Systematic Assessment of Rigid Internal Combustion Engine Dynamic Coupling

D. Geoff Rideout
Assistant Professor
Department of Mechanical Engineering,
S.J. Carew Building, Memorial University,
St. John's, NL, A1B 3X5, Canada
e-mail: gridoute@engr.mun.ca

Jeffrey L. Stein
Professor
Automated Modeling Laboratory,
Department of Mechanical Engineering,
University of Michigan,
G029 Auto Lab, 1231 Beal Avenue,
Ann Arbor, MI 48109-2121
e-mail: stein@umich.edu

Loucas S. Louca
Lecturer
Department of Mechanical and Manufacturing Engineering,
University of Cyprus,
75 Kallipoleos Street,
P.O. Box 20537,
1678 Nicosia, Cyprus
e-mail: lsouca@ucy.ac.cy

1 Introduction

Accurate prediction of engine vibration is important for the design of engine mounts, for prediction of loads transmitted to the vehicle frame, and for assessing whether vibration affects customer perceptions of quality.

Reciprocating engine components move within a block that has six degrees of freedom on its compliant mounts. Angular velocity of the block creates gyroscopic forces and torques on the reciprocating components within. These gyroscopic effects subsequently affect the forces and torques back on the block from the reciprocating components, the resulting motion of the block, and the mount forces transmitted to the vehicle frame. A "fully coupled" model, in which the reciprocation occurs within a block moving in three dimensions, can be computationally unwieldy [1]. To reduce model complexity and simulation time, mount forces have traditionally been predicted with a decoupled model as follows. First, reciprocating dynamics are simulated with a model in which the block is stationary. The required constraint forces and torques to keep the block stationary are then applied to a constant-inertia nonrunning model of the engine on its mounts.

Use of one-way coupled models is described in Refs. [1–3], where the modeler is cautioned against the use of "constant-inertia" models for predicting shaft speed and acceleration for all but low-speed or low-bandwidth control applications. Single- and six-cylinder models were constructed in Ref. [4] to show that the one-way coupled model should be restricted to well-balanced or low-speed engines, and only when the engine block is not subject to large pitch or yaw excitation by the vehicle.

Proper modeling [5] has been formally defined as the systematic determination of the model of minimal complexity that (a) satisfies the modeling objectives and (b) retains physically meaningful parameters and variables for design. Algorithms have been developed to help automate the production of proper models of dynamic systems. Recent work [6,7] has focused on systematically determining if partitions exist in a model, so that the use of one-way coupled models can be validated. Using an energy-based metric, an arbitrary, fully coupled model can be assessed to determine whether or not partitioning is possible without reliance on an assumed form of a one-way coupled model. Decoupled submodels, where appropriate, are automatically generated. If the system parameters or external inputs change, the intensity of the decoupling can be monitored and models of intermediate complexity can be automatically generated. If, for example, cylinder-to-cylinder variations are introduced in a low-speed balanced engine, or if the engine is simulated in a moving vehicle, the efficacy of a one-way coupled model can be monitored. If two-way coupling is detected, the algorithm identifies the specific constraint equations in which the decoupling breaks down, thus increasing physical insight into the system dynamics. Thus, it is the hypothesis of this...
work that application of these existing decoupling algorithms can lead to systematic decoupled engine models for specific user-defined conditions.

The paper is organized as follows. Section 2 describes the decoupling search and model partitioning algorithm, and the bond graph formalism that facilitates its execution, with the aid of an illustrative example. Section 3 describes the in-line six-cylinder engine fully coupled model (FCM), and Sec. 4 shows the results of applying the partitioning algorithm to a balanced and an unbalanced engine. Discussion and conclusions follow in Secs. 5 and 6.

2 Decoupling Search and Model Partitioning

As described in Ref. [6], partitions are defined as one-way coupled groups of dynamic elements that are created when negligible constraint equation terms are removed from a model. Individual constraint terms are assessed by examining their “activity” [8] relative to the maximum term activity in the equation. Activity over a time interval $t_1$ to $t_2$ is defined as

$$A = \int_{t_1}^{t_2} |P| dt$$

(1)

where $P$ is the instantaneous power, the product of generalized “effort” (e.g., force, torque, voltage, pressure) and “flow” (e.g., velocity, angular velocity, current, volume flow rate). Activity is always positive, and either constant or monotonically increasing, over a given time interval. Because of the use of the power-based activity metric, and the need to systematically search all model constraint equations for inactive terms, bond graphs are a convenient (but not required) formalism to generate the engine model prior to application of the partitioning algorithm.

2.1 Bond Graph Modeling Formalism. In bond graphs [9], generalized inertias and capacitances store energy as a function of the system state variables, sources provide inputs from the environment, and generalized resistors remove energy from the system. The state variables are generalized momentum and displacement for inertias and capacitances, respectively. The time derivatives of generalized momentum $p$ and displacement $q$ are generalized effort $f$ and flow $f$. Table 1 expresses the generalized power (effort and flow) variables and energy (momentum and displacement) variables in the terminology of common engineering disciplines.

Power-conserving elements allow changes of state to take place. Such elements include power-continuous generalized transformer (TF) and gyrator (GY) elements that algebraically relate elements of the effort and flow vectors into and out of the element. In certain cases, such as large motion of rigid bodies in which coordinate transformations are functions of the geometric state, the constitutive laws of these power-conserving elements can be state modulated. Dynamic force equilibrium and velocity summations in rigid body systems are represented by power-conserving elements called 1 and 0 junctions, respectively.

Sources represent ports through which the system interacts with its environment. The power-conserving bond graph elements—TF, GY, 1 junctions, 0 junctions, and the bonds that connect them—are collectively referred to as “junction structure.” Table 2 defines the symbols and constitutive laws of sources, storage and dissipative elements, and power-conserving elements in scalar form. Bond graphs may also be constructed with the constitutive laws and junction structure in matrix-vector form, in which case the bond is indicated by a double line.

Power bonds contain a half-arrow that indicates the direction of algebraically positive power flow, and a causal stroke normal to the bond that indicates whether the effort or flow variable is the input or output from the constitutive law of the connected elements. The constitutive laws in Table 2 are consistent with the placement of the causal strokes. Full arrows are reserved for modulating signals that represent powerless information flow such as orientation angles that determine the transformation matrix between a body-fixed and inertial reference frame.

The engine bond graphs that follow will contain bonds and elements with both scalar and vector governing equations. The reader is referred to Ref. [9] for a more thorough development of bond graphs.

Once a bond graph model of a system has been constructed, “conditioning” and “partitioning” algorithms are applied as summarized below with the aid of an illustrative example.

2.2 Model Conditioning and Partition Search Using Relative Activity. The algorithm (described in detail in Ref. [6]) consists of the following general steps, as depicted in Figs. 1 and 2.

Step 0. Construct a bond graph model of the system that can be considered the “full” model inasmuch as its complexity captures
the dynamics of interest. Record the outputs of interest from the full model for later comparison with reduced or partitioned models.

**Step 1: Relative Activity Calculation.** Calculate the activity of each bond in the graph, and compare, at each junction, the activity of each connected bond to the junction maximum. Low relative activity of bond $i$ at a junction implies that

i. for a 0 junction with $n$ bonds, the flow $f_i$ can be neglected in the flow constraint equation

$$\sum_{j=1}^{n} f_j = 0$$

ii. for a 1 junction with $n$ bonds, the effort $e_i$ can be neglected in the effort equation

$$\sum_{j=1}^{n} e_j = 0$$

Locally inactive bonds are defined as those with an activity ratio falling below a user-defined threshold:

$$\frac{A_i}{\max(A_j)} < \varepsilon$$

**Step 2: Conditioning.** “Condition” the bond graph by converting the bonds with negligible activity to modulated sources. Table 3 gives examples of the conversion implications for bonds depending on the junctions or elements to which they are attached. The variables $f_i$ and $e_i$ represent the flow and effort of the bond with label $i$. For the internal bond case, if the activity of Bond 1 is low compared to the other bonds at the 0 junction, but is on the order of the other bond activities at the 1 junction, then the flow input to the 0 junction can be eliminated. The effort $e_1$ input to the 1 junction is delivered by a modulated effort source. Conditioning the bond into the 0 junction removes a term from the flow constraint equation as shown. The presence of the modulated source does not affect the 1-junction equations in either example. If the bond activity is instead locally inactive at the 1 junction, then the bond is converted to a modulated flow source imposing $f_1$ on the 0 junction.

The table also shows examples of TF and GY bond conversion. As power-conserving elements, TF and GY elements have the same activities for Bonds 1a and 1b.

**Step 3: Subgraph Identification.** Identify bond subgraphs (collections of bond graph elements) that are connected by modulating signals instead of power bonds. Whereas a power bond is a conduit for two-way flow of information, the one-way modulating signals carry information from a locally “driving” element to a locally “driven.” If removing modulating signals from the model results in two or more separate bond graphs, then subgraphs have resulted from bond conditioning and the most important prerequisite for partitioning has been met.

**Step 4: Partitioning.** Identify driving and driven partitions—subgraphs between which all modulating signals carry information in the same direction (i.e., from subgraph $i$ to subgraph $j$). If desired, the analyst can reduce the model as follows. If the only outputs of interest are in driving partitions, eliminate the driven partitions. If an output of interest is associated with a driven partition element, replace its driving partition(s) with the necessary inputs (time histories) to excite the driven partition.

### 2.3 Illustrative Example

Figure 1 shows two mass-spring-damper subsystems connected by a lever (power-conserving TF). The lever provides both a velocity and a force constraint:

$$v_2 = v_1, \quad F_1 = \frac{b}{a} F_2$$

(2)

Writing Newton’s second law for $m_A$, and recognizing that the relative velocity of the endpoints of $k_A$ is equal to the mass velocity $v_A$ gives

$$m_A \ddot{v}_A + R_A v_A + k_A v_A + F_1 = 0$$

(3)

The preceding equation contains four force terms that are associated with common velocity $v_A$, suggesting a bond graph 1 junction with four bonds.

The following equation defines the relative velocity $v_{kb}$ of the endpoints of the spring $k_B$ (and parallel damper $R_B$):

$$v_{kb} = v_2 - v_B$$

(4)

These three velocities are associated with the combined spring and damper force $F_2$, suggesting a 0 junction with three bonds. Finally, we can associate the following effort equation terms with the spring velocity $v_{kb}$:

$$F_2 = k_B \int v_{kb} \, dt + R_B v_{kb}$$

(5)
Suppose the relative values of \( k_B, R_B, k_A, R_A \), and the lever ratio \( b/a \) render the lever force on \( m_A \) negligible compared to the inertial, spring, and damper forces. This would result in low relative activity of the \(-F_1\) term in Eq. (3). Suppose further that the transformed velocity \( v_A \) is essential for defining the velocities of \( m_B \) and \( x_B \) and thus the dynamic response of \( m_B \). The term \( v_2 \) would be locally active in Eq. (4). The implications for bond activity are described in Fig. 3.

A one-way coupled model would result, in which \( k_A, R_A \), and \( m_A \) could be simulated in isolation to predict the response of \( m_A \) and to generate \( v_A \). The velocity \( v_2 = (b/a)v_A \) could then be extracted from the \( m_A, k_A, R_A \) subsystem to excite the dynamics of \( m_B, k_B, R_B \). Figure 4 shows the conditioned bond graph and partitions resulting from low activity of the bond associated with the \(-F_1\) term in Eq. (3).

The partitioned state equations can be derived from the bond graph and are given below: Driven, 

\[
\begin{pmatrix}
\dot{v}_A \\
\dot{x}_A \\
\dot{v}_B \\
\dot{x}_B
\end{pmatrix} = 
\begin{bmatrix}
\frac{R_A}{m_A} - \frac{k_A}{m_A} \\
1 & 0 \\
\frac{R_B}{m_B} + \frac{k_B}{m_B} & 1 \\
0 & 1
\end{bmatrix} \begin{pmatrix}
v_A \\
x_A \\
v_B \\
x_B
\end{pmatrix}
\]

(7)

Driving,

\[
\begin{pmatrix}
\dot{v}_A \\
\dot{x}_A \\
\dot{v}_B \\
\dot{x}_B
\end{pmatrix} = 
\begin{bmatrix}
\frac{R_B}{m_B} & \frac{k_B}{m_B} \\
0 & 1 \\
-b & 0 \\
0 & 1
\end{bmatrix} \begin{pmatrix}
v_A \\
x_A \\
v_B \\
x_B
\end{pmatrix}
\]

(8)

The parameters \( b/a, m_B, k_B, R_B \) do not appear in Eq. (7). The driving output \( v_A \) is extracted from Eq. (7) and input to the driving subsystem in Eq. (8).

The partitions can be simulated sequentially or in a parallel computing environment, with each partition having a lower-dimension state space than the original model.

### 3 Engine Model Development

A Detroit Diesel Series 60 in-line six-cylinder four-stroke direct injection engine was modeled by Hoffman and Dowling in Ref. [4] to study the ability of a one-way coupled model to predict engine mount forces during steady-state operation, compared to a FCM. The one-way coupled model (OWCM) mount forces in Ref. [4] were generated by (1) simulating the engine with the block rigidly constrained to the inertial frame, (2) measuring the rigid constraint forces and torques, and (3) applying the constraint forces and torques to the nonrunning engine on its mounts. This application is revisited here as a test case for the conditioning and partitioning algorithms, given that a set of realistic engine parameters and measured combustion forces is available. The engine is run at low speed as in Ref. [4], without a large effective crankshaft inertia to represent the vehicle mass and gearing. Such a scenario would be useful for the study of engine and body vibrations in an idling or parked running vehicle, e.g., at a rest stop. An overall rotary damping rate was used to achieve the desired steady-state speed.

The in-line six-cylinder architecture is inherently balanced for the first and second engine rotational speed harmonics. The dominant component of the engine mount forces for an engine on a test stand is therefore third order. The OWCM in Ref. [4] showed single-digit percentage errors, compared to a FCM, in predicting the third-order force component for an engine with no cylinder-to-cylinder variation in mechanical parameters or combustion forces. As imbalance was added or actual measured combustion forces were used in Ref. [4], model predictive ability was reduced.

In order to predict mount forces without relying on assumption and intuition about the feasibility of decoupling, a new FCM of the engine was developed by the authors using the bond graph formalism, and was subjected to the conditioning and partitioning...
algorithms to

1. verify that decoupling could be systematically found without assuming the form of the OWCM
2. compare the resulting partitioned model with the OWCM in Ref. [4], and demonstrate the physical insight gained
3. quantify and locate the sites where decoupling became

validated by a significant cylinder-to-cylinder variation that would create imbalance.

The vector bond graph formulation for multibody systems [10] was chosen, in which velocity vectors are defined with respect to coordinate frames affixed to individual bodies. Using this formalism, a model based on the full model of Hoffman and Dowling [4] to represent the engine vibrations was constructed. The basic building blocks of this model are given in Ref. [11]. Figures 5–7 show schematics of the engine, individual cylinder slider-crank mechanisms, and the crankshaft. The respective body-fixed coordinate system origins and orientations are as shown. The block and crankshaft origins were both located at point O, and their respective coordinate systems were initially coincident, with the z axis upward through the bore of Cylinder 1. The “front” mount is located at the front of the engine (near Cylinder 1) along the vertical centerline, and the “left-rear” and “right-rear” mounts are located to the left and right of the block-fixed x-z plane beyond Cylinder 6. The mounts are modeled as three orthogonal linear springs with directions parallel to the block-fixed system.

These forces, when applied to the engine with an internal damping value of 30 Nm/s/rad at the main bearing, produced a steady-state speed of approximately 65 rad/s (620 rpm), as shown in Fig. 8.

The model parameters and combustion forces are from Ref. [12]. For the present case study, the Cylinder 1 combustion force versus crank angle curve was used for all cylinders, with the curves for Cylinders 2–6 shifted along the crank angle axis to mimic the firing order.

Parasitic elements (stiff spring and damper elements in parallel) were used in the model at all pin joints. This is a standard procedure to allow a small violation of the ideal rotational joint constraint such that an explicit set of ordinary differential equations can be derived [13]. Parasitic stiffnesses were tuned to suppress constraint violation displacements below 0.025 mm. In the description that follows, the left superscript indicates the reference frame number according to Fig. 6. The position or velocity of a body’s center of gravity is indicated by a numerical subscript, e.g., r1 locates the crankshaft center of gravity with respect to point O, in crankshaft-fixed coordinates. The position or velocity of an arbitrary point is indicated by a letter subscript, e.g., r C in Fig. 6.

Numerical simulations were performed within the 20SIM [14] bond graph simulation environment on a Pentium IV computer using the Vode–Adams variable-step stiff system integrator. Absolute and relative integration tolerances were set at 10⁻⁶. The Appendix lists model parameters.

4 Simulation Results for Balanced Engine

After applying the conditioning algorithm (Sec. 2.2, Steps 1 and 2) to determine local activities, partitions in the balanced engine model were identified for a local activity threshold of 0.3% of the maximum equation activity.

4.1 Balanced Engine Partitions. The partitions are summarized below, are shown schematically in Figs. 9 and 10, and are consistent with the notion of reciprocation about a fixed crankshaft axis as per the OWCM in Ref. [4].

“Reciprocating” (driving partition) contains the following:

• crankshaft spin axis degree of freedom and velocity \( \dot{\omega}_1 \)
• internal friction
• crankpin translational velocity in the crankshaft y-z plane
• coordinate transformations between crankshaft, connecting rods, and pistons in the y-z reciprocation plane
• connecting rod spin axis degree of freedom
• connecting rod inertial forces in the y-z plane
• piston motion and inertial forces in the z direction
• combustion forces
The absence of the $y$ and $z$ components of the crankshaft angular velocity implies that the crankshaft spin axis (and thus the connecting rod $y$-$z$ reciprocation planes) may be considered fixed. "Extended block" (driven partition) contains the following:

- complete engine block and main bearings from FCM
- crankshaft translational velocity; mass and inertial forces in the $x$, $y$, $z$ directions
- crankshaft $\omega_y$ and $\omega_z$ degrees of freedom
- connecting rod $\omega_y$, $\omega_z$, $v_x$ degrees of freedom; mass and inertial force in the $x$ direction
- piston angular motion; piston mass and inertial forces in the $x$ and $y$ directions
- sliding joint forces and moments on block in the $x$ and $y$ directions as a function of piston vertical displacement

95 driving outputs (and thus driven partition inputs) result from...
conversion of power bonds to modulating signals.

Figure 9 shows a “word bond graph” of the driving partition. One of the cylinder submodels is shown in its entirety, but the omitted cylinders are identical. Two-way power bonds have half-arrows, and one-way signals have full arrowheads. The block arrows represent signals from conditioned bonds, and carry the power variables associated with eliminated constraint terms. These are the outputs of the driving partition that are used as inputs to the driven partition. These outputs are typically written to a data file during simulation of the reciprocating dynamics, and then can be used later as inputs to the driven block partition.

The dominant power flow path is from the combustion force source, through the piston-fixed $z$ axis ($\mathbf{V}_{Dp}$), to translation of the connecting rod in its $y$-$z$ plane ($\mathbf{V}_{Cr}$) and rotation about its $x$ axis ($\mathbf{\omega}_{Cr}$), to rotation of the crankshaft about its longitudinal axis ($\mathbf{\omega}_{1}$), and transmission of forces to the block $y$-$z$ plane (“crank pin force $y,z$”).

The crankshaft and connecting rod orientation angles can be generated by integrating an angular velocity vector containing the spin component only. The physical interpretation is that the piston and connecting rod move in a reciprocating body-fixed $y$-$z$ plane that can be assumed fixed to the inertial $y$-$z$ frame.

The power flow paths within the driven partition are shown schematically via the partial bond graph of Fig. 10. Some individual elements are removed for clarity, including parasitic constraining springs and some coordinate transformation juncture structure. Again, double-lined power bonds are vector bonds with which velocity and force/torque vectors are associated, while single-lined bonds carry scalar power variables. Some causal strokes are shown to indicate where forces and torques are being imposed upon the block and crankshaft. Given that some bond graph structure is not included, the reader is cautioned against assigning causal strokes to all bonds of the graph.

The engine block, main bearing, and engine mounts submodels are unchanged from the FCM. The driven partition differs from the OWCM in Ref. [4] in that it is not a constant-inertia model of the block with a stationary crankshaft, connecting rods, and pistons. For example, the piston displacement $d$ is a required input to the driven partition. The varying piston height significantly changes the moments on the block about its $x$ and $y$ axes. In a constant-inertia driven OWCM, the piston displacements with respect to the block would remain constant.

The crankshaft translates with the block in the driven partition, as indicated by the vector bonds from the crankshaft velocity vector node ($\mathbf{V}_{1}$) to the mass and gravity force source. The crankshaft angular velocity components in the body-fixed $y$ and $z$ directions ($\mathbf{\omega}_{1y}$, $\mathbf{\omega}_{1z}$), are present in the driven partition, and create relative velocity contributions to the crankshaft and main bearings through the $\mathbf{\omega}_{1} x$ and $\mathbf{\omega}_{1} y$ block.

Table 4 describes the inactive terms according to the numbers inside triangles next to the block arrows in Fig. 9, and gives the physical interpretation of the resulting reduced constraint equation. Negligible terms are struck out in gray in the table. When interpreting Table 4, recall that an inactive bond at a velocity node...
represents a negligible term in a force/torque equation and vice versa. Bond conditioning removes the force/torque term from the equation and moves it to the driven dynamics. The node velocity then modulates the driven dynamics.

Figures 11 and 12 compare the front- and left-rear steady-state mount force predictions of the fully coupled and partitioned models. The partitioned model forces were generated by running the driving partition, saving the required driving signals to a file, and using the file to excite the driven partition containing the block and mounts. The right-rear mount forces are not shown, but are qualitatively similar to the left rear.

The front mount is characterized by lower vibration amplitudes due to its location in the x-z plane. The partitioned model predicts all the major vibrating components accurately, and also matches the dc offset of all components. Table 5 shows single-digit percentage differences between the fully coupled and partitioned models in predicting the magnitude of the dominant third-order vibration amplitudes.

4.2 Misfire Mount Force Prediction. Next, an imbalance-inducing combustion event is simulated by setting the Cylinder 1 combustion force to zero at all crank angles. Figure 13 shows the resulting engine speed variation. As a result of the misfire, the engine block motions are severe enough to affect the reciprocating forces and moments in the block-fixed frame and compromise decoupling. Relative activity calculation and bond graph conditioning, using the same 0.3% threshold as for the balanced engine, show that creation of two completely decoupled partitions is no longer possible. For illustrative purposes, Figs. 14 and 15 are provided to show that the original partitioned model nonetheless predicts several mount force components accurately. However, significant deviations in the predictions of the vertical (z) component of the front mount force arise. The dc component is closely predicted, but the variation about the mean is not. As expected, given the nature of the imbalance, first-order vibration components dominate the response as shown in the figures.

The relative activity measure also heightens physical insight by allowing the analyst to determine systematically which new constraint terms are required to predict the front mount z component effectively in the face of misfire. Consider the crankshaft, where the equations defining the velocities of the crank pins are partitioned for the balanced engine. Expanding the vector equation

\[ \dot{v}_C = \dot{v}_1 + \omega_1 \times r_C \]  

in scalar form, eliminating negligible terms, and partitioning, the components of crank pin velocity \( v_C \) are defined by Driving.

<table>
<thead>
<tr>
<th>Mount force component</th>
<th>Freq. (Hz)/Magnitude (N)</th>
<th>% error (Mag.)</th>
<th>Full model</th>
<th>Partitioned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x )</td>
<td>31/0.055</td>
<td>31/0.058</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td>31/78</td>
<td>31/80</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>( z )</td>
<td>31/4.2</td>
<td>31/4.25</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>63/0.69</td>
<td>62/0.69</td>
<td>3.5</td>
<td></td>
</tr>
<tr>
<td>Left rear</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x )</td>
<td>31/0.81</td>
<td>31/0.87</td>
<td>7.9</td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td>31/53</td>
<td>31/55</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>( z )</td>
<td>31/60</td>
<td>31/61</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>Right rear</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x )</td>
<td>31/0.90</td>
<td>31/0.97</td>
<td>8.0</td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td>31/53</td>
<td>31/55</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>( z )</td>
<td>31/62</td>
<td>31/63</td>
<td>1.7</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 13 Engine speed with misfire

Fig. 12 Partitioned model: left-rear mount forces

<table>
<thead>
<tr>
<th>Engine Speed (rpm)</th>
<th>Time (s)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Transactions of the ASME
Driven,

\[ \begin{pmatrix} v_{Cy} \\ v_{Cz} \end{pmatrix} = \begin{pmatrix} -r_{Cz} \\ r_{Cy} \end{pmatrix} \omega_{1z} \]  \hspace{1cm} (10) 

Note again the elimination of the crankshaft translational velocity components in Eq. (10). Introduction of the misfire requires that crankshaft translation terms be reinstated to predict the crank pin absolute velocities. Equation (10) expands to

\[ \begin{pmatrix} v_{Cy} \\ v_{Cz} \end{pmatrix} = \begin{pmatrix} v_{1y} \\ v_{1z} \end{pmatrix} + \begin{pmatrix} -r_{Cz} \\ r_{Cy} \end{pmatrix} \omega_{1z} \]  \hspace{1cm} (12)

as the relative activities of the translation terms increase from an average of 0.21% to 1.16%.

Local activity also shows that in the presence of misfire, the motion of the piston relative to the block becomes coupled to off-axis velocity components to a greater extent in Cylinder 1 than in other cylinders. The front mount is most affected given its proximity to Cylinder 1.

Note that conditioning the misfire model (i.e., removing unnecessary constraint equation terms without trying to separate the model into separate, decoupled partitions) using the original 0.3%
threshold can improve predictions of the unbalanced mount forces. Obviously, fewer bonds can be conditioned than in the balanced model, and complete partitioning is not possible. Figures 16 and 17 show the front- and left-rear mount force predictions for the misfire engine, using a model conditioned with a 0.3% threshold. Note the improvement over Figs. 14 and 15. While two-way coupling exists, many sites of local coupling are removed by elimination of negligible constraint terms. Model size is reduced and computational savings result, as described in Sec. 4.3

4.3 Model Size and Computation Time Comparison. Table 6 compares the model size and number of computation steps for the full, conditioned, and partitioned models of the balanced engine. Figure 18 shows the relative computation times. For the partitioned model, the individual times for sequential simulation of the driving and driven partitions sum to approximately one-half the computation time of the FCM. The small computation time for the driving partition is highlighted in the figure. The Vode–Adams absolute and relative integration tolerances were $10^{-6}$ for all runs.

<table>
<thead>
<tr>
<th>Model</th>
<th>Equations</th>
<th>Variables</th>
<th>States</th>
<th>Vode–Adams integration steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fully coupled</td>
<td>5682</td>
<td>6501</td>
<td>279</td>
<td>989,618</td>
</tr>
<tr>
<td>Conditioned</td>
<td>4881</td>
<td>5863</td>
<td>294</td>
<td>908,492</td>
</tr>
<tr>
<td>Partitioned</td>
<td>1688</td>
<td>1999</td>
<td>77</td>
<td>60,973</td>
</tr>
<tr>
<td>Partitioned driven</td>
<td>3452</td>
<td>4111</td>
<td>197</td>
<td>854,084</td>
</tr>
</tbody>
</table>

Fig. 16 Conditioned misfire model: front mount forces

Fig. 17 Conditioned misfire model: left-rear mount forces
Identification of partitions reduces the size of individual simulation models, which will typically bring computational gains as shown. Less obvious, but potentially very useful, is the fact that simply conditioning the model can break algebraic loops, reduce state dependencies, and improve computation time even without partition identification and elimination. The misfire mount forces of Figs. 16 and 17, generated with a conditioned model, reduced computation time by 39% compared to a FCM with misfire.

5 Discussion

The primary contention of this paper is that a recently developed partitioning algorithm can be used in the context of engine mount force prediction to yield, systematically, insight about the relative importance of different physical phenomena contained in a conventional vehicle engine model. It has been shown, based on a 0.3% relative activity threshold, that a decoupled model for a balanced engine predicts similar engine forces as the traditional OWCM in Ref. [4]. Though the systematically generated model is slightly more complicated than the traditional OWCM, it appears to be more robust to imbalance due to the larger number of inputs to the driven subsystem. For example, providing piston displacement to the driven partition allows forces on the cylinder walls to be calculated more accurately than in a nonrunning engine model with pistons constrained to their initial positions.

The existence of partitions depends on the arbitrary threshold value used. Loosening this tolerance (making the threshold numerically higher) would necessarily result in a more completely partitioned model. Setting a local activity threshold a priori can be difficult, meaning that some iteration may be necessary to find an acceptable trade-off between activity threshold and accuracy of the partitioned model. Unfortunately, an analytical determination of this threshold is unknown at this time. Future work will combine the conditioning algorithm with a quantitative model accuracy algorithm [15], with the goal of threshold setting based on tolerances on points of engineering relevance such as overshoot and rise time that can be identified from the full model output plots.

A major point of this paper is that the algorithms allow the decoupling to be easily rechecked for whatever scenario is of particular interest to the modeler. Given any set of engine parameters and inputs of interest, one can simply rerun the model and recalculate the bond relative activities. Had the engine been connected to a rolling vehicle model through a gearbox with internal friction, instead of to an idling vehicle or test stand, the engine speed variations of Figs. 8 and 13 would be lessened. Reduction of engine speed variation and filtering out of the effects of staggered combustion events would likely reduce coupling between reciprocating dynamics and block motion even further. Running the engine at higher speeds, with more severe combustion events, while filtering out engine speed variations with a large effective crankshaft inertia and damping, may create higher local activities in some of the partition boundary bonds. As in the misfire scenario, continued existence of partitions, and locations where decoupling breaks down, can be easily checked.

Computational savings are reported in the case study. Computational savings become especially significant for design processes in which an optimization algorithm may execute the model thousands of times. Given that as the engine parameters and running condition change, the partition boundary bonds may become active; the user may wish to recheck a reduced model periodically by resimulating the full system. Aggregate computation time can still be significant even if an occasional run must be made with a model that is needlessly complex, but is used to adjust the reduced design model.

The physical insight gained by the proposed easy-to-use systematic partitioning algorithm is also a valuable attribute of the technique. For engine vibration, the specific constraint equation terms related to partitionability, or lack thereof, can be identified. A spectrum of models of intermediate complexity is suggested by “reinstated” power bonds as imbalance and/or speed increase, and the modeler can get a physical sense of where decoupling erodes. In the traditional OWCM which has an a priori assumed form, the jump from full to reduced model is made with nothing in between. Intermediate models which exhibit some two-way coupling, but which have many conditioned bonds (eliminated constraint terms), are shown in the case study to be useful and efficient.

Future engine vibration case studies can now be done with the partitioning method, for example, a study of the effect of vehicle motion on the coupling between engine reciprocating dynamics and block motion. The authors in Ref. [4] note the potential for vehicle pitch motion to cause gyroscopic coupling in engines with heavy crankshafts that are aligned with the vehicle’s longitudinal axis.

The algorithm was developed using the bond graph modeling language. While facilitating the calculation and comparison of activity, elimination of negligible terms, reformulation of the model, and identification of partitions, the use of graphical modeling formalisms is not a theoretical prerequisite for applying the algorithm.

For the large model in question, equation formulation by the SIM processor was easier when parasitic elements (stiff springs) were used to create explicit ordinary differential equations. Accuracy of the model with parasitic elements was ensured by tuning the stiffnesses so that no joint constraint violation exceeded 0.025 mm. As reported in Ref. [16], parasitic springs can be tuned by suppressing their relative activity below a tight threshold so that the elements do not participate noticeably in the system dynamics. Bonds to these elements would obviously not be subject to conditioning. Individually tuning the elements for maximum compliance will minimize numerical stiffness and help reduce computation time. Note, however, that the relative activity partitioning method can just as easily be used (and has been used) for models with ideal joint constraints that yield a set of differential-algebraic equations.
6 Summary and Conclusions

In response to the ongoing need to systematically generate accurate models with minimal complexity for simulation-based engineering design of engines and other vehicle systems, an energy-based partitioning algorithm has been applied to the problem of determining whether or not recoupling dynamics can be decoupled from engine block motion in predicting mount forces.

An in-line six-cylinder Diesel engine model was constructed, first with perfect balance and then with a simulated misfire. Activity, an aggregate measure of power flow, was used to identify negligible constraint equation terms within the model. The bond graph formalism facilitated the removal of negligible terms, the subsequent creation of local one-way coupling sites, and the search for separate partitions between which one-way information flow occurred. For the balanced engine, a partitioned model was generated that was able to predict mount forces accurately and with significant computational savings. The partitioned model was qualitatively similar to traditional ad hoc OWCMs; however, its varying inertia made it more robust for mount force predictions in the face of imbalance. Using the partitioned model to predict unbalanced engine mount forces gave better results than the ad hoc models. The algorithm suggested that partitions no longer existed based on the original constraint activity threshold, and indeed some mount force component predictions were significantly degraded. The algorithm pinpointed specific locations within the equation model where decoupling broke down. Running the algorithm after system inputs were changed automatically generated a model of intermediate complexity compared to the fully coupled and partitioned models. The intermediate model was more computationally efficient than the FCM despite the fact that partitions were not created.

The partition search method described herein facilitates monitoring of decoupling strength throughout the design process as parameters and inputs change. Physical insight into the system dynamics is heightened, and computation time can be reduced even when system dynamics cannot be entirely decoupled.

Appendix: Model Parameters

Table 7 Mass properties

<table>
<thead>
<tr>
<th>Body</th>
<th>Mass (kg)</th>
<th>Inertia (kg m²)</th>
<th>IXX, IYY,IZZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine block</td>
<td>1207</td>
<td>122.3, 238.6, 233.1</td>
<td></td>
</tr>
<tr>
<td>Crankshaft</td>
<td>300</td>
<td>4.23, 70.38, 70.38</td>
<td></td>
</tr>
<tr>
<td>Connecting rod</td>
<td>6.32</td>
<td>0.0888, 0.0865, 0.0070</td>
<td></td>
</tr>
<tr>
<td>Piston</td>
<td>5.08</td>
<td>0.0120, 0.0120, 0.0005</td>
<td></td>
</tr>
</tbody>
</table>

Table 8 Mount parameters

<table>
<thead>
<tr>
<th>Mount</th>
<th>Stiffness (kN/m)</th>
<th>Damping (N s/m)</th>
<th>Location rel. to o (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kx, Ky, Kz, Rx, Ry, Rz</td>
<td></td>
<td>[x, y, z]</td>
</tr>
<tr>
<td>Front</td>
<td>193.1, 193.1, 1410</td>
<td>2340, 2340, 6340</td>
<td>[-0.318, 0, -0.197]</td>
</tr>
<tr>
<td>Left Rear</td>
<td>314.5, 314.5, 157.0</td>
<td>2990, 2990, 2110</td>
<td>[1.000, -0.318, 0]</td>
</tr>
<tr>
<td>Right Rear</td>
<td>314.5, 314.5, 157.0</td>
<td>2990, 2990, 2110</td>
<td>[1.000, 0.318, 0]</td>
</tr>
</tbody>
</table>

Table 9 Geometric parameters

<table>
<thead>
<tr>
<th>Body</th>
<th>Point</th>
<th>Location rel. to o (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block</td>
<td>A</td>
<td>[-0.0830, 0, 0.0]</td>
</tr>
<tr>
<td>Crankshaft</td>
<td>A</td>
<td>[0.4290, 0.0, 0.0]</td>
</tr>
<tr>
<td></td>
<td>C1</td>
<td>[0.1651, 0.0693, -0.0400]</td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td>[0.4920, 0.0, 0.0]</td>
</tr>
<tr>
<td></td>
<td>C3</td>
<td>[0.4953, -0.0693, -0.0400]</td>
</tr>
<tr>
<td></td>
<td>C4</td>
<td>[0.6604, 0.0693, -0.0400]</td>
</tr>
<tr>
<td></td>
<td>C5</td>
<td>[0.8255, 0, 0.0800]</td>
</tr>
</tbody>
</table>

References