Keywords: Drilling, vibration, lumped-segment model, stick-slip, bit-bounce, optimal control, bit-rock interaction.

Abstract
Oilwell drillstrings sometimes vibrate severely and can twist off in hard rock drilling. Stick-slip particularly predominates when drilling with polycrystalline diamond compact (PDC) bits, which may also excite severe axial and lateral vibrations in the bottom hole assembly, causing damage to the drillstrings and downhole equipment. Controlling these vibrations is essential to improving the efficiency and minimizing the cost of drilling. A bond graph model of a drillstring has been developed that predicts axial vibration, torsional vibration, and coupling between axial and torsional vibration due to bit-rock interaction. Axial and torsional submodels use a lumped-segment approach, with each submodel having a total of 21 segments to capture vibration of the kelly, drill pipes, and drill collars. In addition, the model incorporates viscous damping, hydrodynamic damping, and hydraulic forces due to drilling mud; an empirical treatment of rock-bit interaction, and top drive motor dynamics. The model predicts the expected coupling between weight on bit (WOB), bit speed, and rock-bit interface conditions; and their effect on stick-slip. Low bit speed and high WOB cause stick-slip. Mitigating open-loop measures used in the drilling industry (increasing rotary speed and decreasing WOB through changing derrick cable tension) were applied to the model, and successfully eliminated stick-slip. A linear quadratic regulator (LQR) controller was then implemented which controlled stick slip and eliminated bit bounce.

1 Introduction
Deep wells for the exploration and production of oil and gas are drilled with a rock-cutting tool driven from the surface by a slender structure of pipes, called the drillstring (Fig. 1). Drillstring vibration is one of the major causes of deterioration of drilling performance in deep well applications. Bit-formation interaction has been recognized as a major cause of drillstring vibration. Bit induced vibration occurs in various forms namely whirl, stick-slip, and bit-bounce. However, stick-slip predominates when drilling with drag bits (especially with PDC bits) and stick-slip oscillations induce large cyclic stresses, which can lead to fatigue problems, reduction of bit life, unexpected changes in drilling direction, and even failure of the drillstring. Stick-slip vibration has received considerable attention in recent years with increasing use of PDC bits in harder formations, and has motivated extensive research on this type of drillstring vibration.

Several dynamic formulations have been reported for investigating specific aspects of drillstring vibrational behavior and few of them have tackled stick-slip. One of the major difficulties in modeling stick-slip stems from the inaccurate description of some involved parameters and downhole boundary conditions. Leine et al. [1] presented a stick-slip whirl model which consists of a submodel for the whirling motion and a submodel for the stick-slip motion. The stick-slip whirl model was a simplification of drilling confined in a borehole with drilling mud. Their model was a low-dimensional model and it aimed at explaining the basic nonlinear dynamic phenomena observed in downhole experiments. The model system was analyzed with the discontinuous bifurcations method which indicates physical phenomena such as dry friction, impact and backlash in mechanical systems or diode elements in electrical circuits which are often studied by means of mathematical models with some kind of discontinuity. The disappearance of stick-slip vibration when whirl vibration appears was explained by bifurcation theory. Stick slip was prevalent at low angular velocity and backward whirl was prevalent for high angular velocity consistent with the measurements. They did not consider the effect of axial vibration and rock-bit interaction in the model.

Christoforou and Yigit [2,3] used a simple dynamic model to simulate the effects of varying operating conditions on stick-slip and bit bounce interactions. The equations of motion of such a system were developed by using a simplified lumped parameter model with only one compliance. This model did not account for the effect of

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higher modes, the flow inside and outside the drillpipe and collars, or complicated cutting and friction conditions at the bit/formation interface.

Richard et al. [4,5] studied the self-excited stick-slip oscillations of a rotary drilling system with a drag bit, using a discrete model which takes into consideration the axial and torsional vibration modes of the bit. Coupling between these two vibration modes took place through a bit-rock interaction law which accounted for both frictional contact and cutting processes at the bit-rock interface. The cutting process introduced a delay in the equations of motion which was ultimately responsible for the existence of self-excited vibrations, exhibiting stick-slip oscillations under certain conditions. One of the limitations of their model is that the simulation stops when the bit lifts off. Furthermore, their model reduced the drillstring to a two degree of freedom system and they were working to capture more modes of vibration.

Recently Khulief et al. [6] formulated a finite element dynamic model of the drillstring including the drill pipes and drill collars. The model accounted for the torsional-bending inertia coupling and the axial-bending geometric nonlinear coupling. In addition the model accounted for the gyroscopic effect, the effect of the gravitational force field, and stick-slip interaction forces. Complex modal transformations were applied and reduced-order models were obtained. The finite element formulation was then integrated into a computational scheme for calculating the natural frequencies of the whole drillstring. The computational scheme was extended further to integrate the equations of motion, either in the full-order or the reduced-order form, to obtain the dynamic response. MATLAB™ was used as a simulation tool. They did not consider hydrodynamic damping, due to drilling fluid circulation in the drill pipe and the annular space, in their model. Stick-slip interaction was giving a coupling between axial and torsional vibration but [6] did not have any discussion about the complex effect of bit rotary speed and threshold force on torque on bit.

This paper presents a bond graph model of the whole drillstring including both drill pipes and drill collars. In addition to the axial vibration, torsional vibration, and axial-torsional coupling due to rock-bit interaction, the developed model accounts for the self weight effect, the associated tension and compression fields, viscous damping, hydrodynamic damping, and hydraulic forces due to drilling mud within the drillstring; an empirical treatment of rock-bit interaction, and top drive motor dynamics. The main contribution of this work is a model suitable for parametric study of the effect of table rotary speed and weight on bit on stick-slip vibration, and the coupling between stick-slip and bit bounce. Active control of the rotary table to suppress stick-slip and bit bounce is also implemented.

2 Modeling of Oilwell Drilling System

The system being modeled consists of drill pipes, the drill collar assembly (made up of heavier collar pipes), the drill bit at the end of the collar assembly and the rock (formation). Drilling fluid is circulated in the drill pipe and the annular space between the drill pipe and the well bore. The drilling fluid is characterized by the flow rate developed by the mud pumps. The top of the drillstring is subject to a tension force, applied through the surface cables. Rotary motion is applied by an armature-controlled motor, through a gearbox, to the rotary table via the kelly (a square, hexagonal or octagonal shaped tubing that is inserted through and is an integral part of the rotary table that moves freely vertically while the rotary table turns it). In this study, a DC motor with winding inductance and resistance is assumed. The essential components of the oilwell drilling system and the necessary geometry used for the model are shown in Fig. 1. A lumped-segment approach is used in the axial and torsional dynamic models. In the lumped segment approach, the system is divided into a number of elements, interconnected with springs. This method is a more cumbersome bond graph representation and the accuracy of the model depends on the number of elements considered; however, analytic mode shapes and natural frequencies need not be determined.

Fig. 1 Oilwell drilling system (adapted from [1])
2.1 Modeling of Axial Dynamics

A total of 21 segments are used in the dynamic model to capture the first eight axial natural frequencies of the whole drillstring. One segment is used for the relatively short kelly, and the kelly model is shown in Fig. 2. For both drill pipe and collar, 10 segments are used in the model, and a drill pipe/collar bond graph model segment is shown in Fig. 3. Hydraulic forces are included at the top of the drill collar and bottom of the drillstring to capture the effect of drilling mud density. Hydrodynamic damping, due to drilling fluid circulation in the drill pipe and the annular space, is considered in the drill pipe and collar model instead of viscous damping [8].

2.2 Modeling of Torsional Dynamics

Similarly a total of 21 segments are used in the dynamic model to capture the first eight torsional natural frequencies of the whole drillstring. The number of segments for the kelly, drill pipe and drill collar is the same as in the torsional model. Fig. 4 and 5 depict torsional dynamic submodels for kelly and drill pipe/collar segments. The drill pipe and drill collar dynamic models consider viscous damping which results from the contact between drillstring surfaces and drilling fluid [8].

2.3 Coupling Between Axial and Torsional Dynamics

The bit-rock interaction provides coupling between axial and torsional drillstring dynamics. In this present work a quasi-static rock-bit model is used instead of a computationally intensive and difficult-to-parameterize complete dynamic representation. Yigit and Christoforou [2,3] have shown a static rock-bit interaction model in a drillstring represented using only two inertias and one compliance for both axial and torsional vibration. Their model is modified as described below. The original model in [3] assumed both friction and cutting torque regardless of whether or not dynamic weight on bit was sufficient to create penetration and cuttings. Depth of cut was a function of average rather than instantaneous rotation speed, along with rate of penetration. Rate of penetration was a function of average rotation speed and a constant applied weight on bit (WOB), rather than dynamic weight on bit. This paper incorporates threshold force and the effect of instantaneous WOB and bit rotation speed on cutting torque on bit (TOB). Below a threshold force \( W_f \), the drill tool does not penetrate into the rock, leaving only friction as a source of TOB. The model equations are presented in two parts. First, the dynamic WOB, which is the axial force applied at the bit under dynamic conditions is given as in [3]

\[
WOB = \begin{cases} 
 k_c (x - s) & \text{if } x \geq s \\
 0 & \text{if } x < s 
\end{cases} 
\]  

(1)

where \( k_c \) and \( s \) indicate formation contact stiffness and bottom-hole surface profile. Surface profile is given as [3]

\[ s = s_0 f(\theta) \]  

(2)
The formation elevation function \( f(\phi) \) is chosen to be sinusoidal as in [3], \( f(\phi) = \sin b \phi \), where \( b \) indicates bit factor which depends on the bit type. The term \( \phi \) indicates rotational displacement of the bit.

The total torque on bit (TOB) is related to frictional and cutting conditions, and dynamic WOB. When bit rotary speed is in the positive direction then TOB can be written as

\[
\text{TOB} = \begin{cases} 
\text{TOB}_f + \text{TOB}_c & \text{WOB} > W_{fs} \\
\text{TOB}_f & \text{WOB} \leq W_{fs}
\end{cases}
\] (3)

In the case of zero bit rotary speed

\[
\text{TOB} = \begin{cases} 
\text{TOB}_c & \text{WOB} > W_{fs} \\
0 & \text{WOB} \leq W_{fs}
\end{cases}
\] (4)

Finally for negative bit rotary speed

\[
\text{TOB} = \text{TOB}_f
\] (5)

where \( \text{TOB}_f \) and \( \text{TOB}_c \) represent frictional and cutting torque on bit and both are calculated as below,

\[
\text{TOB}_f = (\text{WOB}) r_b \mu(\dot{\phi})
\] (6)

\[
\text{TOB}_c = (\text{WOB}) r_b \frac{\delta_c}{r_b}
\] (7)

The term \( \dot{\phi} \) indicates instantaneous bit rotary speed, and the function \( \mu(\dot{\phi}) \) characterizes the friction process at the bit and it is given as [3]

\[
\mu(\dot{\phi}) = \mu_0 \left( \tanh \frac{\phi}{\delta_c} + \frac{\alpha \phi}{1 + \beta \phi^2} + \gamma \phi \right)
\] (8)

where \( \mu_0, \alpha, \beta, \gamma, \) and \( v \) are the experimentally-determined parameters of the frictional model. In equation (7) the terms \( \eta_c \) and \( \delta_c \) indicate bit radius and depth of cut per revolution, the latter given as

\[
\delta_c = \frac{2\pi \text{ROP}}{\dot{\phi}}
\] (9)

The instantaneous rate of penetration (ROP) is a function of dynamic WOB, instantaneous bit speed \( \dot{\phi} \), and rock/bit characteristics. The modified ROP equation from [3] can be written as

\[
\text{ROP} = C_1 \text{WOB} \sqrt{\dot{\phi}} + C_2
\] (10)

where \( \xi, C_1 \) and \( C_2 \) characterize the cutting action at the bit and depend on the type of the bit and formation.
3 Simulation Data

The bond graph model of the rotary drilling system is shown in Fig. 6. Table 1 summarizes all relevant data that is used in the current simulation.

<table>
<thead>
<tr>
<th>Drillstring data</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Cable and derrick spring constant</td>
<td>9.3e+06 N/m</td>
</tr>
<tr>
<td>Swivel and derrick mass</td>
<td>7031 kg</td>
</tr>
<tr>
<td>Kelly length</td>
<td>15 m</td>
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<tr>
<td>Kelly outer diameter</td>
<td>0.379 m</td>
</tr>
<tr>
<td>Kelly inner diameter</td>
<td>0.0825 m</td>
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<tr>
<td>Drill pipe length</td>
<td>2000 m</td>
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<tr>
<td>Drill pipe outer diameter</td>
<td>0.101 m (4 in)</td>
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<tr>
<td>Drill pipe inner diameter</td>
<td>0.0848 m (3.34 in)</td>
</tr>
<tr>
<td>Drill collar length</td>
<td>200 m</td>
</tr>
<tr>
<td>Drill collar outer diameter</td>
<td>0.171 m (6.75 in)</td>
</tr>
<tr>
<td>Drill collar inner diameter</td>
<td>0.0571 m (2.25 in)</td>
</tr>
<tr>
<td>Drill string material</td>
<td>Steel</td>
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<tr>
<td>Wellbore diameter</td>
<td>0.2 m</td>
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</table>

<table>
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<tr>
<th>Drift bit-rock data</th>
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</tr>
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<tbody>
<tr>
<td>Bit type</td>
<td>PDC (Single cutter)</td>
</tr>
<tr>
<td>Drill bit diameter</td>
<td>0.2 m (7.875 in)</td>
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<tr>
<td>Drill bit mass</td>
<td>65 kg</td>
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<td>Rock stiffness</td>
<td>1.16e+09 N/m</td>
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<td>Rock damping</td>
<td>1.5e+05 N.sec/m</td>
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<td>Surface elevation amplitude $s_0$</td>
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<td>Bit factor, $b$</td>
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<tr>
<td>Cutting coefficient $\xi$, $C_1$, $C_2$</td>
<td>1, 1.35e-08, -1.9e-4</td>
</tr>
<tr>
<td>Frictional coefficient $\mu_0, \alpha, \beta, \gamma$ &amp; $\nu$</td>
<td>0.06, 2, 1, 1 &amp; 0.01</td>
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<td>Threshold force, $W_{fs}$</td>
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<table>
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<tr>
<td>Mud fluid density</td>
<td>1198 kg/m³</td>
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<tr>
<td>Mud flow rate, $Q$</td>
<td>$Q_m + Q_s \sin(\omega t)$</td>
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<td>Mean mud flow rate, $Q_m$</td>
<td>0.022 m³/sec</td>
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<td>Mud flow pulsation amplitude, $Q_a$</td>
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<tr>
<td>Freq. of variation in mud flowrate, $q$</td>
<td>25.13 rad/sec</td>
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<td>Equivalent fluid viscosity for fluid resistance to rotation $\mu_e$</td>
<td>30e-03 Pa.sec</td>
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<td>Weisbach friction factor outside drill pipe or collar, $\alpha_a$</td>
<td>0.045</td>
</tr>
<tr>
<td>Weisbach friction factor inside drill pipe or collar, $\alpha_p$</td>
<td>0.035</td>
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<th>Motor data</th>
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</thead>
<tbody>
<tr>
<td>$L$, $K_m$, $n$ and $R_m$</td>
<td>0.005 H, 6 V/s, 7.2 and 0.01 $\Omega$</td>
</tr>
</tbody>
</table>

4 Stick-slip and Bit-bounce Interaction

The main objective of the current simulations is to study stick-slip vibrations and the effect of this vibration on bit-bounce. During bit-bounce the drill bit alternately separates from and impacts the rock surface in the longitudinal direction during drilling. When the bit is off-bottom the critical frequencies for axial resonances are found to be 2.3, 8.7, 15.7, 20.9, 25.7, 32.7, 40.0, 47.3 rad/sec...etc.; and for torsional resonances are found to be 1.2, 5.3, 10.1, 14.9, 19.8, 24.4, 28.9, 33.3 rad/sec...etc. When the bit is in contact with rock (on-bottom condition) the critical frequencies for axial resonance are found to be 7.7, 15.1, 20.5, 25.1, 31.8, 37.9, 42.1, 48.0 rad/sec...etc. It was found that for axial vibrations the frequencies 31.8, 37.9 and 42.1 rad/sec gave the greatest increases in dynamic forces at the bit. When the bit rotary speed reached that critical frequency range then high dynamic forces at the bit or bit bounce resulted as in Figs. 7-10.

Fig. 7 High stick-slip vibrations with bit-bounce at 13 rad/sec rotary table speed and 100 kN applied WOB

Fig. 8 Stick-slip with high bit-bounce at 30 rad/sec rotary table speed and 100 kN applied WOB

Fig. 7 shows the simulation results when the desired rotary table speed is 13 rad/sec with 100 kN applied WOB. Table speed is outside the critical frequency range mentioned above. Though the motor appears to control the rotary table speed as desired, the bit experiences large fluctuations evolving into a limit cycle. As mentioned earlier, this behavior is known as stick-slip oscillation [1-6]. The stick-slip vibration of the drillstring is characterized by alternating stops (during which the bit sticks to the rock)
and intervals of large angular speed of the bit. When the bit speed fluctuation approaches the critical speed range mentioned above, bit bounce occurs as demonstrated in Fig. 8 where dynamic WOB periodically becomes zero. Reducing the applied WOB and increasing rotary table speed is a standard technique to help alleviate torsional problems [3].

Fig. 8 shows the simulation results when the desired table speed is 30 rad/sec at 100 kN applied WOB. Although stick-slip vibrations appear in the figure the time interval of stick decreases and the bit speed experiences smaller fluctuation as a proportion of the mean. The peak speed of the bit is approximately two times the desired speed whereas in Fig. 7 it was approximately three times the desired speed. Bit-bounce appears more predominant than in the previous figure due to bit speed entering the critical speed range mentioned above.

Fig. 9 shows both stick-slip and bit-bounce completely eliminated by increasing rotary table speed to 75 rad/sec. A nearly constant steady-state bit rotation speed is attained. This is due to the positive slope of the friction behavior curve discussed earlier [3]. At very low speed the transition from static to kinetic friction coefficient causes a drop in the frictional torque and the negative slope causes instability in torsional motion. At high speed the slope of frictional torque is found to be positive and suppresses torsional instability. Bit-bounce is eliminated due to bit rotary speed being out of the critical speed range mentioned above.

Fig. 10 shows the simulation results when the applied WOB is 50 kN at 30 rad/sec rotary table speed. Stick-slip vibration reduces due to decreasing the applied WOB, but it increases the bit-bounce vibrations compared to Fig. 8.

From simulation results it is found that by decreasing applied WOB and increasing desired table rotary speed beyond a threshold it may possible to eliminate stick-slip vibrations. The results obtained are in excellent agreement with the actual drilling optimization workflow in the field [9]. By avoiding critical speed ranges it may possible to eliminate bit-bounce. However, increasing the rotary speed may cause lateral vibration problems, such as backward and forward whirling. Decreasing applied WOB may not be a desirable solution as it will result in reduced rate of penetration (ROP). Active control will be investigated in the next section as a means of eliminating stick-slip and bit bounce without affecting drilling performance or worsening other modes of vibration.

5 Controller Design

In control design an optimal controller using the Linear Quadratic Regulator (LQR) method can be designed in which the state feedback gain matrix [K] is selected to eliminate stick-slip vibration in drillstring. To reduce the dimension of the state vector and to maximize the number of states that could be physically measured, a simplified lumped parameter torsional model (Fig. 11) is used instead of taking 21 segments. The state space equation of the simplified model in Fig. 11 is

$$\dot{X} = AX + BV_c$$

where $X$, $A$, and $B$ are the state vector, coefficient, and input matrices, respectively:

$$A = \begin{bmatrix}
-\frac{R_m}{L} & 0 & -\frac{nK_m}{L} & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
\frac{nK_m}{J_k} & 0 & -C_v & 0 & 0 \\
\frac{1}{L} & 0 & \frac{1}{L} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{K_t}{J} & -\frac{C_v}{J}
\end{bmatrix}$$

The control matrix $B$ is given by:

$$B = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}$$

The vector $V_c$ contains the control actions to be applied.

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\[ X^T = \begin{bmatrix} I & \Theta_{rt} & (\Theta_{rt} - \Theta) & \Theta \end{bmatrix} \]

\[ B^T = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \]

(13)

(14)

The main goal here is not to place the new closed loop poles at an exact specified location at any cost, but to minimize the vibration state variables and the control effort. The control problem is to find out the necessary gain vector \([K]\) that will minimize the following performance index [2]:

\[ C = \frac{1}{2} \int_0^x (x^T Q x + r V_c^2) \, dt \]

where \(Q\) is a weighting matrix chosen to reflect the relative importance of each state and \(r\) is a weighting factor to adjust the control effort. If the desired state vector is \(x_d\) then the resulting optimal control input (rotary table motor voltage) can be written as

\[ V_c = V_{ref} - K(x - x_d) \]

(16)

The gain matrix, \(K\) can be written as

\[ K = r^{-1} B^T P \]

(17)

where \(P\) is the symmetric, positive-definite solution matrix of the algebraic Riccati equation given by

\[ A^T P + PA - r^{-1} P BB^T P + Q = 0 \]

(18)

If \(V_{ref}\) is a constant reference voltage applied to maintain the desired speed \(\omega_d\) at steady state in the absence of any disturbance then the control voltage necessary to keep the torsional vibrations zero while maintaining a desired bit and rotary table speed is given by [2]:

\[ V_c = V_{ref} - K_1 l - K_2 (\Theta_{rt} - \omega_l t) - K_3 (\Theta_{rt} - \Theta) - K_4 (\Theta_{rt} - \Theta) - K_5 (\Theta - \omega_d) \]

(19)

Using MATLAB, the state feedback gain matrix \(K\) is calculated and the result is

\[ K = [0.0285 \quad 4.588 \quad 1.809 \quad 35.165 \quad 10.74] \]

(20)

where \(R = 950\)

\[ Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2000 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 80000 & 0 \\ 0 & 0 & 0 & 0 & 950000 \end{bmatrix} \]

Fig. 11 (a) Physical schematic of model used for control design. (b) Bond graph torsional model using simplified lumped parameter model

Fig. 12 Stick-slip and bit-bounce vibrations eliminated by rotary table control at 13 rad/sec table speed and 100 kN applied WOB

5.1 Results

The gains from equation (20) are used in the high order model for simulation. Fig. 12 shows the response of the high order combined axial-torsional model discussed earlier, when rotary table controller is active at the simulation time of 40 second, for the case of 100 kN applied WOB and a desired speed of 10 rad/sec. As can be seen,
stick-slip vibration is controlled and a smooth drilling condition is achieved. At the same time the controller eliminates high dynamic force at the bit compared to Fig. 7, and there is no bit bounce.

Except for the bit speed, all other quantities in the controller can easily be measured or determined. The bit speed measurement requires downhole equipment and while feasible, it may be the most challenging controller implementation task as discussed in [2, 3].

6 Conclusions

Development and application of a bond graph model of a drillstring using a lumped segment approach has been presented. The proposed dynamic model includes the mutual dependence of axial and torsional vibrations, and coupling between axial and torsional vibration due to bit-rock interaction. While the top drive motor dynamics

\[ K_t = \text{Torsional stiffness, N-m-rad} \]

\[ L = \text{Motor inductance, H} \]

\[ n = \text{Gear ratio} \]

\[ R_m = \text{Armature resistance, } \Omega \]

\[ C_v = \text{Viscous damping coefficient, N-m-s-rad} \]

\[ \dot{\omega}_r = \text{Rotary table speed, rad/sec} \]

\[ \phi_r = \text{Rotary table angular displacement, rad} \]

\[ r_w = \text{Wellbore radius, m} \]

\[ r_0 = \text{Drill pipe/collar outer radius, m} \]

\[ r_i = \text{Drill pipe/collar inner radius, m} \]

\[ \omega_n = \text{rotary speed of } n^{th} \text{ cell, rad/sec} \]

use of a different type of motor would require rederiving the LQR matrices and gains; however, the procedure outlined herein would still apply. Simulation results show the same qualitative trends as field observations regarding stick-slip oscillations and their relationship to rotary speed, weight on bit, and bit bounce. These vibrations are self excited, and they generally disappear as the rotary speed is increased beyond a threshold value and the applied weight on bit decreases. However, increasing rotary speed may cause lateral problems and decreasing applied weight on bit decreases rate of penetration. The model also includes a state feedback controller that was designed based on a linear quadratic regulator (LQR) technique. It has been shown that the proposed control can be effective in suppressing stick-slip oscillations once they are initiated, and provides a possibility to drill at lower speeds. The simulation time is very fast compared to high order finite-and discrete-element models, making the model suitable as a tool for design and sensitivity analysis. An experimental drilling facility, currently under development, will be used to parameterize the model for various types of rock and bottom-hole pressure conditions, thereby increasing its predictive ability for design and optimization of the drillstring, vibratory tool, shock absorbers, and controllers.

Nomenclature

\[ C_{rt} = \text{Equivalent viscous damping coefficient, N-m-s-rad} \]

\[ I = \text{Current, A} \]

\[ J = \text{Drillstring mass moment of inertia, kg-m}^2 \]

\[ J_k = \text{Inertia of kelly, kg-m}^2 \]

\[ J_m = \text{Inertia of motor, kg-m}^2 \]

\[ J_{rt} = \text{Inertia of rotary table, kg-m}^2 \]

\[ K_m = \text{Motor constant, V-s} \]

References


APPENDIX A

EQUATIONS USED IN THE MODEL

**Fluid Drag Force/Damping for Axial Model**

Assuming the nonlaminar Newtonian flow formulations and ignoring any eccentric location of the drillstring in the wellbore, the pressure drop in the annulus between the borehole and a stationary drillpipe can be written as below [8]

\[
\Delta P = \frac{\alpha_a \rho_m Q^2 \, dx}{4 \pi^2 (r_w - r_0)(r_w^2 - r_0^2)^2} \quad (21)
\]

The resulting longitudinal force, \(F_A\) (positive down) exerted on the drillstring segment which is moving with velocity, \(V_n\) can be written as below [8]

\[
F_A = - \left( \frac{\alpha_a \rho_m \pi}{4} \right) \left( \frac{Q}{\pi (r_w^2 - r_0^2)} \right) + V_n \left\{ \left( \frac{Q}{\pi (r_w^2 - r_0^2)} \right) + V_n \right\} \quad (22)
\]

And the drag force on the drillstring due to flow in the drillpipe is given by [8]

\[
F_p = - \left( \frac{\alpha_p \rho_m \pi}{4} \right) \left( \frac{Q}{\pi r_i^2} \right) - V_n \left\{ \left( \frac{Q}{\pi r_i^2} \right) - V_n \right\} \quad (23)
\]

**Fluid Friction Resistance/Viscous Damping to Rotation**

Again ignoring any nonconcentric drillpipe location in the borehole, a simple expression for the fluid torque is given by [2, 8]

\[
T_{R_n} = \left( \frac{2 \pi \mu e}{r_w - r_0} \right)_n dx_n \omega_n \quad (24)
\]