

**MEMORIAL UNIVERSITY OF NEWFOUNDLAND**  
**FACULTY OF ENGINEERING AND APPLIED SCIENCE**

**Engineering: 5434 — Applied Mathematical Analysis**

**SOLUTION OF LINEAR SIMULTANEOUS EQUATIONS**

**Algorithm for Gaussian Elimination (Backward Substitution)**

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = a_{1,n+1}$$

Given:  $a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = a_{2,n+1}$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n = a_{n,n+1}$$

Input consists of the augmented matrix  $A = [a_{ij}]$ , where,  $1 \leq i \leq n$ ,  $1 \leq j \leq n + 1$ .

Required: Solution  $x_i$  for values of  $1 \leq i \leq n$ .

Step 1 For  $i = 1, 2, \dots, n - 1$ , do steps 2-7

*Elimination Process*

Step 2 Find  $p$  such that  $i \leq p \leq n$  and  $a_{ip} \neq 0$ .

*non-zero pivot.*

If  $p$  can not be found, then

PRINT "No Unique Solution Exists - Zero Pivot for  $i =$ ",  $i$

STOP

*Unsuccessful solution.*

Step 3 If  $p \neq i$  then

For  $1 \leq j \leq n + 1$ , Set  $b_j = a_{pj}$

*Store the row*

For  $1 \leq j \leq n + 1$ , Set  $a_{pj} = a_{ij}$

*Interchange the row*

For  $1 \leq j \leq n + 1$ , Set  $a_{ij} = b_j$

*Eliminate the zero pivot*

Step 4 For  $j = i + 1, \dots, n$  do steps 5 to 7

Step 5 If  $a_{ji} \neq 0$  do Steps 6 and 7

Step 6 Set  $m_{ij} = a_{ji} / a_{ii}$

Step 7 For  $1 \leq k \leq n + 1$ , Set  $a_{jk} = a_{jk} - m_{ji}a_{ik}$  *Forward Elimination*

Step 8 If  $a_{nn} = 0$  then

PRINT "No Unique Solution Exists - Zero Pivot for ",  $n$

STOP

*Unsuccessful solution.*

Step 9 Set  $x_n = a_{n,n+1} / a_{nn}$

*Begin backward substitution*

Step 10 For  $i = n - 1, \dots, 1$  Set  $x_i = \frac{a_{i,n+1} - \sum_{j=i+1}^n a_{ij}x_j}{a_{ii}}$

Step 11 PRINT  $x_1, x_2, \dots, x_n$

STOP

*Successful solution.*