Break-even analysis is used to compare processes by finding the volume at which two different processes have equal total costs. Break-even point is the volume at which total revenues equal total costs. Variable costs (c) are costs that vary directly with the volume of output. Fixed costs (F) are those costs that remain constant with changes in output level.

- "Q" is the volume of customers or units, "c" is the unit variable cost, F is fixed costs and p is the revenue per unit.
- cQ is the total variable cost.
- Total cost = F + cQ
- Total revenue = pQ
- Break-even is where pQ = F + cQ

(Total revenue = Total cost)

Break-Even Analysis can tell you...

- If a forecast sales volume is sufficient to break even (no profit or no loss)
- How low variable cost per unit must be to break even given current prices and sales forecast.
- How low the fixed cost need to be to break even.
- How price levels affect the break-even volume.

Hospital Example

Example A.1

A hospital is considering a new procedure to be offered at $200 per patient. The fixed cost per year would be $100,000, with total variable costs of $100 per patient.

What is the break-even quantity for this service?

Q = F / (p - c) = 100,000 / (200-100) = 1,000 patients
### Sensitivity Analysis Example A.2

#### Forecast = 1,500

\[
pQ - (F + cQ)
\]

- **Total annual revenue**: \(200(1500) - (100,000 + 100(1500)) = 50,000\)
- **Total annual cost**: \(200,000\)
- **Profit**: \(50,000\)
- **Break-even quantity**: \(200(1500) - 200,000 = 300\)
- **Fixed costs**: \(200,000\)
- **Profit Loss**: \(50,000\)
- **Total annual costs**: \(200,000\)

#### Application A.1

The Denver Zoo needs to decide whether to move to the polar bears to SeaWorld or build a special exhibit for them and the zoo. The expected increase in attendance is 200,000 patrons. The costs are:

- **Revenues per Patron for Exhibit**
  - Gates/entrance $4
  - Concessions $5
  - Licensed apparel $10

- **Estimated Fixed Costs**
  - Exhibit construction $2,400,000
  - Salaries $220,000
  - Food $30,000

- **Estimated Variable Costs per Person**
  - Gates/entrance $2
  - Concessions $2

Is the predicted increase in attendance sufficient to break even?
Application A.1

Solution

b. Algebraic solution of Denver Zoo problem

\[ pQ = F + qQ \]

\[ Q = \frac{F}{p - q} \]

\[ Q = \frac{2,400,000}{4 - 3} = 200,000 \]

<table>
<thead>
<tr>
<th>( Q )</th>
<th>( TR = pQ )</th>
<th>( TC = F + qQ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0</td>
<td>$2,400,000</td>
</tr>
<tr>
<td>200,000</td>
<td>$6,000,000</td>
<td>$5,600,000</td>
</tr>
</tbody>
</table>

Where

\[ p = 4 \quad q = 15 \quad F = 2,400,000 \quad q = 30,000 \quad c = 2 \quad q = 11 \]

Two Processes and Make-or-Buy Decisions

- Breakeven analysis can be used to choose between two processes or between an internal process and buying those services or materials.
- The solution finds the point at which the total costs of each of the two alternatives are equal.
- The forecast volume is then applied to see which alternative has the lowest cost for that volume.

Application A.2

At what volume should the Denver Zoo be indifferent between buying special sweaters from a supplier or have zoo employees make them?

<table>
<thead>
<tr>
<th></th>
<th>Buy</th>
<th>Make</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed costs</td>
<td>$0</td>
<td>$300,000</td>
</tr>
<tr>
<td>Variable costs</td>
<td>$9</td>
<td>$7</td>
</tr>
</tbody>
</table>

\[ Q = \frac{F_m - F_b}{c_b - c_m} = \frac{300,000 - 0}{9 - 7} = 150,000 \]

Preference Matrix

- A Preference Matrix is a table that allows you to rate an alternative according to several performance criteria.
- The criteria can be scored on any scale as long as the same scale is applied to all the alternatives being compared.
- Each score is weighted according to its perceived importance, with the total weights typically equaling 100.
- The total score is the sum of the weighted scores (weight \times score) for all the criteria. The manager can compare the scores for alternatives against one another or against a predetermined threshold.
Decision Theory

- Decision theory is a general approach to decision making when the outcomes associated with alternatives are often in doubt.
- A manager makes choices using the following process:
  1. List the feasible alternatives
  2. List the chance events (states of nature).
  3. Calculate the payoff for each alternative in each event.
  4. Estimate the probability of each event.
  5. Select the decision rule to evaluate the alternatives.

Decision Rules

- Decision Making Under Uncertainty is when you are unable to estimate the probabilities of events.
  - Minimax Regret: Minimizing your regret (also pessimistic)
  - Laplace: The alternative with the best weighted payoff using assumed probabilities.
- Decision Making Under Risk is when one is able to estimate the probabilities of the events.
  - Expected Value: The alternative with the highest weighted payoff using predicted probabilities.

MaxiMin Decision Example A.6 a.

<table>
<thead>
<tr>
<th>Events (Uncertain Demand)</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small facility</td>
<td>100</td>
<td>270</td>
</tr>
<tr>
<td>Large facility</td>
<td>160</td>
<td>800</td>
</tr>
<tr>
<td>Do nothing</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

1. Look at the payoffs for each alternative and identify the lowest payoff for each.
2. Choose the alternative that has the highest of these. (the maximum of the minimums)
### MaxiMax Decision
**Example A.6 b.**

- **Events** (Uncertain Demand)
  - **Alternatives**
    - Low
      - Small facility: 200
      - Large facility: 160
      - Do nothing: 0
    - High
      - Small facility: 270
      - Large facility: 800
      - Do nothing: 0

1. Look at the payoffs for each alternative and identify the “highest” payoff for each.
2. Choose the alternative that has the highest of these. (the maximum of the maximums)

### Laplace
*(Assumed equal probabilities)*
**Example A.6 c.**

- **Events** (Uncertain Demand)
  - **Alternatives**
    - Low
      - Small facility: 200
      - Large facility: 160
      - Do nothing: 0
    - High
      - Small facility: 270
      - Large facility: 800
      - Do nothing: 0

Multiply each payoff by the probability of occurrence of its associated event.

Select the alternative with the highest weighted payoff.

### MiniMax Regret
**Example A.6 d.**

- **Events** (Uncertain Demand)
  - **Alternatives**
    - Low
      - Small facility: 200
      - Large facility: 160
      - Do nothing: 0
    - High
      - Small facility: 270
      - Large facility: 800
      - Do nothing: 0

Look at each payoff and ask yourself, “If I end up here, do I have any regrets?”

Your regret, if any, is the difference between that payoff and what you could have had by choosing a different alternative, given the same state of nature (event).

**Example A.6 d. continued**

- **Events** (Uncertain Demand)
  - **Alternatives**
    - Low
      - Small facility: 200
      - Large facility: 160
      - Do nothing: 0
    - High
      - Small facility: 270
      - Large facility: 800
      - Do nothing: 0

Building a large facility offers the least regret.

### MiniMax Regret
**Example A.6 d. continued**

- **Events** (Uncertain Demand)
  - **Regret Matrix**
    - **Alternatives**
      - Low
        - Small facility: 50
        - Large facility: 40
      - High
        - Small facility: 530
        - Large facility: 80
        - Do nothing: 200

### Expected Value
**Decision Making under Risk**
**Example A.7**

- **Events** (Uncertain Demand)
  - **Alternatives**
    - Low
      - Small facility: 200
      - Large facility: 160
      - Do nothing: 0
    - High
      - Small facility: 270
      - Large facility: 800
      - Do nothing: 0

Multiply each payoff by the probability of occurrence of its associated event.

Select the alternative with the highest weighted payoff.
Expected Value Analysis

Example A.7

Decision Trees

- Decision Trees are schematic models of alternatives available along with their possible consequences.
- They are used in sequential decision situations.
- Decision points are represented by squares.
- Event points are represented by circles.

After drawing a decision tree, we solve it by working from right to left, starting with decisions farthest to the right, and calculating the expected payoff for each of its possible paths.
- We pick the alternative for that decision that has the best expected payoff.
- We “saw off,” or “prune,” the branches not chosen by marking two short lines through them.
- The decision node’s expected payoff is the one associated with the single remaining branch.

Decision Trees

Drawing the Tree

Example A.8 continued
Decision Making
Solving Decision #1
Example A.8

Application A.6
GM Explorer Solution

Decision Making