy is also frequently used in the following form using enthalpy:

\[
\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W}_{CV} + \sum_{\text{inlets}} m_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_{\text{exits}} m_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)
\]

- Steady State

\[
\dot{Q}_{CV} - \dot{W}_{CV} + \sum_{\text{inlets}} m_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_{\text{exits}} m_e \left( h_e + \frac{V_e^2}{2} + gz_e \right) = 0
\]

- Closed System

\[
\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W}_{CV} \quad \text{or} \quad \Delta E_{CV} = Q_{CV} - W_{CV}
\]

\[
E = KE + PE + U
\]
Conservation of Energy

• We can apply conservation principle to components and systems (assemblages of components):
Example - 3

• Re-consider Example 1 earlier, but this time determine the work done by the superheated steam passing through the turbine. Assume the turbine is insulated, such that there is no heat loss to the surroundings. Also, consider the effect of neglecting the kinetic energy.
Example - 4

• A stream of liquid water at 300 kPa and 20°C is mixed with steam at 300 kPa and 250°C in an adiabatic mixing chamber. The steam has a flow rate of 90 kg/s and the mixture leaves at a pressure of 300 kPa. Determine the mass flow rate of the liquid entering the chamber and the mass flow rate of the mixture leaving the chamber.
Turbines

- We can apply conservation energy to turbines:
  - Derivation in class notes
Pumps and Compressors

• We can apply conservation of energy to pumps and compressors:
Example - 5

• A pump is used to raise water steadily at a volume flow rate of 50 L/s through an elevation change of change of 100 m. The pump inlet has a diameter of 15 cm and the exit a diameter of 18 cm. The water is drawn from a supply at 20°C and is discharge to atmospheric pressure of 100 kPa. Assuming that the pump is well insulated and that frictional heating effects are small, and assuming that the exit temperature is approximately the same as the inlet, what is the power (work) required for this pump?
Example - 6

• A compressor is used to compress atmospheric air at 100 kPa and 20°C to a pressure of 1 MPa. The compressor loses approximately 10% of the energy input as work through heat loss to the surroundings. Air enters the compressor at 50 m/s though an inlet having a diameter of 10 cm and leaves at 120 m/s through an exit having a diameter of 2.5 cm. Determine the exit air temperature and the input power required to the compressor.
Heat Exchangers

• We can apply conservation of energy to heat exchangers:
Example - 7

- A heat exchanger is designed to use exhaust steam from a turbine to heat air for space heating. Steam enters the heat exchanger with a flow rate of 1.2 kg/s a pressure of 200 kPa and a temperature of 200 °C. The steam leaves the heat exchanger as a saturated vapor at 200 kPa. The air enters the heat exchanger at 20 °C and 101 kPa with a flow rate of 3 kg/s, and leaves at a pressure of 101 kPa. Assuming that losses to the surroundings are negligible, calculate the exit air temperature.
Throttling Valves

- We can apply conservation of energy to throttling valves:
Example - 8

- A throttling valve is used in a refrigeration system. Saturated liquid Refrigerant 22 at 10 bar is passed through a throttling valve and discharged at 1 bar. Determine the inlet and exit temperatures and the exit quality assuming that heat transfer and kinetic energy effects are negligible.
Unsteady Processes

• We will now consider two unsteady thermodynamic processes:
  – Charging (filling) of a rigid vessel
  – Discharging (emptying) of a rigid vessel

• Apply conservation of mass and energy to each case develop expressions for each process:
  – Derivation in class notes
Example - 9

High pressure CO$_2$ at 1.0 MPa and 57°C enters an initially evacuated rigid tank. The gas stops flowing when the pressure in the tank reaches that of the supply line. Assuming that the process occurs quick enough so that heat transfer with the surroundings is negligible, determine the final temperature of the gas inside the vessel if the volume of the tank is 0.5 m$^3$. How much mass entered the vessel?
Example - 10

- Air at 5 bar and 25 C is discharged from a vessel with a volume of 0.5 L. The gas stops flowing when the pressure in the vessel is reduced to 4.0 bar. Assuming that the process occurs quick enough so that heat transfer with the surroundings is negligible, determine the final temperature of the gas remaining inside the vessel. How much mass exited the vessel?
Thermodynamic Cycles

• We will now consider two fundamental thermodynamic cycles
  – Simple steam (vapor) power cycle
  – Simple gas power cycle
  – Simple vapor refrigeration cycle

• Basic steps in a cycle analysis:
  – Establish the thermodynamic states at inlet/exit of major components
  – Use energy/mass balances to close out the analysis as required
Steam Power Cycle

1. Boiler (\(Q_{in}\))
2. Turbine (\(\dot{W}_t\))
3. Condenser (\(\dot{Q}_{out}\))
4. Pump (\(\dot{W}_p\))

Cooling water flows through the system.
Gas Power Cycle

Compressor - Heat exchanger - Turbine

\[ \dot{Q}_{\text{in}} \]

\[ \dot{Q}_{\text{out}} \]

\[ W_{\text{cycle}} \]
Vapor Refrigeration Cycle

Faculty of Engineering and Applied Science
Memorial University of Newfoundland
St. John’s, Newfoundland, Canada
Measures of Performance

- Simple cycles are evaluated using an index of performance:
  - Efficiency (for power cycles)
  - Coefficient of Performance (refrigeration cycles)

\[
\eta_T = \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{W_{\text{turbine}} - W_{\text{pump}}}{Q_{\text{in}}}
\]

\[
COP = \frac{Q_{\text{in}}}{W_{\text{in}}} = \frac{Q_{\text{evaporator}}}{W_{\text{compressor}}}
\]

- In Chapter 5/6 we will address issues of maximum theoretical performance for a cycle
Example - 11

- A simple steam cycle uses water as a working fluid which circulates at 80 kg/s. The boiler pressure pressure is 6 MPa and the condenser pressure is 10 kPa. Steam enters the turbine at 600 °C and is discharged with a quality of 0.85 at 10 kPa. Saturated liquid at 10 kPa, is discharged from the condenser. Determine the power developed by the cycle and the thermal efficiency.
Example - 12

- Air at 100 kPa and 37°C enters an ideal adiabatic compressor with a compression ratio of 10:1 and a mass flow rate of 10 kg/s. The high pressure air is then heated in a combustion process at constant pressure which raises its temperature to 1227°C. The hot gas is then expanded through a turbine to 100 kPa and leaves at 497°C. Determine the net power generated and the thermal efficiency for this cycle.
Example - 13

• Refrigerant 22 enters the compressor of an ideal refrigeration cycle with a flow rate of 15 m³/min, and is compressed from a saturated vapor at 1 bar to a pressure of 9 bar and temperature of 60 °C. The refrigerant leaves the condenser as a saturated liquid and is expanded through a throttling valve to a pressure of 1.0 bar. Determine the compressor power, refrigeration capacity and coefficient of performance for the cycle.