Fourier Series and Harmonic Analysis

Practice Problems

Q1. Assume that $f(x)$ has a period 2π. Sketch the graph of $f(x)$ and find its Fourier series.

a) $$f(x)=\begin{cases}1 & -\pi/2 \le x < \pi/2 \\ -1 & \pi/2 \le x \le 3\pi/2\end{cases}$$

b) $$f(x)=\begin{cases}0 & -\pi \le x < 0 \\ x & 0 \le x < \pi/2 \\ \pi/2 & \pi/2 \le x \le \pi\end{cases}$$

Q2. Assume that $f(t)$ has a period $\pi/2$. Sketch the graph of $f(t)$ and find its Fourier series. Check if the series is even or odd first and use the information.

$$f(t)=\begin{cases}t & -\pi/8 \le t < \pi/8 \\ \pi/4-t & \pi/8 \le t \le 3\pi/8\end{cases}$$

Q3. The monthly average river flow rate in m^3/sec of Kimberly River in Mt. Pearl based on several years of record is as follows:

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
60.3	46.5	43.3	33.5	18.0	9.8	7.9	11.9	25.0	33.1	42.4	52.4

Estimate the number and coefficients of harmonics to be fitted to model the periodicity in the average monthly flows of the Kimberly river. Graphically check the fit of your model to the observed values.

Answers:

1. a) $$\frac{4}{\pi}\left[\cos x-\frac{1}{3}\cos 3x+\frac{1}{5}\cos 5x-.....\right]$$

   b) $$\frac{3\pi}{16}-\frac{1}{\pi}\cos x+\left(\frac{1}{2}+\frac{1}{\pi}\right)\sin x-\frac{1}{2\pi}\cos 2x-.....$$

2. $$\frac{1}{\pi}\left[\sin 4t-\frac{1}{9}\sin 12t+\frac{1}{25}\sin 20t.....\right]$$

3. Period T=12, a_0=32, a_1=21.04, b_1=10.44, R_1^2=97%