Eng. 5434
Applied Mathematical Analysis

Introduction and Review of Previous Work

Outline:

Posted at: www.engr.mun.ca/~adluri (and if possible, at L:/courses/5434/ on the faculty network).

1. –Midterm and final
2. -Assignments to be done manually and in EXCEL (can be done in MATLAB with permission)
3. -Major areas of emphasis:
    - Several application areas in civil engineering that need Numerical Solution (usually leading to Differential Equations of various kinds) -mostly linear problems.
    - Fourier Analysis –both continuous and discrete problems
    - This is not a mathematics course (at least we all should will try make it so)!

Quick Review of earlier material

Differential Equations:

Equations containing independent variable(s), dependent variable(s), and the derivative(s) of the dependent variable(s).

How do we get differential equations?
Most of the time, the differential equations of interest to Civil/Mechanical engineers crop up while

- describing the behaviour of a physical system,
- applying the principles of nature such as
    - the principle of equilibrium,
    - The principle of minimum potential energy,
    - Principle of conservation –e.g. energy, mass, momentum, ….
- Any problem relating two or more variables that change with respect to each other in an orderly manner (even if it is a complicated order).

Examples of simple differential equations:

The speed of an automobile is: dx/dt=100 kmph

Equilibrium of the internal and external moments on a beam gives

$$EI\, d^2y/dx^2 = M$$

Vibration of a portal frame obtained through equilibrium of internal and external forces

$$m\, d^2x/dt^2 + kx = F$$

Deflection of a plate under uniform load

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial x^4}\right) = q$$

What do you need to solve a differential equation?

- The equation itself along with the relevant physical data such as E, I, etc.
- Initial and/or boundary conditions
- A technique(s) to solve the problem

Classifying the Differential Equations

- Order of a differential equation
- Degree of a differential equation
- Linear and nonlinear differential equations.
- Ordinary and Partial differential equations, etc.

Solving the Differential Equations

Classical and numerical methods:

Classical methods of solving differential equations are well developed and give good, elegant and very useful closed form solutions.

However, most differential equations of practical interest cannot be solved using classical methods because of complex domains and properties. Hence, we resort to approximate numerical methods.

What is a numerical method?
----Technique that gives an approximate solution at discrete numerical points in the domain of interest.
----The material is somewhat introduced in the first numerical methods course.

Numerical solutions become attractive when

- Classical solutions are not possible or overly difficult to obtain or are not available in the desired form
- Cheap computational power is feasible
- Reasonable solutions can be obtained with required accuracy

Primary focus of the numerical methods is on

- Computational effort
- Accuracy of the solution

Numerical methods can be direct (e.g., Gauss elimination) or iterative (e.g., subspace iteration). Direct methods may not be feasible for large systems or systems where rounding is a problem. On the other hand, iterative methods must be able to converge reasonably fast and reasonably accurately.

Most intractable problems where classical solutions are very difficult (e.g., nonlinear differential equations) can be solved using numerical methods.

First order ordinary differential equations: Review

Euler method

- Graphical interpretation
- Taylor series interpretation

An example of Euler method with error analysis

Modified Euler (Huen) method

- Graphical interpretation
- Taylor series interpretation

An example of Modified Euler method

Runge-Kutta (4th Order) method

- Graphical interpretation

An example of Runge-Kutta method