MEMORIAL UNIVERSITY OF NEWFOUNDLAND
FACULTY OF ENGINEERING AND APPLIED SCIENCE

Engineering: 5434 —– Applied Mathematical Analysis

Conversion Of A Higher Order Differential Equation To A System Of First Order Differential Equations :

Given: $\dfrac{d^n y}{dx^n} = f\left(x, y, y', y'', y''', \ldots\ldots y^{(n-1)}\right) \qquad x_o \le x \le x_n$

Required: n first order equations to be solved simultaneously in place of the one *n*th order differential equation given.

Step 1 Define a new set of variables in the following manner:

$$z_1 = y; \quad z_2 = dy/dx; \quad z_3 = d^2y/dx^2; \quad z_4 = d^3y/dx^3; \cdots\cdots \quad z_n = d^{n-1}y/dx^{n-1}$$

Step 2 Write the given *n*th order equation as, $dz_n/dx = f(x, z_1, z_2, z_3, \cdots, z_{n-1})$.

Step 3 The given equation is thus converted to a set of simultaneous eqns. as below:

$$\begin{aligned} z_1' &= z_2 \\ z_2' &= z_3 \\ z_3' &= z_4 \\ \vdots \quad & \quad \vdots \\ z_{n-1}' &= z_n \\ z_n' &= f\left(x, z_1, z_2, z_3, \cdots\cdots, z_{n-1}\right) \end{aligned}$$

Example:

The following two simultaneous second order equations represent the motion of a particular two storey building under wind load. Convert them to a set of first order equations.

$$m_1\ddot{x}_1 + c\dot{x}_1 + k_2(x_1 - x_2) + k_1x_1 = 0$$
$$m_2\ddot{x}_2 + k_2(x_2 - x_1) + k_3x_2 = p\sin(\omega t)$$

where the constants represent the following,
m_1 and m_2 = masses,
k_1, k_2, and k_3 = stiffnesses of the columns,
c = damping constant,
p = force amplitude,
ω = natural frequency
The dot on top of x represents differentiation with respect to time t.

Solution:

The dependent variables in the problem are x_1 and x_2. The independent variable is t.

Step 1 Introduce the following new variables:

$$z_1 = x_1; \quad z_2 = dx_1/dt;$$
$$z_3 = x_2; \quad z_4 = dx_2/dt$$

Step 2 The given equations then become:

$$m_1\dot{z}_2 + cz_2 + k_2(z_1 - z_3) + k_1 z_1 = 0$$
$$m_2\dot{z}_4 + k_2(z_3 - z_1) + k_3 z_3 = p\sin(\omega t)$$

Step 3 Rearranging,

$$dz_1/dt = z_2;$$
$$dz_2/dt = \frac{-1}{m_1}\left(cz_2 + k_2(z_1 - z_3) + k_1 z_1\right);$$
$$dz_3/dt = z_4;$$
$$dz_4/dt = \frac{-1}{m_2}\left(k_2(z_3 - z_1) + k_3 z_3 - p\sin(\omega t)\right)$$

The solution of these four first order equations is identical to the solution of the original two second order equations.