Geophysical Navigation of Autonomous Underwater Vehicles Using Geomagnetic Information

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Abstract: This paper addresses the general problem of Autonomous Underwater Vehicle navigation by exploiting the availability of terrain and geophysics-related data. Geophysical navigation algorithms are derived to estimate the position and velocity of an AUV in the presence of unknown ocean currents and sensor biases. The current implementation merges acoustic and magnetic measurements with dead-reckoning data. One of the main contributions of the paper is the utilization of maps of invariant gradients that can be calculated from prior maps of the geomagnetic field. The results obtained through computer simulations with real data show the effectiveness of the GN approach proposed.

Keywords: Geophysical navigation, terrain navigation, autonomous underwater vehicles, geopotential fields, geomagnetism.

1. INTRODUCTION

True autonomous navigation is still a challenging problem in underwater robotics. Without access to external sources of position information like the Global Positioning System (GPS) and in the absence of artificial beacons, a robotic underwater vehicle must rely on dead-reckoning techniques to estimate its position. The limitations of the dead-reckoning approach are well-known and become specially relevant in missions where autonomous underwater navigation must proceed for long periods of time without the aid of external information. The need to develop cost-effective systems for multipurpose oceanic applications is not compatible with the costs of navigation grade Inertial Navigation Systems (INS) or the deployment of acoustic baseline systems. As a consequence, there has been an increasing interest in the development of reliable navigation systems for autonomous underwater vehicles (AUVs) that rely on information extracted from the surrounding environment without incurring in the cost of deploying artificial beacons.

Among the methods proposed, the by now classical terrain aided navigation (TAN) approach has already demonstrated its great potential for the development of a new generation of navigation systems. This paper addresses the problem of using TAN complemented with geophysical information to estimate the position and velocity of an AUV in the presence of unknown ocean currents. We propose a geophysical navigation algorithm based on a Particle Filter that integrates bathymetric and geomagnetic measurements to sequentially estimate the position and velocity of the AUV. The navigation filter derived relies essentially on the AUV kinematics.

2. PRIOR WORK AND MAIN CONTRIBUTIONS OF THIS PAPER

The work reported herein relies on navigation methods developed previously by the authors Teixeira and Pascoal (2005, 2007); Teixeira (2007). The interested reader can find in the references cited a comprehensive list of references on the following topics: sequential Monte Carlo methods, terrain information and Cramér-Rao Lower Bound (CRLB) for TAN, geophysical navigation, geomagnetic data acquisition, and signal processing of geopotential fields’ data. We refer the reader to Teixeira (2007) for an in depth study of the issues addressed in the this paper.

As one of the main contribution of this work, we present here the results of simulations obtained with our geophysical navigation algorithm using real bathymetric and geomagnetic data collected at the offshore of Aveiro, in the north-western coast of Portugal. These results evidence the great potential of geophysical navigation based on geomagnetic information.

3. PROBLEM FORMULATION AND MODELING

The following notation will be used in the sequel: \( \{I\} \) represents an inertial coordinate frame, \( \{B\} \) denotes the body-fixed frame that moves with the vehicle, \( \mathbf{p} = [x, y, z]^T \) is the position of the origin of \( \{B\} \) measured in \( \{I\} \), \( \lambda = [\phi, \theta, \psi]^T \) represents roll, pitch and yaw angles that parameterize locally the orientation of \( \{B\} \) relative to \( \{I\} \), and \( \omega = [p, q, r]^T \) represents the angular velocity of \( \{B\} \) w.r.t. \( \{I\} \), expressed in \( \{B\} \).

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3.1 Problem formulation and Modeling

Sensor suite

The sensor suite proposed includes a Doppler Velocity Logger (DVL) that measures the velocity of the AUV relative to the water, and an attitude and heading unit (AHU). The topographic measurements are supplied by a sonar altimeter plus two sonars (forward-looking and side-looking) that provide measurements of the distance of the AUV to the sea bottom (Figure 1). A magnetic gradiometer provides measurements of the vertical gradient of the environmental magnetic field.

Problem formulation and assumptions

We use the vehicle kinematics based on measurements supplied by the DVL and the AHU to perform dead reckoning. However, there are unknown ocean currents that introduce a velocity bias, $\mathbf{b}$, that must be estimated. The terrain navigation problem consists of estimating the position and velocity of the vehicle referred to the inertial frame by matching the measured depth and ranges, and magnetic measurements with a priori terrain maps.

It is assumed that the vehicle is leveled horizontally and stabilized in roll and pitch, i.e. $\phi = \theta = 0$ such that $r = \psi$. The velocity vector of the current is considered constant or varying very slowly in time.

Terrain acquisition and Modeling

The high-resolution bathymetric data used in one of the scenarios presented below was acquired using a 0.8 deg mechanically scanning pencil beam sonar integrated with an attitude unit. The data collected has associated an uncertainty value to each measurement thus allowing for the incorporation of map uncertainty in the estimation algorithms. The terrain elevation map $m(x,y)$ is represented by a matrix of gridded nodes with 1 meter spacing. The depth values $m(x,y) = h(x,y) + z$ are obtained by bilinear interpolation in the map. The low-resolution bathymetry used in the second scenario does not have a measure of the uncertainty associated to the data. It is represented by a grid with 10m resolution.

Process and measurement models

The continuous-time process model is:

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}_{\mathbf{u},k}\mathbf{u}_k + \mathbf{L}\xi_k$$

where $\mathbf{F}$ is the state transition model, $\mathbf{G}_{\mathbf{u},k}$ is the input coupling matrix $G_{u,k}$ is a function of the input vector $\mathbf{u}_k = [u_\mathbf{x}, u_\mathbf{v}, u_\psi, r, z]^T$, and $\mathbf{L}$ is the rotation matrix from $\mathbf{b}$ to $\mathbf{I}$ which is parameterized only by $\psi$ because we assumed $\phi = \theta = 0$. $V = [v_u, v_v, v_w]^T$ represents the linear velocity of $\{B\}$ relative to the water, supplied by the Doppler, expressed in $\{B\}$. $\mathbf{b} = [b_x, b_y, b_z]^T$ represents the velocity bias introduced by ocean currents expressed in $\{I\}$. $\xi_k$ is a white noise process and $\xi_k$ is zero or a white noise process with very small intensity. We treat $z$ as an input (not as a state variable) since we can obtain measurements of depth with very high accuracy. This has the advantage of reducing the dimension of the state vector which is crucial to the performance of the Particle Filter estimator. We, thus, reduce the position and velocity estimation to a 2D estimation problem. The corresponding discrete-time process model is

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{G}_{\mathbf{u},k}\mathbf{u}_k + \mathbf{L}\xi_k$$

where $\mathbf{x} = [x, y, b_x, b_y]^T$ is the state vector and $\xi_k$ represents the process noise sequence. The input coupling matrix $G_{u,k}$ is a function of the input vector $\mathbf{u}_k = [u_\mathbf{x}, u_\mathbf{v}, u_\psi, r, z]^T$.

Assuming additive measurement noise, where $\eta$ models the measurement errors which are a function of the 3D position $p_k$, the discrete-time measurement model is

$$y_k = f(x_k) + \eta(p_k)$$

where $y_k$ is a vector that represents the simultaneous measurements taken at each iteration. A detailed derivation of this model can be found in Teixeira (2007).

Noise models

The process noise variables specified in (1) are assumed mutually independent Gaussian white processes, with process noise intensity represented by the diagonal matrix $\Xi$. The discrete-time counterpart of $\Xi$ is $Q = \Xi\Delta t$ and the corresponding discrete-time independent sequences $\xi_k$ are also assumed Gaussian.

The measurement noise variables represented in the vector $\eta$ are considered mutually independent and are characterized by the time-varying measurement noise intensity matrix $R_k$ (see Teixeira (2007)).
PF, decomposing the state vector into two parts: \( \mathbf{x} = [\mathbf{x}^{pf}, \mathbf{x}^{kf}]^T \), where \( \mathbf{x}^{pf} = [x, y]^T \) represents the part of the state vector estimated by the PF and \( \mathbf{x}^{kf} = [b_x, b_y]^T \) is the part of the state vector estimated by a Kalman Filter (KF). We decompose similarly the state transition matrix \( \mathbf{F} \), the input coupling matrix \( \mathbf{G}_{u,k} \), the noise coupling matrix \( \mathbf{L} \), the input and noise vectors \( \mathbf{u}_k \) and \( \mathbf{\zeta}_k \), and the discrete-time process noise intensity matrix \( \mathbf{Q} \).

In the sequel we use \( \mathbf{u}_k = [v_{u_k}, v_{v_k}, v_{w_k}, \psi_k, r_k, z_k]^T = [V_k, \psi_k, r_k, z_k]^T \). The RB PF formulation becomes:

**Prediction**

\[
\begin{align*}
\mathbf{x}_{k+1}^{pf,i} &= \mathbf{x}_k^{pf,i} + F^{pf} \mathbf{x}_k^{kf,i} + G_{u,k}^{pf} \mathbf{v}_k + \zeta_k^{pf} \\
\mathbf{x}_{k+1}^{kf,i} &= F^{kf} \left[ \mathbf{x}_{k|k-1}^{kf,i} + K_k \mathbf{v}_k \right]
\end{align*}
\]

where \( K_k \) denotes the Kalman filter gain,

\[
\nu_k = \mathbf{x}_{k+1}^{pf,i} - \mathbf{x}_k^{pf,i} = \left( F^{pf} \mathbf{x}_{k|k-1}^{kf,i} + G_{u,k}^{pf} \mathbf{v}_k \right), \quad \text{and}
\]

\[
\zeta_k^{pf} \sim \mathcal{N} \left( 0, F^{pf} \mathbf{P}_{k|k-1}^{pf} (F^{pf})^T + L^{pf} \mathbf{Q}^{pf} (L^{pf})^T \right)
\]

**Update - The Smooth Particle Filter**

One of the problems that we face in the context of TAN is that of applying a multi-modal likelihood function in the updating of the weights according to the conventional expression \( \omega_k \propto \omega_{k-1} \cdot p(y_k|\mathbf{x}_k) \). Due to terrain symmetry several states can exist with similarly high likelihoods. This possibility leads to situations where outliers (process model-implausible observations) can have their weights greatly amplified relatively to model-plausible observations. The PF approach to TAN becomes even more sensitive to this effect when techniques such as *roughening* (see Gordon et al. (1993)) or *jittering* (Teixeira (2007)) are employed since they increase the probability of generating extreme outliers associated to important weights.

In simulations this proved to be the main cause of filter divergence. Thus the need for a method that will attenuate the weights of extreme outliers, while maintaining the possibility of using *jittering* and other ad-hoc techniques. The PF version that we introduce here achieves this objective. Notice that the Auxiliary Particle Filter (APF) introduced by Pitt and Shephard (1999) does not address this problem adequately when the process noise is large or when we add synthetic noise to avoid particles’ degeneracy. Our filter updates the weights according to:

\[
\omega_k \propto \omega_{k-1} \cdot p(y_k|\mathbf{x}_k) \cdot p\left( \mathbf{x}_k | \mathbf{x}_{k-1}^{i} \right)
\]

where \( p\left( \mathbf{x}_k | \mathbf{x}_{k-1}^{i} \right) = N\left( \mathbf{y}_k - \mathbf{f}(\mathbf{x}_k)|0, R_k \right) \) and \( p\left( \mathbf{x}_k | \mathbf{x}_{k-1}^{i} \right) \) is a smoothing kernel applied at the update step to each predicted particle \( \mathbf{x}_k^{i} \). We designate this version of the filter as the Smooth Kernel Particle Filter (SKPF). The results obtained with this version of the filter are clearly superior to those obtained with a classical PF implementation; see Teixeira (2007); Teixeira and Pascoal (2007).

**Point estimates**

A point estimate of the current state \( \hat{x}_k \) and the associated covariance matrices \( P_k \) can be obtained trough:

\[
\begin{align*}
\hat{x}_k^{MMS} &\simeq \sum_{i=1}^{N} \omega_k^{i} \mathbf{x}_k^{i} \\
P_k^{pf} &= \sum_{i=1}^{N} \omega_k^{i} \left( \mathbf{x}_k^{pf,i} - \mathbf{x}_k^{MMS} \right) \left( \mathbf{x}_k^{pf,i} - \mathbf{x}_k^{MMS} \right)^T \\
P_k^{pf} &= P_k^{pf} + \sum_{i=1}^{N} \omega_k^{i} \left( \mathbf{x}_k^{tf,i} - \mathbf{x}_k^{MMS} \right) \left( \mathbf{x}_k^{tf,i} - \mathbf{x}_k^{MMS} \right)^T
\end{align*}
\]

5. IMPLEMENTATION OF GEOPHYSICAL NAVIGATION

5.1 Geophysical measurements and maps

The terrain based information used by the navigation algorithms consists of acoustic measurements of range to bottom reflectors and geomagnetic gradient measurements. The main advantage of measuring the gradient of a potential field instead of its magnitude is the elimination of common mode noise and biases that normally affect geomagnetic measurements of the total field. We assume the AUV is sufficiently stable in roll and pitch to ensure that measurements of vertical gradients of the geomagnetic field can be taken without significant errors due to onboard sensor attitude. The horizontal gradients of the total magnetic field can be computed from grids of total field intensity data. To obtain the magnetic maps used by the GN algorithm, computation of the vertical gradient from gridded data can be done through Fourier transformation of the total field data or by applying Hilbert transforms to the horizontal derivatives, see e.g. Nabighian (1984) and Nelson (1986). However, it is preferable to obtain the prior maps by direct measurement of the field gradient to avoid the errors introduced by these computations.

5.2 Mapping geomagnetic gradients in 3D - Invariant gradient maps

It is important to notice that the geomagnetic field scales non-linearly with the altitude at which it is measured. To obtain geomagnetic maps at altitudes different from the measuring level it is necessary to calculate upward or downward continuations of the field. Downward continuation is an "unstable" operation, very sensitive to data noise, that requires complex signal processing to ensure reliable transformations of the data; see Cooper (2004). A navigation approach based on correlating measurements of a potential field with previously mapped values requires the utilization of a 3D grid of magnetic data with enough resolution to allow accurate localization. To accomplish this, a collection of 2D maps, calculated with a fixed vertical separation \( \Delta Z_{\text{map}} \), can be stacked to approximate a 3D map; see Fig. 2. A desirable alternative to the stacking of a collection of 2D maps is the derivation of a map that is invariant under vertical scaling for a significant range of altitudes. We have shown (Teixeira (2007)) that for a potential field that verifies the Euler’s homogeneity equation we can determine a set of points in a horizontal plane...
where the vertical gradient of the geomagnetic field is invariant under vertical scaling. This is illustrated in Fig. 3. Hence, we can use this map at different altitudes without incurring in the computationally complex problem of field continuations. Since invariant gradient maps are sparse (see Fig. 3), these maps convey less information than the corresponding 3D stacks of 2D gradient maps. In spite of this limitation, the approach based on invariant gradient maps simplifies dramatically the 3D map representation and becomes computationally much less demanding than its counterpart based on 3D grids. We have shown in previous works (Teixeira and Pascoal (2007)) that it is possible to achieve accurate localization in areas of low topographic excitation, using invariant gradient maps provided that the geomagnetic field in the area of operation presents enough information.

5.3 Integration of bathymetric and geomagnetic information

We implement the integration of bathymetric and geomagnetic information by fusing the acoustic and the geomagnetic gradient measurements in the filtering step of the Smooth Kernel Particle Filter, using the joint likelihood of the measurements according to equation (7). A more detailed discussion of this topic can be found in Teixeira (2007).

6. RESULTS OF SIMULATIONS

6.1 Scenarios and configurations used in simulations

In this work we used two scenarios that differ both in scale and type of terrain. The first scenario (Scenario 1) represents the D. João de Castro volcanic seamount in the Azores archipelago. It corresponds to an area of less than 1$Km^2$, characterized by areas of high terrain excitation intercalated with other, much smoother areas (see Fig. 4.a). The high-resolution bathymetry of this areas has been acquired using a mechanically scanned narrow beam sonar. The magnetic data used for this scenario has been modeled using GravMag software (Pedley et al. (1993)).

The second scenario (Scenario 2) includes geophysical data in an area of 10Km x 10Km resultant from a larger scale survey executed offshore northern Portugal; see Teixeira (1997). In topographical terms it is representative of the Portuguese continental shelf characterized by a very gentle slope and by the absence of prominent bathymetric anomalies. The low-resolution bathymetric map of the area has been obtained employing a standard echo-sounder. The geomagnetic data has been acquired near the sea-surface using a proton magnetometer. This magnetic information is characterized by strong magnetic anomalies with typical wavelengths of 1Km to 5Km that are probably associated with geological faults or anomalous geological bodies that are not revealed by the topography of the sea-bottom; see Fig. 5.

6.2 Bayesian setup used in the simulations

The results presented next are averaged over a set of close but independent randomly generated tracks. At each MC run, the estimated initial state vector is a random vector drawn from a normal distribution parameterized in the

Fig. 2. Stack of 2D geomagnetic maps used in GN to approximate a 3D map.

Fig. 3. Location of nodal points (black dots) corresponding to positions of invariant vertical gradient in the modeled magnetic field of D. João de Castro Seamount.

Fig. 4. (a) Bathymetry (graphical vertical exaggeration $\sim 10x$) and (b) modeled magnetic anomaly at D. João de Castro seamount, Azores.
Fig. 5. (a) Bathymetry (graphical vertical exaggeration ≃ 10x) and (b) magnetic anomalies on the continental shelf offshore Aveiro, Portugal.

Table 1. Parameters used in Scenario 1

<table>
<thead>
<tr>
<th>Type of trajectory</th>
<th>Trajectory A</th>
<th>Trajectory B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of vehicle operation (m)</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Speed of ocean current (knts)</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>Vehicle’s velocity rel. water (knts)</td>
<td>3.6</td>
<td>4.9 (max)</td>
</tr>
<tr>
<td>Duration of simulation (sec)</td>
<td>150</td>
<td>425</td>
</tr>
<tr>
<td>Number of Monte Carlo runs</td>
<td>500</td>
<td>250</td>
</tr>
</tbody>
</table>

following way: \( \mathbf{x}_0^m \sim N(\mathbf{x}_0^MC, \mathbf{P}_0^MC) ; m = 1 \ldots N_{MC} \). \( \mathbf{x}_0^MC \) is the initial state vector used in each MC simulation. This is also a random value generated according to: \( \mathbf{x}_0^MC \sim N(\mathbf{x}_0, \mathbf{P}_0) \), where \( \mathbf{x}_0 \) is the nominal initial state vector and \( \mathbf{P}_0 = \text{diag}(100, 100, 1, 1) \). \( N_{MC} \) is the number of Monte Carlo runs executed in each simulation; see Tables 1 and 3. In these tables the Type of Map used is denoted by B (bathymetric), G (vertical magnetic gradient), IG (invariant magnetic gradient) and 3G (3-axial magnetic gradient). The localization of the trajectories used in Scenarios 1 and 2 are represented in Figs. 6 and 7 respectively.

Table 2. Summary of results for Scenario 1

<table>
<thead>
<tr>
<th>Type of map used</th>
<th>Trajectory A</th>
<th>Trajectory B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Num. M.C. runs</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Percent. runs diverged</td>
<td>0</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 3. Summary of parameters and results for Scenario 2

<table>
<thead>
<tr>
<th>Type of trajectory</th>
<th>Trajectory C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of trajectory</td>
<td>lawn-mower</td>
</tr>
<tr>
<td>Type of map used</td>
<td>B.3G</td>
</tr>
<tr>
<td>Direction and speed of ocean current</td>
<td>SW-NE, 2.75 knts</td>
</tr>
<tr>
<td>Vehicle’s velocity rel. water (knts)</td>
<td>4.75 (max)</td>
</tr>
<tr>
<td>Depth of vehicle operation (m)</td>
<td>1</td>
</tr>
<tr>
<td>Duration of simulation (sec)</td>
<td>3500</td>
</tr>
<tr>
<td>Number of Monte Carlo runs</td>
<td>25</td>
</tr>
<tr>
<td>Percent. runs diverged</td>
<td>0</td>
</tr>
</tbody>
</table>

6.3 Discussion of the simulations’ results

The main parameters of the simulations executed in Scenario 1 are presented in Table 1 and the results of the simu-
The simulations executed in this work show great potential of geophysical navigation based on fusion of topographic and geomagnetic data. In particular, it has been shown that geophysical navigation can be addressed at different scales and with different types of terrain information.

The implementation of magnetic based GN as proposed here, requires precisely georeferenced data and the utilization of very accurate sensors. The practical validation of these methods will be performed in a near future by the acquisition of geophysical data and tests of geophysical navigation at sea.

REFERENCES


