Municipal Engineering - 8713
Assignment #1. 2007

1) Population for all metropolitan areas

1991 - 17,164,331
1996 - 18,413,521
2001 - 19,620,366

$$P_F = P_0 + \left(\frac{\Delta P}{\Delta t}\right) t$$

$$19{,}620{,}366 = 18{,}413{,}521 + \left(\frac{1{,}206{,}845}{5}\right) 5$$

$$P_F = 19{,}620{,}366 + \left(\frac{1{,}206{,}845}{5}\right) 10$$ 1

$$= \underline{\underline{22{,}034{,}056 \text{ in } 2011.}}$$

3)

1991 17,164,331
1996 18,413,521
2001 19,620,366

$$\ln P_F = \ln P_0 + K'\Delta t$$
$$\ln 19{,}620{,}366 = \ln 18{,}413{,}521 + K'(5)$$
$$16.79208 = 16.72860 + K'5$$
$$K' = 0.06348/5$$
$$K' = 0.012697$$ 1

$$\ln P_F = \ln 19{,}620{,}366 + 0.012697(10)$$
$$= 16.79208 + 0.12697$$
$$= 16.91905$$

$$\underline{\underline{P_F = 22{,}276{,}658 \text{ in } 2011}}$$ 1

3) $P_0 = 17,164,331$
$P_1 = 18,413,521$
$P_2 = 19,620,366$

$$P_{sat} = \frac{2P_0P_1P_2 - P_1^2(P_0+P_2)}{P_0P_2 - P_1^2}$$

numerator $= 2 \times 17,164,331 \times 18,413,521 \times 19,620,366 - 18,413,521^2 (17,164,331 + 19,620,366)$

$= 1.2402 \times 10^{22} - 1.2472 \times 10^{22}$

$= -0.0070 \times 10^{22}$ ①

denominator $= 17,164,331 \times 19,620,366 - 18,413,521^2$

$= 3.3677 \times 10^{14} - 3.39058 \times 10^{14}$

$= -0.02288 \times 10^{14}$

$$P_{sat} = \frac{0.007 \times 10^{22}}{0.02288 \times 10^{14}} = 30,594,406$$ ①

$$a = \ln\left(\frac{P_{sat} - P_0}{P_0}\right) = \ln\left(\frac{30,594,406 - 17,164,331}{17,164,331}\right) = \ln(0.78244)$$

$= -0.245337$ ①

$$b = \frac{1}{n} \ln \frac{P_0 (P_{sat} - P_1)}{P_1 (P_{sat} - P_0)}$$

$$= \frac{1}{5} \ln \frac{17,164,331 (30,594,406 - 18,413,521)}{18,413,521 (30,594,406 - 17,164,331)}$$

$$= \frac{1}{5} \ln \frac{2.0908 \times 10^{14}}{2.4729 \times 10^{14}} = -3.3569 \times 10^{-2} = -0.033569$$ ①

3) continued. $P_F = \frac{P_{sat}}{1 + e^{a + b\Delta t}}$

$$P_F = \frac{30,594,406}{1 + e^{(-0.24534 + (-0.033569) \times 40)}} = \frac{30,594,406}{1 + e^{-1.5881}}$$

$$= \frac{30,594,406}{1 + 0.20431} =$$

$$= \underline{\underline{25,404,095 \text{ in } 2031}}$$ ①

$$P_F = \frac{30,594,406}{1 + e^{(-0.24534 + (-0.033569) \times 30)}} = \frac{30,594,406}{1 + e^{-1.25241}}$$

$$= \frac{30,594,406}{1 + 0.285815}$$

$$= \underline{\underline{23,793,785 \text{ in } 2021}}$$ ①

4) 1991 – 17,164,331 use P_{sat} from problem #3.
1996 – 18,413,521
2001 – 19,620,366

$$K'' = -\frac{1}{n} \ln \left(\frac{P_{sat} - P_1}{P_{sat} - P_0} \right) = -$$

$$= -\frac{1}{n} \ln \left(\frac{30,594,406 - 18,413,521}{30,594,406 - 17,164,331} \right)$$

$$= -\frac{1}{5} \ln \left(\frac{12,180,885}{13,430,075} \right)$$

$$= 0.019526$$ ①

) continued. $K'' = -\frac{1}{n} \ln \left(\frac{30,594,406 - 19,620,366}{30,594,406 - 18,413,521} \right)$

$= -\frac{1}{5} \ln \left(\frac{10,974,040}{12,180,885} \right)$

$= 0.020867$ ①

average $K'' = \frac{0.019526 + 0.020867}{2} = 0.040393 \div 2$

$= 0.0201965$ ①

$P_F = P_o + (P_{sat} - P_o)(1 - e^{-K'' \Delta t})$

$= 17,164,331 + (30,594,406 - 17,164,331) \times$

$(1 - e^{-0.0201965 \times 20})$

$= 17,164,331 + (13,430,075)(1 - \underbrace{e^{-0.40393}}_{0.332309})$

$= \underline{\underline{21,627,268.}}$ ①

or

$P_F = 19,620,366 + (30,594,406 - 19,620,366) \times$

$(1 - e^{-0.0201965 \times 10})$

$= 19,620,366 + (10,974,040)(0.182876$

$= \underline{\underline{21,627,254}}$ or virtually the same answer.

5) a) 2006 population estimated on data of:

utility connections: $28{,}209 \times \frac{8633}{7152} = 34{,}050$

school enrollment: $28{,}209 \times \frac{5{,}398}{4{,}478} = 34{,}004$

telephone connections: $28{,}209 \times \frac{6{,}748}{5{,}589} = 34{,}059$

$$\text{average} = (34{,}050 + 34{,}004 + 34{,}059)/3$$
$$= \underline{\underline{34{,}038}}$$

b) Populations for cities A, B, C and D are plotted. The point at which they equal 34,038 is assumed to occur in 2006, and for city X. Averages of all projected populations are used to estimate populations for city X.

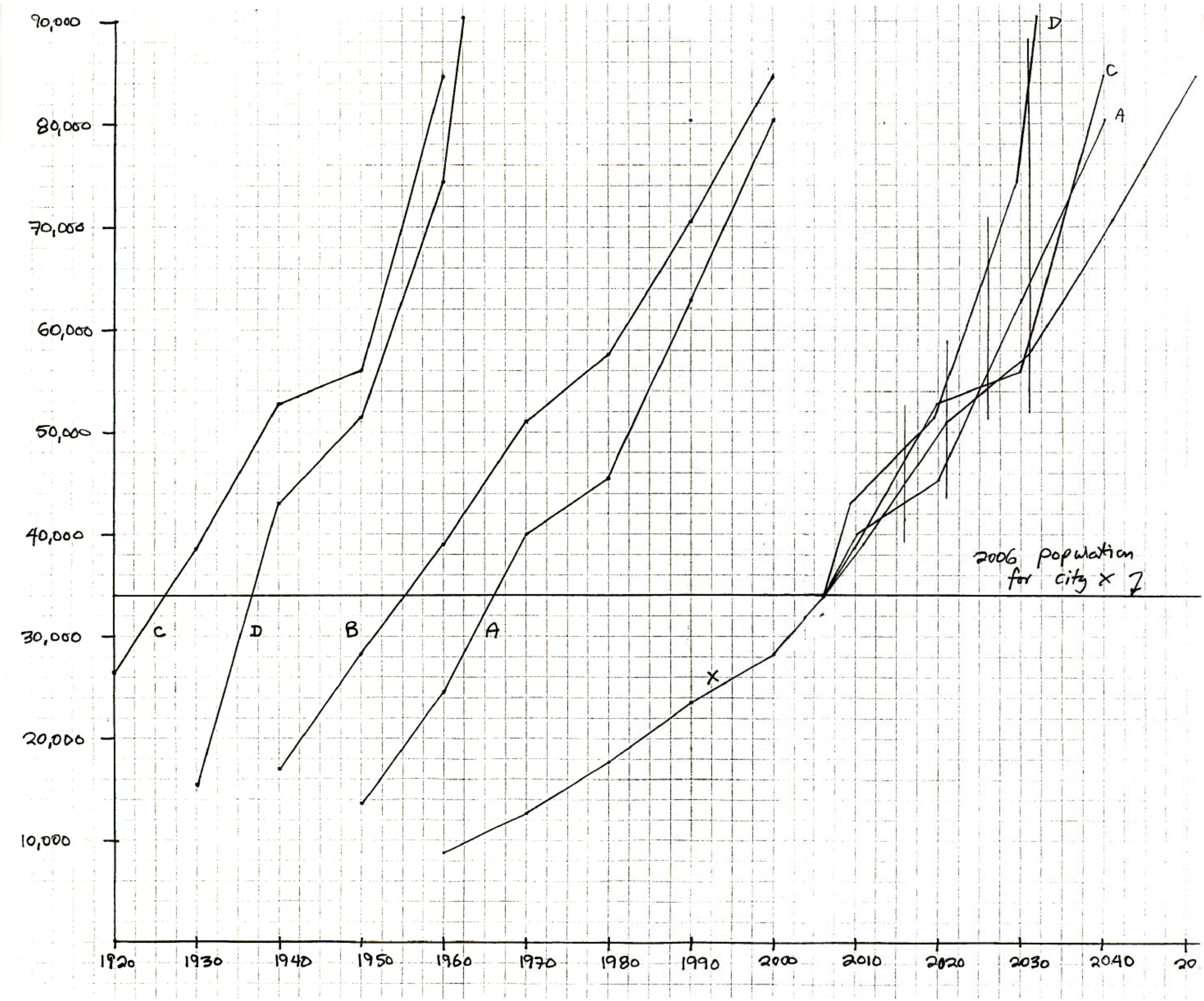
From graph:

$$2016: (43{,}100 + 44{,}900 + 47{,}200 + 48{,}300)/4$$
$$= \underline{\underline{45{,}875}}$$

$$2021: (47{,}000 + 51{,}000 + 53{,}200 + 55{,}200)/4$$
$$= \underline{\underline{51{,}600}}$$

$$2026: (54{,}300 + 54{,}700 + 56{,}000 + 68{,}200)/4$$
$$= \underline{\underline{58{,}300}}$$

$$2031: (57{,}800 + 59{,}000 + 64{,}600 + 83{,}700)/4$$
$$= \underline{\underline{66{,}275}}$$


90,000
80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
2020
2030
2040
20
C
D
B
A
X
D
C
A
2006 population for city X

6)

$$\left(\quad P_0 \quad \right)$$

1965	15,200	$= P_0$
1985	98,600	$= P_1$
2005	324,000	$= P_2$

$$P_{sat} = \frac{2(15{,}200)(98{,}600)(324{,}000) - 98{,}600^2\,(15{,}200 + 324{,}000)}{(15{,}200)(324{,}000) - 98{,}600^2}$$

$$= \frac{9.7117055 \times 10^{14} - 3.297688 \times 10^{15}}{4{,}924{,}800{,}000 - 9{,}721{,}960{,}000}$$

$$= \frac{-2.3265 \times 10^{15}}{-4{,}797{,}160{,}000}$$

$$= 484{,}974$$

$$a = \ln\left(\frac{P_{sat} - P_0}{P_0}\right) = \ln\left(\frac{484{,}974 - 15{,}200}{15{,}200}\right)$$

$$= \ln(30.906) = 3.431$$

$$b = \frac{1}{n} \ln \frac{P_0}{P_1} \left(\frac{P_{sat} - P_1}{P_{sat} - P_0} \right)$$

$$= \frac{1}{20} \ln \frac{15,200}{98,600} \left(\frac{484,974 - 98,600}{484,974 - 15,200} \right)$$

$$= \frac{1}{20} \ln \frac{15,200}{98,600} \left(\frac{386,374}{469,774} \right)$$

$$= \frac{1}{20} \ln 0.12679 \qquad = -0.10326$$

$$P_F = \frac{P_{sat}}{1 + e^{a + b\Delta t}}$$

1965 to 2015 = 50 years.

$$= \frac{484,974}{1 + e^{3.431 + (-0.10326)(50)}}$$

-5.163

-1.732

$$= \frac{484,974}{1.17693}$$

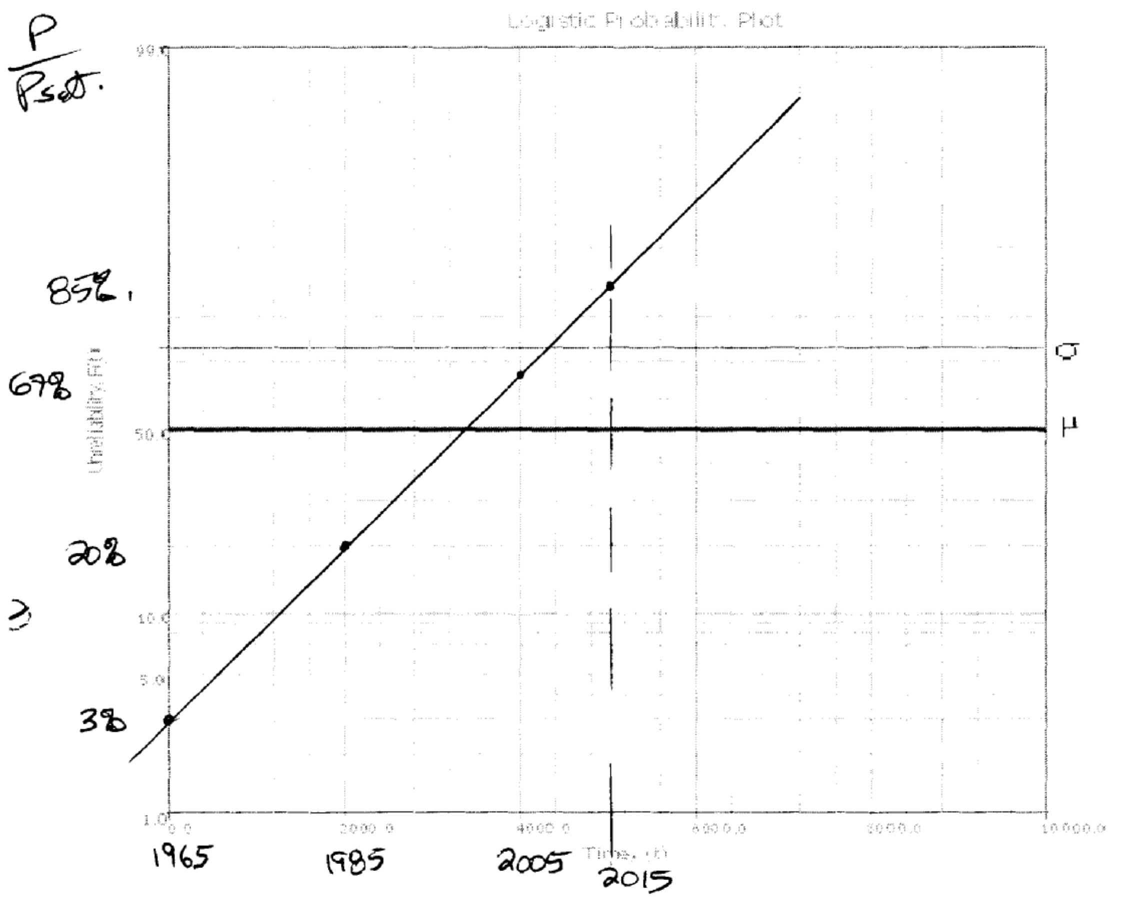
$$= \underline{\underline{412,067}}$$

$$\frac{15,200}{484,974} = 0.03$$ or 3%.

$$\frac{98,600}{484,974} = 0.20$$ or 20%.

$$\frac{324,000}{484,974} = 66.8\%.$$

$$\frac{412,067}{484,974} = 85.0\%.$$



http://www.weibull.com/LifeDataWeb/the_logistic_distribution.htm#probability_paper

"The Logistic Distribution"

Since the data forms an approximately straight line on the logistic graph paper, the calculated population for 2015 is correct.