Structural Steel Design
Flexural Members

Dr. Seshu Adluri
Beams in Buildings
Beams in Bridges

Hibbeler; Wallarah Creek Bridge; eastbound on I-64, approaching I-295 overpasses

Flexural members - Dr. Seshu Adluri
Beams in Bridges

Flexural members - Dr. Seshu Adluri
Beams in Bridges
Beams in Bridges

Canary Wharf, Docklands
East India Beam footbridge (curved beam)
Cascade Creek valley

Flexural members - Dr. Seshu Adluri
Beam construction!

Then  [engineering necessity]

Now  [Television (CSI) vanity]!

The high-flying cast of 'CSI: NY's' fourth season 2007

Construction workers take a lunch break on a steel beam atop the RCA Building at Rockefeller Center, New York, Sept. 29, 1932. In the background is the Chrysler Building.
Beam construction!

Building the Empire State, 1930. Lewis Hine (two workers)
Acrobats Jarley Smith (top), Jewell Waddek (left), and Jimmy Kerrigan (right) perform a delicate balancing act on a ledge of the Empire State Building in Manhattan. August 21, 1934

Structural worker on the Empire State Building, 1930
The new telephone company skyscraper at Vesey Street and the North River under construction. Photo shows the iron workers perched on a girder twenty stories in the air and enjoying their lunch -New York
Beam construction

Construction workers eat their lunches atop a steel beam 800 feet above ground, at the building site of the RCA Building in Rockefeller Center, 1930's
Beam installation
Beam profiles
Beam geometry

- Stand alone beams
- Frame beams
Beam Loading

\[ M_{\text{max}} = \frac{PL}{4} \]

\[ M_{\text{max}} = \frac{wL^2}{8} \]
### Beam diagrams

<table>
<thead>
<tr>
<th>Loading</th>
<th>Shear Diagram $\frac{dV}{dx} = -w$</th>
<th>Moment Diagram $\frac{dM}{dx} = V$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram 1" /></td>
<td><img src="image2" alt="Diagram 2" /></td>
<td><img src="image3" alt="Diagram 3" /></td>
</tr>
<tr>
<td>Downward force $P$ causes $V$ to jump downward from $V_1$ to $V_2$.</td>
<td>Constant slope changes from $V_1$ to $V_2$.</td>
<td></td>
</tr>
<tr>
<td><img src="image4" alt="Diagram 4" /></td>
<td><img src="image5" alt="Diagram 5" /></td>
<td><img src="image6" alt="Diagram 6" /></td>
</tr>
<tr>
<td>No change in shear since slope $w = 0$.</td>
<td>Constant positive slope. Counterclockwise $M_0$ causes $M$ to jump downward.</td>
<td></td>
</tr>
<tr>
<td><img src="image7" alt="Diagram 7" /></td>
<td><img src="image8" alt="Diagram 8" /></td>
<td><img src="image9" alt="Diagram 9" /></td>
</tr>
<tr>
<td>Constant negative slope.</td>
<td>Positive slope that decreases from $V_1$ to $V_2$.</td>
<td></td>
</tr>
<tr>
<td><img src="image10" alt="Diagram 10" /></td>
<td><img src="image11" alt="Diagram 11" /></td>
<td><img src="image12" alt="Diagram 12" /></td>
</tr>
<tr>
<td>Negative slope that increases from $-w_1$ to $-w_2$. Positive slope that decreases from $V_1$ to $V_2$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image13" alt="Diagram 13" /></td>
<td><img src="image14" alt="Diagram 14" /></td>
<td><img src="image15" alt="Diagram 15" /></td>
</tr>
<tr>
<td>Negative slope that decreases from $-w_1$ to $-w_2$. Positive slope that decreases from $V_1$ to $V_2$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image16" alt="Diagram 16" /></td>
<td><img src="image17" alt="Diagram 17" /></td>
<td><img src="image18" alt="Diagram 18" /></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Flexural members - Dr. Seshu Aduri
Bending moment diagrams
Flexure

- Plane section remains plane
  - Strain varies linearly
Bending stress

\[ M = 22.5 \text{ kN} \cdot \text{m} \]
Stress for yield moment $M_Y$
Yield moment $M_Y$

- If the stress due to applied moment is at or below the yield, the stress distribution is linear.
- Max moment for which the stress is linearly varying is the yield moment $M_Y$.
- $C = T = (bh/2)(\sigma_y/2)$
- $M_Y = C \times (\text{lever arm}) = C \times (2h/3) = (bh^2/6)\sigma_y = S\sigma_y$
- Elastic section modulus for rectangle, $S = (bh^2/6)$
- Section modulus is listed for other sections.
Stress for moment $M > M_Y$
Progression of stress with load increase

Figure 4. Transition from elastic to plastic state of a cross-section in bending.
The maximum moment that the c/s can take occurs when the entire c/s is yielded in compression or tension.

- $C = T = (bh/2)\sigma_y$
- Moment, $M_p = C \times \text{(lever arm)} = (bh^2/4)\sigma_y$
- Plastic modulus for rectangle c/s, $Z = bh^2/4$
- Plastic modulii are listed in Section properties tables
Moment curvature relationship

Figure 5 Moment-curvature (M-\(\phi\)) relationship for a rectangular cross-section in bending
Moment curvature relationship

Figure 8  Moment-curvature (M-\(\phi\)) behaviour of I-sections
Plastic hinge

Figure 1  Behaviour of a simply supported beam
Plastic hinge

Figure 6 Load-deflection curve for a statically indeterminate steel beam
Beams in Shear
Shear

(a) Before deformation

(b) After deformation

Transverse shear stress

Longitudinal shear stress

Shear–stress distribution
Shear

Shear-stress distribution

Intensity of shear-stress distribution (profile view)
Shear flow
Shear flow

Shear-flow distribution
Shear flow

$q_{\text{max}}$

$q_{\text{max}}$

Shear-flow distribution

$(q_{\text{max}})_w$

$(q_{\text{max}})_f$

Shear flow distribution

Shear flow $q$
Shear centre

(a) Flexural members - Dr. Seshu Adluri

Shear-flow distribution

(b) Hibbeler

(c) Shear centre

(d) Shear force distribution

(e) Flexural members - Dr. Seshu Adluri
Shear flow effect
Shear centre
Shear centre
Bending terminology

- Moment of inertia \( I_x = \int_A y^2 \, dA \)
- Parallel axis theorem \( I_x' = I_x + A x^2 \)
- Flexural stress \( \sigma = \frac{M_y}{I} \)
- Average shear stress \( = \frac{V_f}{hw} \)
- Yield moment, \( M_Y \)
  - Elastic Section modulus, \( S \)
- Plastic moment, \( M_P \)
  - Plastic section modulus, \( Z \)
- Beam (slab load) vs. Girder (load from beams)
Beams and Girders

- **Steel flexural members**
  - Beams in building frames
  - Elements carrying lateral loads
  - Equipment, etc.

- Useful in pure bending as well as in beam-columns

- **Design Clauses: CAN/CSA-S16**
  - Bending strength as per Clauses 13.5, 6 & 7
  - Shear strength as per Clause 13.4
  - Local buckling check: Clause 11 (Table 2)
  - Special provisions: Clause 14
  - Deflection limits: Appendix D
Lateral-Torsional Buckling

- Instability due to buckling of compression half of the beam (perpendicular to the plane of loading).

Figure 2  Response of a slender cantilever beam to vertical loading: lateral-torsional buckling
Lateral-Torsional Buckling

- Instability due to buckling of compression half of the beam (perpendicular to the plane of loading).

Stress distribution in flanges due to bending

Compression flange treated as an equivalent strut
Lateral-Torsional Buckling

Strut buckling

\[
\frac{EA}{L} > \frac{EA_2}{L^3}
\]

Beam (lateral-torsional) buckling

\[
\begin{align*}
& E_{I_y} > E_{I_z} \\
& E_{I_y} > GJ
\end{align*}
\]

Figure 3  Similarity between strut buckling and beam buckling
Lateral-Torsional Buckling

- Lateral support
- Characteristic length, $L_u$
Lateral-Torsional Buckling

- Lateral support
- Characteristic length, $L_u$

Beam loaded by cross beams which provide lateral support to points B & C

Plan view of buckled shape

Figure 8 Buckling of beams provided with lateral bracing
Beam plastic hinge mechanisms

- Capacity is much higher than that predicted by simple section analysis (max. moment in the structure < moment capacity of the section)

- Only possible if the sections allow plastic hinge formation and redistribution beyond the initial plastic hinge formation.
## Classification of beams

<table>
<thead>
<tr>
<th>Classification of beams</th>
<th>Model of Behaviour</th>
<th>Moment Resistance</th>
<th>Rotation Capacity</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="#" alt="Graph" /></td>
<td>Plastic moment on full section</td>
<td>Sufficient</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td><img src="#" alt="Graph" /></td>
<td>Plastic moment on full section</td>
<td>Limited</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td><img src="#" alt="Graph" /></td>
<td>Elastic moment on full section</td>
<td>None</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td><img src="#" alt="Graph" /></td>
<td>Elastic moment on effective section</td>
<td>None</td>
<td>4</td>
</tr>
</tbody>
</table>

**Graphs:**

- **Local buckling**
- **Plastic moment on full section**
- **Elastic moment on full section**
- **Moment on effective section**

**Sufficient**

**Limited**

**None**
Classification of beams

- **Class 1** – Full plastification and redistribution are allowed
- **Class 2** – Full plastification is allowed but no redistribution. More curvature will result in section buckling.
- **Class 3** – Only $M_Y$ is allowed to be developed. Section will buckle after that.
- **Class 4** – Not even $M_Y$ will develop. Section will fail in elastic lateral-torsional buckling.
Plate buckling

- Different types of buckling depending on
  - b/t ratio
  - Webs and flanges have different limits
  - End conditions for plate segments
  - Use Table 2 for beams and beam-columns

Figure 1 Types of plate buckling
Local (Plate) buckling of Flange
Local (Plate) buckling of Web

Figure 2. Vertical web buckling.

Figure 9. Deformed configuration of the beam with $\alpha = 2.86^\circ$ (1/20).
Web buckling
Real beams - Factors for consideration

- Residual stresses in Hot-rolled shapes (idealized)
Real beams - Factors for consideration

- Residual stresses in Hot-rolled shapes
  - Idealized
  - Measured
Beam columns

(a) Compression

(b) Bending

(c) Combined
Plate Girders

- Slender webs buckle easily due to shear or bending

(a) Bending

(b) Shear

(c) Compression
Plate Girders

- Slender webs buckle easily due to shear or bending
  - Use reduced effective c/s
  - Use reduced capacity

Figure 11 Reduced effective cross-section
Stiffeners are Important

June 1970, Milford Haven bridge over the River Cleddau in the UK
References

- [http://www.esdep.org/members/samplecourse/toc.htm](http://www.esdep.org/members/samplecourse/toc.htm)
- [http://www.roadstothefuture.com/I64-I295-Interchange-Mod.html](http://www.roadstothefuture.com/I64-I295-Interchange-Mod.html)
- [http://international fhwa dot gov/prefab_bridges/chapter_two_c.htm](http://international fhwa dot gov/prefab_bridges/chapter_two_c.htm)
- [http://www.bardaglea.org.uk/bridges/bridge-types/bridge-types-beam.html](http://www.bardaglea.org.uk/bridges/bridge-types/bridge-types-beam.html)
- [http://www.playlsi.com/Products/SportsandFitness/CurvedBalanceBeam/](http://www.playlsi.com/Products/SportsandFitness/CurvedBalanceBeam/)
- [http://www.monorails.org/tmspages/switch.html](http://www.monorails.org/tmspages/switch.html)
- [http://911research.wtc7.net/mirrors/guardian2/fire/SCI.htm](http://911research.wtc7.net/mirrors/guardian2/fire/SCI.htm)
- [http://911research.wtc7.net/mirrors/guardian2/fire/SLamont.htm](http://911research.wtc7.net/mirrors/guardian2/fire/SLamont.htm)

- Most of the pictures in the preceding were gathered from various sources. To the extent possible, the source has been indicated at the bottom of the slides. The text and some of the figures are done by the instructor, Dr. Seshu Adluri.
- No copy right for pictures that have been obtained from other sources is claimed or implied by the instructor. The material is for class room instruction only.