Formulas for the Final Exam:

Some of the relevant formulae are given below. Note that the format of the formulae that you know could be different from that given below, e.g., the integration limits for Fourier expansions or the variables *x, y*, etc.

Faddeev-Leverrier Method Equations: Eigen value polynomial equation is given by:

$$\left|\underline{\underline{A}} - \lambda \underline{\underline{I}}\right| = \lambda^n + \alpha_{n-1}\lambda^{n-1} + \alpha_{n-2}\lambda^{n-2} + \ldots\ldots + \alpha_1\lambda + \alpha_0,$$

$$\underline{\underline{D}}_1 = \underline{\underline{I}}; \qquad \alpha_{n-1} = -trace\left(\underline{\underline{A}}\,\underline{\underline{D}}_1\right); \qquad \underline{\underline{D}}_i = \underline{\underline{A}}\,\underline{\underline{D}}_{i-1} + \alpha_{n-i+1}\underline{\underline{I}}; \qquad \alpha_{n-i} = -\frac{1}{i}trace\left(\underline{\underline{A}}\,\underline{\underline{D}}_i\right)$$

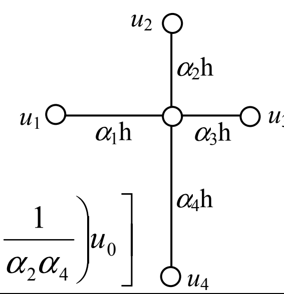
The eigen vector is given by any column of the matrix C: $\underline{\underline{C}} = \sum_{i=1}^{n} \lambda^{n-i}\underline{\underline{D}}_i$

For harmonic analysis, $R_m^2 = \dfrac{\frac{1}{2}\sum_{i=1}^{m}\left(a_i^2 + b_i^2\right)}{\frac{1}{T}\sum_{j=1}^{p}\left(f(t) - \overline{\sigma}\right)^2}$; where, m=the number of harmonics considered, p=the number of values in the function f, and $\overline{\sigma}$=the mean of the data.

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos(nwx) + b_n\sin(nwx)\right); \quad a_0 = \frac{1}{T}\int_0^T f(x)dx; \; a_n = \frac{2}{T}\int_0^T f(x)\cos(n\omega x)dx; \quad \omega = \frac{2\pi}{T}$$

For 1-D, $\dfrac{d^3x}{dy^3} = \dfrac{-x_{i-2} + 2x_{i-1} - 2x_{i+1} + x_{i+2}}{2h^3}$, $\dfrac{d^4x}{dy^4} = \dfrac{x_{i-2} - 4x_{i-1} + 6x_i - 4x_{i+1} + x_{i+2}}{h^4}$

For two dimensions, $\nabla^2 w_{i,j} = \left(\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2}\right)_{i,j} = \dfrac{w_{i,j}}{h^2}\begin{Bmatrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{Bmatrix}$



For irregular grid,

$$\nabla^2 u = \frac{2}{h^2}\left[\frac{u_1}{\alpha_1(\alpha_1 + \alpha_3)} + \frac{u_2}{\alpha_2(\alpha_2 + \alpha_4)} + \frac{u_3}{\alpha_3(\alpha_1 + \alpha_3)} + \frac{u_4}{\alpha_4(\alpha_2 + \alpha_4)} - \left(\frac{1}{\alpha_1\alpha_3} + \frac{1}{\alpha_2\alpha_4}\right)u_0\right]$$

For any $w' = f(x, w)$, Runge-Kutta method at any step i,

$K_1 = hf(x_i, w_i); \qquad K_2 = hf(x_i + h/2, w_i + K_1/2)$

$K_3 = hf(x_i + h/2, w_i + K_2/2); K_4 = hf(x_i + h, w_i + K_3) \quad w_{i+1} = w_i + (K_1 + 2K_2 + 2K_3 + K_4)/6$

For implicit Euler, $w_{i+1} = w_i + hf(x_{i+1}, w_{i+1})$

For shooting, $guess_3 = guess_1 + \dfrac{guess_2 - guess_1}{result_2 - result_1}(t\arg et - result_1)$

For a general PDE $a\dfrac{\partial^2 u}{\partial x^2} + b\dfrac{\partial^2 u}{\partial x \partial t} + c\dfrac{\partial^2 u}{\partial t^2} = f$, the characteristic lines have $\dfrac{dt}{dx} = \dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$

The solution of the PDE reduces to solving $a\left(\dfrac{dt}{dx}\right)\delta\left(\dfrac{\partial u}{\partial x}\right) + c\,\delta\left(\dfrac{\partial u}{\partial t}\right) - f(dt) = 0$

For cubic equation $z^3 + a_2 z^2 + a_1 z + a_0 = 0$, let $Q = \dfrac{3a_1 - a_2^2}{9}, R = \dfrac{9a_2 a_1 - 27a_0 - 2a_2^3}{54}$

$\theta = \cos^{-1}\left(\dfrac{R}{\sqrt{-Q^3}}\right)$, Roots become,

$$z_1 = 2\sqrt{-Q}\cos\left(\frac{\theta}{3}\right) - \frac{1}{3}a_2; \quad z_2 = 2\sqrt{-Q}\cos\left(\frac{\theta + 2\pi}{3}\right) - \frac{1}{3}a_2$$

$$z_3 = 2\sqrt{-Q}\cos\left(\frac{\theta + 4\pi}{3}\right) - \frac{1}{3}a_2$$