

$$\textcircled{1} \quad R_{\log} = \log\left(\frac{C_F}{C_P}\right) = 2.5$$

$$10^{2.5} = 316.228 = \frac{C_F}{C_P}$$

$$1 - \frac{C_P}{C_F} = 1 - \frac{1}{316.228} = 0.99684$$

$$\textcircled{2} \quad 99.96\% \text{ is } 0.9996 = 1 - \frac{C_P}{C_F}$$

$$\frac{C_P}{C_F} = 1 - 0.9996 = 0.0004$$

$$\frac{C_F}{C_P} = \frac{1}{0.0004} = 2,500$$

$$R_{\log} = \log\left(\frac{C_F}{C_P}\right) = \log(2,500) = 3.398$$

$$\textcircled{3} \quad \begin{aligned} Q_{\text{winter}} &= 10,400 \text{ m}^3/\text{d} && \rightarrow \text{used for design} \\ Q_{\text{summer}} &= 15,600 \text{ m}^3/\text{d} \end{aligned}$$

$$TMP_{\text{max}} = 152 \text{ kPa}$$

$$TMP_{\text{operating}} = 0.75 (TMP_{\text{max}}) = 114 \text{ kPa}$$

$$R_m = 2.94 \times 10^{12} \text{ m}^{-1}$$

$$T_{\text{winter}} = 4^\circ\text{C}$$

$$T_{\text{summer}} = 20^\circ\text{C}$$

$$\mu = 1.567 \text{ mPa}\cdot\text{s}$$

$$\mu = 1.002 \text{ mPa}\cdot\text{s}$$

Check membrane area needed for winter and compare it to membrane area needed in the summer

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3) continued

$$J = \frac{\Delta P}{\mu R_m} = \frac{114 \text{ kPa}}{(1.567 \times 10^{-6} \text{ kPa} \cdot \text{s})(2.94 \times 10^{12} \text{ m}^{-1})}$$
$$= 2.4745 \times 10^{-5} \text{ m/s} \times \frac{60 \times 60 \times 24 \text{ s}}{\text{d}}$$
$$= 2.138 \text{ m/d or m}^3/\text{m}^2 \cdot \text{d}$$

$$A = \frac{Q}{J} = \frac{10,400 \text{ m}^3/\text{d}}{2.138 \text{ m}^3/\text{m}^2 \cdot \text{d}} = 4,864 \text{ m}^2$$

This is the membrane area required for winter operation.

$$J = \frac{114 \text{ kPa} \times 86,400 \text{ s/d}}{(1.002 \times 10^{-6} \text{ kPa} \cdot \text{s})(2.94 \times 10^{12} \text{ m}^{-1})}$$
$$= 3.344 \text{ m}^3/\text{m}^2 \cdot \text{d}$$

$$A = \frac{15,600 \text{ m}^3/\text{d}}{3.344 \text{ m}^3/\text{m}^2 \cdot \text{d}} = 4,665 \text{ m}^2$$

Since the membrane area required for Summer operation is less than for Winter operation, the winter operation design will also be enough for the summer.

4)

$$Q_{max} = 30,300 \text{ m}^3/\text{d}$$

$$J = 0.127 \text{ m}^3/\text{h} \cdot \text{m}^2$$

$$J_{max \text{ allowable}} = 0.170 \text{ m}^3/\text{h} \cdot \text{m}^2$$

Cleaning cycle = 30 min.

Time off-line to clean = 150 s. = 2.5 min.

Membrane area per module =  $15 \text{ m}^2$

modules per rack = 90

Determine # racks needed if 2 off-line at a time

$$\frac{Q_{max}}{J} = \frac{30,300 \text{ m}^3/\text{d}}{0.127 \text{ m}^3/\text{h} \cdot \text{m}^2} \times \frac{\text{d}}{24\text{h}} = 9,941 \text{ m}^2 \text{ total area}$$

$$\frac{9941 \text{ m}^2}{15 \text{ m}^2/\text{module}} = 662.7 \text{ r } 663 \text{ modules}$$

$$\frac{663 \text{ modules}}{90 \text{ modules/rack}} = 7.36 \text{ or } 8 \text{ racks}$$

$$\frac{\text{cleaning cycle } 30 \text{ min}}{\text{time off-line to clean, } 2.5 \text{ min}} = 12 \text{ racks could be cleaned in 1 cycle}$$

8 racks need to be operating at a time  
 With 2 racks off-line, a total of 10 racks  
 would be needed.

Since 12 racks can be cleaned in a cycle  
 theoretically, cleaning the 10 racks should  
 be possible.

However, to provide redundancy, probably better  
 to have 11 racks in total.

5)  $Ca^{2+} = 67.2 \text{ mg/L}$   
 $CO_3^{2-} = 0.72 \text{ mg/L}$   
 Alkalinity = 284.0 mg/L as  $CaCO_3$   
 pH = 7.6 units.

Find permeate recovery rate.

$$K_{sp} = 4.95 \times 10^{-9}$$

$$\text{For } Ca^{2+}: \frac{67.2 \text{ mg/L}}{40 \times 10^3 \text{ mg/mol}} = 1.68 \times 10^{-3} \text{ mol/L}$$

$$\text{For } CO_3^{2-}: \frac{0.72 \text{ mg/L}}{60 \times 10^3 \text{ mg/mol}} = 1.20 \times 10^{-5} \text{ mol/L}$$

$$K_{sp} = \left[ \frac{A^{+n}}{1-r} \right]^n \left[ \frac{B^{-2}}{1-r} \right] \quad [ ] \text{ denoting mol/L}$$

$$4.95 \times 10^{-9} = \left[ \frac{1.68 \times 10^{-3}}{1-r} \right]^2 \left[ \frac{1.20 \times 10^{-5}}{1-r} \right]$$

$$(1-r)^2 = \frac{2.016 \times 10^{-8}}{4.95 \times 10^{-9}} = 4.0727$$

$$1-r = 2.018$$

$$r = 1 - 2.018 = -1.018 \text{ so no recovery is possible.}$$

6) Find sulfuric acid that will give 75% recovery in previous problem water.

$$(284 \frac{\text{mg}}{\text{L}} \text{ as } \text{CaCO}_3) \left( \frac{61 \text{ g/eq HCO}_3^-}{50 \text{ g/eq CaCO}_3} \right) = 346.48 \frac{\text{mg}}{\text{L}} \text{ as HCO}_3^-$$

$$\frac{346.48 \text{ mg/L as HCO}_3^-}{61,000 \text{ mg/mol}} = 5.68 \times 10^{-3} \text{ mol/L}$$

$$K_{a2} = \frac{[\text{H}^+][\text{CO}_3^{2-}]}{[\text{HCO}_3^-]} \quad ; \quad \text{p}K_{a2} = 10.33 \quad ; \quad K_{a2} = 10^{-10.33}$$

\* see below

$$\text{CO}_3^{2-} = \frac{10^{-10.33} [5.68 \times 10^{-3}]}{[\text{H}^+]}$$

$$K_{sp} = \left[ \frac{A^{p+}}{1-r} \right]^n \left[ \frac{B^{2-}}{1-r} \right]^m$$

$$4.95 \times 10^{-9} = \left[ \frac{1.68 \times 10^{-3}}{0.25} \right] \left[ \frac{10^{-10.33} [5.68 \times 10^{-3}]}{0.25 [\text{H}^+]} \right]$$

$$= \left[ 6.72 \times 10^{-3} \right] \left[ \frac{1.063 \times 10^{-12}}{[\text{H}^+]} \right]$$

$$[\text{H}^+] = \frac{[6.72 \times 10^{-3}][1.063 \times 10^{-12}]}{4.95 \times 10^{-9}} = 1.443 \times 10^{-6}$$

$$\text{pH} = -\log [\text{H}^+] = -\log [1.443 \times 10^{-6}] = \text{pH } 5.84$$

acid must be added to drop to pH 5.84

At pH 5.84 much bicarbonate will have been converted to carbonic acid

$$\begin{aligned} * \text{p}K_{a2} &= 10.33 = -\log [K_{a2}] \\ \log [K_{a2}] &= -10.33 \\ K_{a2} &= 10^{-10.33} \end{aligned}$$

(5)

6) continued

$$K_{a1} = \frac{[H^+][HCO_3^-]}{[H_2CO_3]}$$

At pH 7.6 (at start)  $H_2CO_3$  is negligible

$$pK_{a1} = 6.35 ; K_{a1} = 10^{-6.35} = 4.47 \times 10^{-7}$$

$$4.47 \times 10^{-7} = \frac{[H^+][HCO_3^- - x]}{[0 + x]}$$

$x = [H^+]$  to react with  $HCO_3^-$

$$4.47 \times 10^{-7} = \frac{[1.443 \times 10^{-6}][5.63 \times 10^{-3} - x]}{[0 + x]}$$

$$x = \frac{[1.443 \times 10^{-6}][5.63 \times 10^{-3} - x]}{4.47 \times 10^{-7}}$$

$$x = 1.817 \times 10^{-2} - 3.228x$$

$$x = \frac{1.817 \times 10^{-2}}{4.228} = 4.298 \times 10^{-3} \text{ mol/L}$$

1 mole  $H_2SO_4 \rightarrow 2$  moles  $H^+$

$$\frac{4.298 \times 10^{-3} \text{ mol/L}}{2} = 2.148 \times 10^{-3} \text{ mol/L}$$

$$2.148 \times 10^{-3} \frac{\text{mol}}{\text{L}} \left( 98,000 \frac{\text{mg}}{\text{mol}} \right) = 210 \frac{\text{mg}}{\text{L}}$$

pure acid

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