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ENERGY BASED ICE COLLISION FORCES

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ABSTRACT

Ice collision forces can be determined by energy considerations. A variety of interaction geometry cases are considered. The indentation energy functions for eight different cases are derived and expressed in a common format. The indentation functions are expressed as functions of the indentation model parameters, assuming a pressure-area representation. Two types of collisions are identified; simple impacts which can be treated as equivalent to one-dimensional collisions, and beaching collisions which involve two-dimensional behaviour (indentation and sliding). Solutions for the impact cases are presented for all geometry cases. A solutions procedure is presented for the beaching collision, with an exact solution for a linear case. Design equation development and future directions are discussed.

1. INTRODUCTION

Ice forces on ships and structures are typically the result of collisions. The magnitude of the force is determined by some form of limit (see e.g. Croasdale, 1980, Daley, Tuhkuri and Riska 1998). In some cases the ice strength is the determining factor, while in others the force may be limited by available kinetic energy. In such cases the available kinetic energy is expended in crushing (irrecoverable) and potential (recoverable) energy. Energy methods provide a simple method of determining forces, and have long been used to do so (see Popov et. al. 1967). This paper will summarize the general energy approach, derive some old and new cases and provide examples.

2. GENERAL APPROACH

The problem under discussion is one of impact between two objects. It is assumed that one body is initially moving (the impacting body) and the other is at rest (the impacted body). This concept applies to a ship striking an ice edge, or ice striking an offshore structure. The energy approach is based on equating the available kinetic energy with the energy expended in crushing and potential energy:

$$KE_e = IE + PE \tag{1}$$

The available kinetic energy is the difference between the initial kinetic energy of the impacting body and the total kinetic energy of both bodies at the point of maximum force. If the impacted body has finite mass it will gain kinetic energy. Only in the case of a direct (normal) collision involving one infinite (or very large) mass will the effective kinetic energy be the same as the total kinetic energy. In such a case all motion will cease at the time of maximum force. The indentation energy is the integral of the indentation force F_n on the crushing indentation displacement ζ_c ;

$$IE = \int_0^{\zeta_m} F_n \cdot d\zeta_c \tag{2}$$

The potential energy is the energy that has been expended in recoverable processes, which can be either rigid body motions (pitch/heave) or elastic deformation (of either body). The potential energy is the integral of the indentation force F_n on the recoverable displacement ζ_e :

$$PE = \int_0^{\zeta} F_n \cdot d\zeta_e \tag{3}$$

These equations are the basis of all solutions. Equation (1) can be solved for F_n provided that the required kinematic and geometric values are known. The general approach to determining *IE* and *PE* will be described next, with specific geometric examples further on. After that the determination of collision forces will be discussed.

3. ICE INDENTATION

In order to pose and solve the general energy equations it is necessary to formulate an equation relating force to indentation. By using the pressure-area relationship to describe the ice pressures, it is easy to derive a force-indentation relationship. This assumption means that ice force will depend only on indentation. In this case the maximum force occurs at the time of maximum penetration. The collision geometry is the ice/structure overlap geometry. The average pressure P_{av} in the nominal contact area A is related to the nominal contact area as;

$$P_{av} = P_0 \cdot A^{ex} \tag{4}$$

where Po is the pressure at 1m^2, and ex is a constant.

The ice force is also related to the nominal contact area;

$$F_i = P_{av} \cdot A = P_0 \cdot A^{1+ex} \tag{5}$$

The available kinetic energy may be the total kinetic energy, in the case of a head-on collision, in which all motion ceases at the point of maximum force. Alternatively the available energy may be the ‘normal’ or ‘effective’ kinetic energy, as in the case of a glancing collision.

4. INDENTATION ENERGY

For each geometric case, there is a relationship between the normal indentation ζ_n and normal contact area;

$$A_n = f_A(\zeta_n) \tag{6}$$

where f_A is a function that depends on the contact geometry. This results in a function relating force to indentation;

$$F_n = f_F(\zeta_n) \tag{7}$$

where $f_F = Po\ (f_A(\zeta_n))^{1+ex}$. The next step is to determine the indentation energy *IE*, which is found by integrating the force;

$$IE = \int F_n d\zeta_n = f_{IE}(\zeta_n) \tag{8}$$

where f_{IE} is a function giving the indentation energy.

5. POTENTIAL ENERGY

For each geometric/kinematic case, there may be a relationship between the normal force F_n and potential energy. In the case of ramming the vertical component of the indentation force results in potential energy in pitch/heave. This can be expressed in terms of indentation as;

$$PE = f_{PE}(\zeta_n) \tag{9}$$

where f_{PE} is a function giving the indentation energy.

6. INDENTATION GEOMETRY CASES

The relationship between indentation and nominal crushing area depend on the collision geometry. The following cases of interest apply to both ship-ice and ice-structure collision problems.

6.1 Case 1 : Symmetric V Wedge

Figure 1 shows a symmetric wedge-shaped indentation in a square edge. The indentation energy is derived as follows. The projected areas, vertical, horizontal and normal are;

$$A_v = \zeta_n^2 \frac{\tan(\alpha)}{\cos^2(\gamma)} \tag{10}$$

$$A_h = \zeta_n^2 \frac{\tan(\alpha)}{\cos^2(\gamma)\tan(\gamma)} \tag{11}$$

$$A_n = \zeta_n^2 \frac{\tan(\alpha)}{\cos^2(\gamma)\sin(\gamma)} \tag{12}$$

The normal force is related to the normal area by the pressure/area relation. The average pressure is:

$$\bar{p} = p_o A_n^{ex} \tag{13}$$

Force is:

$$F_n = \bar{p} A_n \tag{14}$$

and hence force can be stated as:

$$F_n = p_o A_n^{1+ex} \tag{15}$$

Substituting (12) into (15) we arrive at:

$$F_n = p_o \left(\frac{\tan(\alpha)}{\cos^2(\gamma)\sin(\gamma)} \right)^{1+ex} \cdot \zeta_n^{2+2ex} \tag{16}$$

The indentation energy is found by substituting (16) into (8), to give:

$$IE = \frac{p_o}{3 + 2 \cdot ex} \left(\frac{\tan(\alpha)}{\cos^2(\gamma)\sin(\gamma)} \right)^{1+ex} \zeta_n^{3+2ex} \tag{17}$$

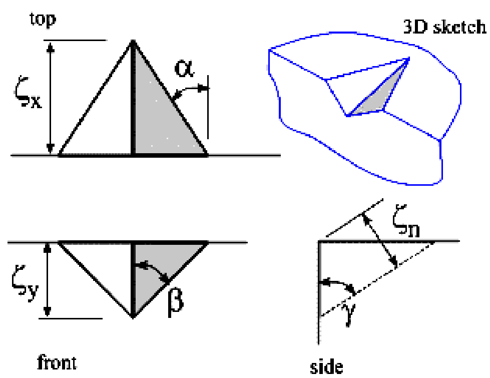

Figure 1. Symmetric V Wedge Indentation

6.2 Case 2 : Symmetric Spoon

Figure 2 shows a symmetric spoon-shaped indentation in a square edge. The indentation energy is derived as follows. The projected areas, vertical, horizontal and normal are;

$$A_v = \left(\frac{\zeta_n}{\cos(\gamma)}\right)^{bex+1} \frac{2 \cdot c}{(bex+1)} \tag{18}$$

$$A_h = \left(\frac{\zeta_n}{\cos(\gamma)}\right)^{bex+1} \frac{2 \cdot c}{(bex+1)\tan(\gamma)} \tag{19}$$

$$A_n = \left(\frac{\zeta_n}{\cos(\gamma)}\right)^{bex+1} \frac{2 \cdot c}{(bex+1)\sin(\gamma)} \tag{20}$$

where $c = Beam/(2\ LB^{bex})$
Beam is the beam of the vessel
LB is the bow length
(note that the equation for the bow waterline is $y = c\ x^{bex}$)
.
Equations (13) to (15) are used again. Substituting (20) into (15) we arrive at:

$$F_n = p_o \left(\left(\frac{1}{\cos(\gamma)}\right)^{bex+1} \frac{2 \cdot c}{(bex+1)\sin(\gamma)} \right)^{1+ex} \cdot \zeta_n^{(1+bex)(1+ex)} \tag{21}$$

The indentation energy is found by substituting (21) into (8), to give:

$$IE = \frac{p_o}{(bex+1)(1+ex)+1} \left(2 \left(\frac{1}{\cos(\gamma)}\right)^{bex+1} \frac{c}{(bex+1)\sin(\gamma)} \right)^{1+ex} \zeta_n^{(bex+1)(1+ex)+1} \tag{22}$$

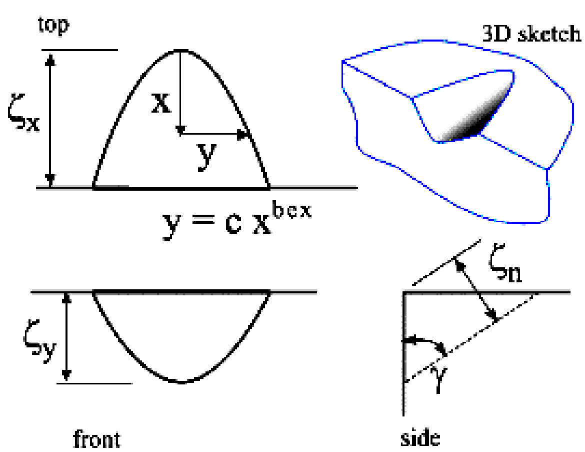

Figure 2. Symmetric Spoon Indentation

6.3 Case 3 : Right-Angle Edge

Figure 3 shows a right-angle wedge indentation. The indentation energy is derived as follows. The projected areas, vertical, horizontal and normal are;

$$A_v = \frac{\zeta_n^2}{\sin(\alpha)\cos(\alpha)\cos^2(\beta`)} \tag{23}$$

$$A_h = \frac{\zeta_n^2}{\sin(\alpha)\cos(\alpha)\sin(\beta`)\cos(\beta`)} \tag{24}$$

$$A_n = \frac{\zeta_n^2}{\sin(\alpha)\cos(\alpha)\sin(\beta`)\cos^2(\beta`)} \tag{25}$$

Equations (13) to (15) are used again. Substituting (25) into (15) we arrive at:

$$F_n = p_o\left(\frac{1}{\sin(\alpha)\cos(\alpha)\sin(\beta`)\cos^2(\beta`)}\right)^{1+ex} \cdot \zeta_n^{2+2ex} \tag{26}$$

The indentation energy is found by substituting (26) into (8), to give:

$$IE = \frac{p_o}{(3+2ex)}\left(\frac{1}{\sin(\alpha)\cos(\alpha)\sin(\beta`)\cos^2(\beta`)}\right)^{1+ex} \cdot \zeta_n^{3+2ex} \tag{27}$$

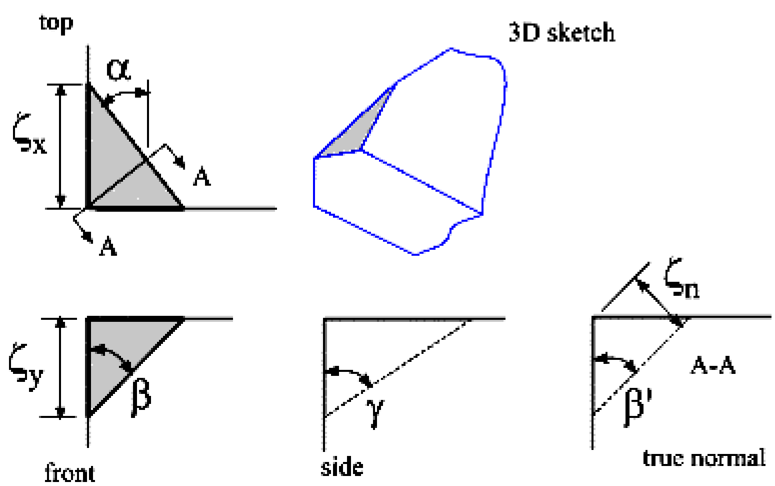

Figure 3. Right-apex Oblique Indentation

6.4 Case 4 : General Wedge (Normal to hull)

Figure 4 shows a general wedge-shaped edge indentation (normal to hull). The indentation energy is derived as follows. The projected areas, vertical, horizontal and normal are;

$$A_v = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\cos^2(\beta`)} \tag{28}$$

$$A_h = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\sin(\beta`)\cos(\beta`)} \tag{29}$$

$$A_n = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\sin(\beta`)\cos^2(\beta`)} \tag{30}$$

Equations (13) to (15) are used again. Substituting (30) into (15) we arrive at:

$$F_n = p_o \left(\frac{\tan(\phi/2)}{\sin(\beta`)\cos^2(\beta`)} \right)^{1+ex} \cdot \zeta_n^{2+2ex} \tag{31}$$

The indentation energy is found by substituting (31) into (8), to give:

$$IE = \frac{p_o}{(3+2ex)} \left(\frac{\tan(\phi/2)}{\sin(\beta`)\cos^2(\beta`)} \right)^{1+ex} \cdot \zeta_n^{3+2ex} \tag{32}$$

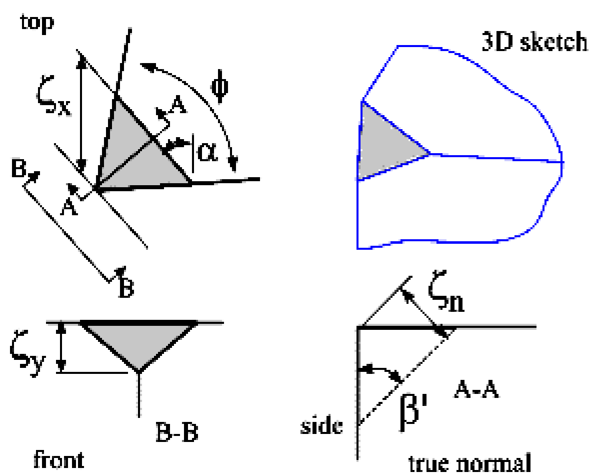

Figure 4. General Wedge-shaped Edge (normal to hull).

6.5 Case 5 : General Round Edge

Figure 5 shows a general round indentation. The indentation energy is derived as follows. The projected areas, vertical, horizontal and normal are;

$$A_v = \frac{4}{3\cdot\cos^{1.5}(\beta`)}\zeta_n^{1.5}\sqrt{2R} \tag{33}$$

$$A_h = \frac{4}{3\cdot\cos^{1.5}(\beta`)\tan(\beta`)}\zeta_n^{1.5}\sqrt{2R} \tag{34}$$

$$A_n = \frac{4}{3\cdot\cos^{1.5}(\beta`)\sin(\beta`)}\zeta_n^{1.5}\sqrt{2R} \tag{35}$$

Equations (13) to (15) are used again. Substituting (35) into (15) we arrive at:

$$F_n = p_o\left(\frac{4}{3\cdot\cos^{1.5}(\beta`)\sin(\beta`)}\sqrt{2R}\right)^{1+ex}\cdot\zeta_n^{1.5+1.5ex} \tag{36}$$

The indentation energy is found by substituting (36) into (8), to give:

$$IE = \frac{p_o}{(2.5+1.5ex)}\left(\frac{4}{3\cdot\cos^{1.5}(\beta`)\sin(\beta`)}\sqrt{2R}\right)^{1+ex}\cdot\zeta_n^{2.5+1.5ex} \tag{37}$$

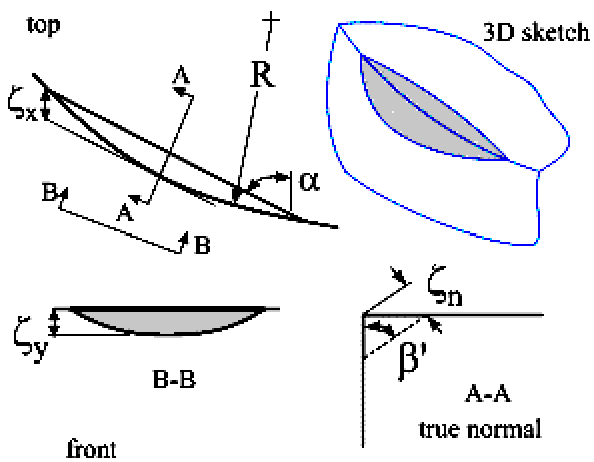

Figure 5. General Round Edge .

6.6 Case 6 : Round Vertical Cylinder

Figure 6 shows a general round indentation. The indentation energy is derived as follows. The projected normal area is;

$$A_n = 2H\sqrt{2R\zeta_n} \tag{38}$$

Equations (13) to (15) are used again. Substituting (38) into (15) we arrive at:

$$F_n = p_o\left(2H\sqrt{2R}\right)^{1+ex} \cdot \zeta_n^{.5+.5ex} \qquad (39)$$

The indentation energy is found by substituting (39) into (8), to give:

$$IE = \frac{p_o}{(1.5+.5ex)}\left(2H\sqrt{2R}\right)^{1+ex} \cdot \zeta_n^{1.5+.5ex} \qquad (40)$$

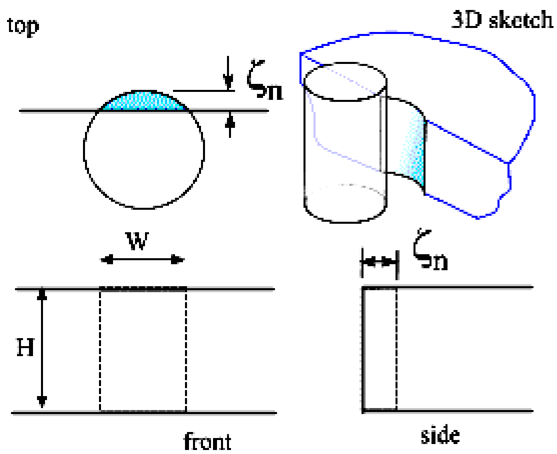

Figure 6. Round Vertical Cylinder

6.7 Case 7 : Rectangular Vertical Cylinder

Figure 7 shows a general rectangular indentation. The indentation energy is derived as follows. The projected normal area is;

$$A_n = HW \qquad (41)$$

Equations (13) to (15) are used again. Substituting (41) into (15) we arrive at:

$$F_n = p_o(HW)^{1+ex} \qquad (42)$$

The indentation energy is found by substituting (42) into (8), to give:

$$IE = p_o(HW)^{1+ex} \cdot \zeta_n \qquad (43)$$

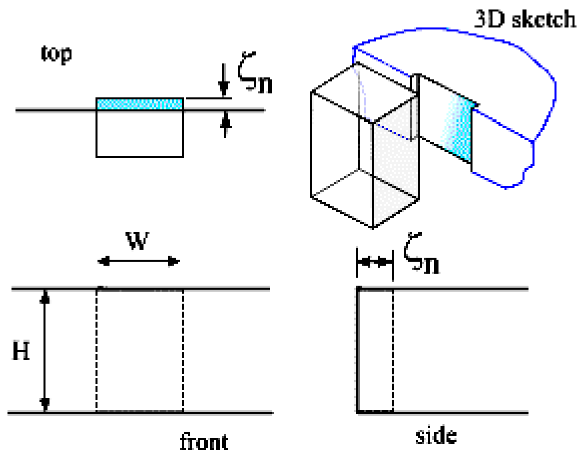

Figure 7. Rectangular Vertical Cylinder

6.8 Case 8 : Spherical Contact

Figure 8 shows a spherical indentation. The indentation energy is derived as follows. The projected normal area is;

$$A_n = 2 \cdot \pi \cdot R \cdot \zeta_n \tag{44}$$

Equations (13) to (15) are used again. Substituting (44) into (15) we arrive at:

$$F_n = p_o (2 \cdot \pi \cdot R)^{1+ex} \cdot \zeta_n^{1+ex} \tag{45}$$

The indentation energy is found by substituting (45) into (8), to give:

$$IE = \frac{p_o}{(2+ex)} (2 \cdot \pi \cdot R)^{1+ex} \cdot \zeta_n^{2+ex} \tag{46}$$

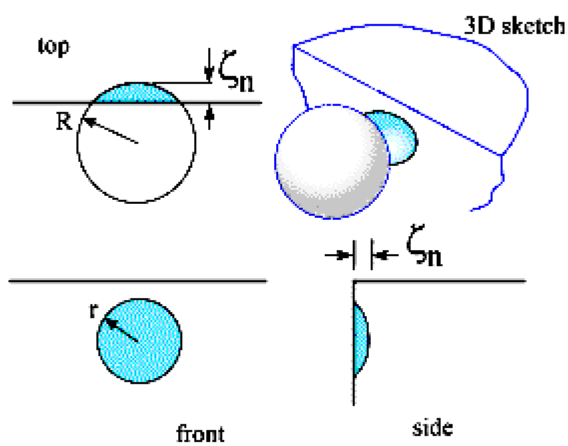

Figure 8. Spherical Contact

7. SUMMARY OF CASES

In each case the force and indentation energy can be stated as;

$$F_n = p_o \cdot fa \cdot \zeta_n^{fx-1} \tag{47}$$

$$IE = \frac{p_o}{fx}\, fa \cdot \zeta_n^{fx} \tag{48}$$

where *fx* is a function of *ex*, and *fa* is a function of the geometric parameters. Table 1 summarizes the *fx* and fa functions for each of the cases.

Table 1 Indentation functions

Geometric Case	fx	fa
Case 1 : Symmetric V Wedge	$fx = (3 + 2 \cdot ex)$	$fa = \left(\frac{\tan(\alpha)}{\cos^2(\gamma)\sin(\gamma)} \right)^{1+ex}$
Case 2 : Symmetric Spoon	$fx = ((bex+1)(1+ex)+1)$	$fa = \left(2\left(\frac{1}{\cos(\gamma)}\right)^{bex+1} \frac{c}{(bex+1)\sin(\gamma)} \right)^{1+ex}$
Case 3 : Right-Angle Edge	$fx = (3 + 2 \cdot ex)$	$fa = \left(\frac{1}{\sin(\alpha)\cos(\alpha)\sin(\beta`)\cos^2(\beta`)} \right)^{1+ex}$
Case 4 : General Wedge (Normal to hull)	$fx = (3 + 2 \cdot ex)$	$fa = \left(\frac{\tan(\phi/2)}{\sin(\beta`)\cos^2(\beta`)} \right)^{1+ex}$
Case 5 : General Round Edge	$fx = (2.5 + 1.5 \cdot ex)$	$fa = \left(\frac{4}{3 \cdot \cos^{1.5}(\beta`)\sin(\beta`)} \sqrt{2R} \right)^{1+ex}$
Case 6 : Round Vertical Cylinder	$fx = (1.5 + 0.5 \cdot ex)$	$fa = \left(2H\sqrt{2R} \right)^{1+ex}$
Case 7 : Rectangular Vertical Cylinder	$fx = 1$	$fa = (HW)^{1+ex}$
Case 8 : Spherical Contact	$fx = (2 + ex)$	$fa = (2 \cdot \pi \cdot R)^{1+ex}$

8. COLLISION TYPES

There are two types of collisions that can (presently) be solved by energy methods. The first is a ‘normal’ type of impact. For general ship collisions this is referred to as a ‘Popov’ (see Popov et. al. 1967) type of impact. The second is a beaching impact. Both will be described and applied to various cases. Figure 9 shows a sketch of the two conditions, as they may exist in a head-on ram. Either force may be larger, depending on the circumstances.

In the normal impact case the collision is idealized as a one-dimensional (normal) impact. The normal kinetic energy is equated to the indentation energy. Potential energy (for example, due to beaching) is ignored. Typically friction from sliding is also ignored. This type of analysis can be used with any of the geometric cases described above, and for ship-ice and ice-structure collisions. The analysis is valid within the range of the assumptions.

The beaching impact is a two-dimensional analysis. The total kinetic energy is equated to the sum of indentation and potential energy. Again, friction is typically ignored. This type of analysis can be used with geometric cases 1 and 2 described above, for ship-ice collisions. And again, the analysis is valid within the range of the assumptions.

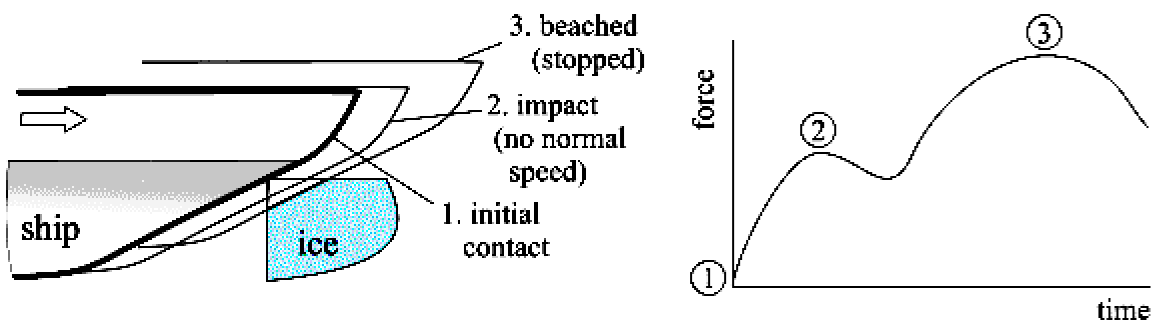

Figure 9. Collision conditions, initial impact and beached condition

9. INITIAL IMPACT COLLISIONS

A wide variety of collision scenarios can be analyzed as ‘normal’ collisions. The general approach is presented followed by the force values that occur for the set of cases in Table 1. Start by equating the normal kinetic energy with the ice crushing energy.

$$KE_e = IE \tag{49}$$

where

$$KE_e = \frac{Me}{2} \cdot Vn^2 \tag{50}$$

which ,using equation (48) can be stated as;

$$KE_e = \frac{p_o}{fx}\, fa \cdot \zeta_n^{fx} \tag{51}$$

Solving for the normal indentation:

$$\zeta_n = \left(\frac{KE_e \cdot fx}{p_o \cdot fa} \right)^{\frac{1}{fx}} \tag{52}$$

The normal force can be found by substituting eqn. (52) into (47) to give

$$F_n = p_o \cdot fa \cdot \left(\frac{KE_e \cdot fx}{p_o \cdot fa} \right)^{\frac{fx-1}{fx}} \tag{53}$$

The values from Table 1 can be substituted into eqn. (53), together with (50) to get impact force equations for each case. Table 2 shows the equations. The effective kinetic energy depends on the nature of the collision. For simple direct collisions the effective kinetic energy (eqn. (50)) is the total kinetic energy. For ship-ice collisions (see Figure 10) , the effective mass and velocity properties at the point of impact are determined as follows (see Appendix for the *lx* and *Co* terms) ;

$$V_n = V_{ship} \cdot lx \tag{54}$$

where V_n is the normal velocity at the point of impact
V_{ship} is the x-direction velocity (all others are zero)
lx is the x-direction cosine

$$M_e = \frac{M_{ship}}{Co} \tag{55}$$

where M_e is effective mass at the point of impact
M_{ship} is the ship's mass (displacement)
Co is the mass reduction factor

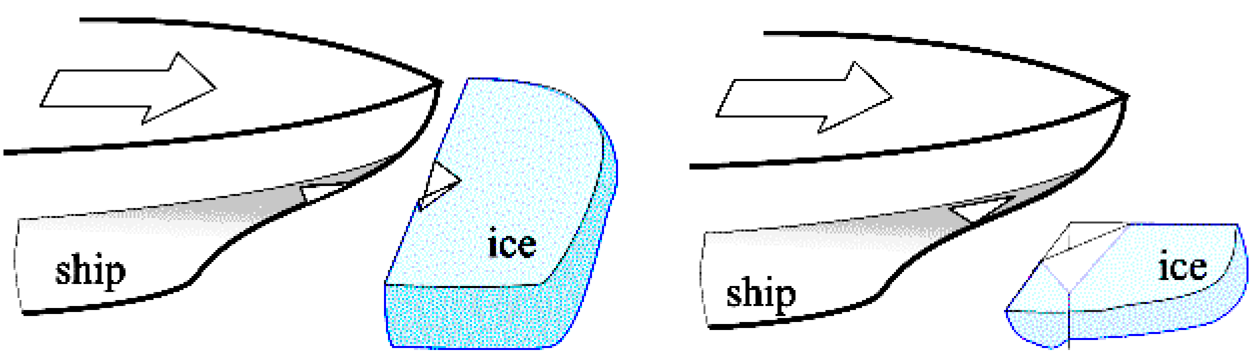

Figure 10. Head-on (Symmetrical) and Shoulder (Oblique) Impacts.

Table 2 Force equations – valid for initial impact (normal impact) conditions

Case 1: $F_n := \left[p_o \cdot \left[\frac{\tan(\alpha)}{\left(\cos(\gamma)^2 \cdot \sin(\gamma)\right)} \right]^{(1+ex)} \right]^{\left[\frac{1}{(3+2 \cdot ex)}\right]} \cdot \left[\frac{1}{2} \cdot Me \cdot Vn^2 \cdot (3+2 \cdot ex) \right]^{\left[\frac{(2+2 \cdot ex)}{(3+2 \cdot ex)}\right]}$
Case 2: $F_n := \left[p_o \cdot \left[2 \cdot \left(\frac{1}{\cos(\gamma)}\right)^{(bex+1)} \cdot \frac{c}{((bex+1) \cdot \sin(\gamma))} \right]^{(1+ex)} \right]^{\frac{1}{(bex+1) \cdot (1+ex)+1}} \cdot \left[\frac{1}{2} \cdot Me \cdot Vn^2 \cdot ((bex+1) \cdot (1+ex)+1) \right]^{\left[(bex+1) \cdot \frac{(1+ex)}{((bex+1) \cdot (1+ex)+1)}\right]}$
Case 3: $F_n := \left[\left[p_o \cdot \left[\frac{1}{\left(\sin(\alpha) \cdot \cos(\alpha) \cdot \sin(\beta') \cdot \cos(\beta')^2\right)} \right]^{(1+ex)} \right]^{\frac{1}{3+2 \cdot ex}} \cdot \left[\frac{1}{2} \cdot Me \cdot Vn^2 \cdot (3+2 \cdot ex) \right]^{\left[\frac{(2+2 \cdot ex)}{(3+2 \cdot ex)}\right]} \right]$
Case 4: $F_n := \left[p_o \cdot \left[\frac{\tan\left(\frac{1}{2} \cdot \phi\right)}{\left(\sin(\beta') \cdot \cos(\beta')^2\right)} \right]^{(1+ex)} \right]^{\frac{1}{3+2 \cdot ex}} \cdot \left[\frac{1}{2} \cdot Me \cdot Vn^2 \cdot (3+2 \cdot ex) \right]^{\left[\frac{(2+2 \cdot ex)}{(3+2 \cdot ex)}\right]}$
Case 5: $F_n := \left[p_o \cdot \left[\frac{4}{\left(3 \cdot \cos(\beta')^{1.5} \cdot \sin(\beta')\right)} \cdot \sqrt{2} \cdot \sqrt{R} \right]^{(1+ex)} \right]^{\frac{1}{2.5+1.5 \cdot ex}} \cdot \left[\frac{1}{2} \cdot Me \cdot Vn^2 \cdot (2.5+1.5 \cdot ex) \right]^{\left[\frac{(1.5+1.5 \cdot ex)}{(2.5+1.5 \cdot ex)}\right]}$
Case 6: $F_n := \left[p_o \cdot \left(2 \cdot H \cdot \sqrt{2} \cdot \sqrt{R}\right)^{(1+ex)} \right]^{\frac{1}{1.5+.5 \cdot ex}} \cdot \left[\frac{1}{2} \cdot Me \cdot Vn^2 \cdot (1.5+.5 \cdot ex) \right]^{\left[\frac{(.5+.5 \cdot ex)}{(1.5+.5 \cdot ex)}\right]}$
Case 7: $F_n := p_o \cdot (H \cdot W)^{(1+ex)}$
Case 8: $F_n := \left[p_o \cdot (2 \cdot \pi \cdot R)^{(1+ex)} \right]^{\frac{1}{2+ex}} \cdot \left[\frac{1}{2} \cdot Me \cdot Vn^2 \cdot (2+ex) \right]^{\left[\frac{(1+ex)}{(2+ex)}\right]}$

10. BEACHING IMPACT COLLISIONS

Head-on collisions between a ship and ice can result in the ship sliding up and beaching on the ice. The general approach to these collisions is presented followed by the force values that occur for the set of cases 1 and 2 in Table 1. To start we assume that initial kinetic energy is equal to the sum of ice indentation (crushing) energy and pitch/heave potential;

$$KE \;= IE + PE \tag{56}$$

The kinetic energy is;

$$KE = 1/2\, M\, V^2 \tag{57}$$

The potential energy, assuming linearity in heave and pitch is;

$$PE = 1/2\, F_v^{\,2}/K_b \tag{58}$$

where Kb is the effective vertical stiffness at the bow;

$$K_b = \rho\, g\, Awp\, /(1+(L/2/\lambda)^2) \tag{59}$$

where
λ is the radius of gyration of the waterplane (i.e. $Iwp = \lambda^2\, Awp$).

Letting $K_h = \rho\, g\, Awp$, and assuming that, for most ships;

$$K_b = K_h\, /5 \tag{60}$$

This gives;

$$PE = \frac{5}{2}\frac{F_v^{\,2}}{K_h} \tag{61}$$

The vertical force and normal force are related as:

$$F_v = F_n\;\; n \tag{62}$$

so that:

$$PE = \frac{5}{2}\frac{F_n^{\,2}\cdot n^2}{K_h} \tag{63}$$

The force equation (42) can be re-written as:

$$F_n = K_{ice}\cdot \zeta_n^{\,fx-1} \tag{64}$$

where

$$K_{ice} = p_o \cdot fa \tag{65}$$

This allows the indentation energy equation to be written as:

$$IE = \frac{K_{ice}}{fx} \zeta_n^{fx} \tag{66}$$

which with (64) can be rearranged to give:

$$IE = \frac{\left(K_{ice}\right)^{\frac{-1}{fx-1}}}{fx} F_n^{\frac{fx}{fx-1}} \tag{67}$$

The general beaching impact equation with (66) and (63) substituted into (56) is:

$$\frac{1}{2} M \cdot V^2 = \frac{5}{2} \frac{F_n^2 \cdot n^2}{K_h} + \frac{\left(K_{ice}\right)^{\frac{-1}{fx-1}}}{fx} F_n^{\frac{fx}{fx-1}} \tag{68}$$

This equation can be solved for F_n . For certain special cases there is an analytical solution. For the general cases, a numerical solution is required. For the simple linear case (for example Case 1, ex= -.5) (68) reduces to:

$$\frac{1}{2} M \cdot V^2 = \frac{5}{2} \frac{F_n^2 \cdot n^2}{K_h} + \frac{F_n^2}{2 \cdot K_{ice}} \tag{69}$$

The force F_n can be solved for :

$$F_n = \sqrt{\frac{1}{5 \cdot n^2 + \frac{1}{\kappa}}} \cdot \sqrt{M} \cdot \sqrt{K_h} \cdot V \tag{70}$$

where: $\kappa = \frac{K_{ice}}{K_k}$

$$K_{ice} = p_o \cdot \left(\frac{\tan(\alpha)}{\cos^2(\gamma)\sin(\gamma)} \right)^{0.5}$$

$$K_h = \rho\, g\, Awp$$

It is obvious that the beaching force, for the linear case, is proportional to velocity and the square root of mass. It is also weakly dependant on the ice strength parameter p_o. Note that this is just the solution for the beaching condition. The equivalent case for the initial impact (with linear assumptions) is:

$$F_n = \sqrt{K_{ice}} \cdot \sqrt{\frac{M}{Co}} \cdot V \cdot l \tag{71}$$

Comparison of (70) and (71) indicates that the beaching force is less dependent on hull form than is the impact force.

11. DESIGN EQUATIONS

For longitudinal strength assessment equation (68) may be used as a simple check. It does not include the effects of initial impact or any dynamics. Nevertheless, for cases which are primarily beaching (i.e. large ships) the equation is valid. It is not analytically solvable for most values of *fx*. A design equation could be formed by using equations for both impact and beaching, covering a range of possible conditions. An equation of the form:

$$F_v = C_1 \cdot \kappa^a \cdot n^b \cdot \sqrt{M \cdot K_h} \cdot V \tag{72}$$

could be determined. Such an equation was first proposed by Riska (1994) (see also Daley and Riska 1994). The constants C_1, a, b would be determined by fitting the calculated results.

For oblique collisions equations from Table 2 may be used. A design equation based on Case 4 has been suggested for Polar Rule Harmonization work (see Kendrick and Daley 1998, Daley 1999). The design equation for force has the form:

$$F_n = fa \cdot p_o^{.36} \cdot M^{.64} \cdot V^{1.28} \tag{73}$$

where:

$$fa = \left(.097 - .68 \left(\frac{x}{L} - .15 \right)^2 \right) \frac{\alpha}{\sqrt{\beta`}} \tag{74}$$

12. CONCLUSION

A variety of ice force equations have been presented. All are based on energy methods. These results should be viewed as useful approximate values of force. More accurate results may be obtained by solving the interaction equations directly, as a time series for instance. Nevertheless, these energy solutions give insight into the process, particularly for cases in which the energy balance governs the outcome.

The next important step required is the general solution of the oblique collision, with sliding motions taken into account. While the impact idealization is essentially one-dimensional, and the beaching collision is two-dimensional, the sliding-oblique collision is three-dimensional.

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APPENDIX: Description of the Mass Reduction Coefficient Co

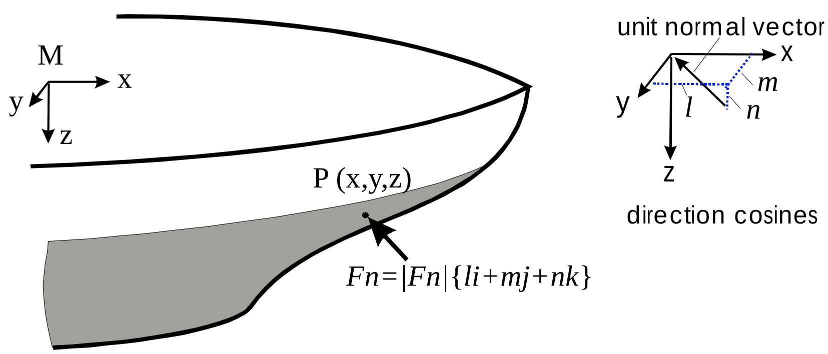

Figure A1 Collision point geometry

A collision taking place at point 'P' (see Figure A1), will result in a normal force *Fn*. Point P will accelerate, and a component of the acceleration will be along the normal vector, with a magnitude a_n. The collision can be modeled as if point P were a single mass (a 1 degree of freedom system) with an equivalent mass *Me* of;

$Me = Fn/a_n$

The equivalent mass is a function of the inertial properties (mass, radii of gyration, hull angles and moment arms) of the ship. The equivalent mass is linearly proportional to the mass (displacement) of the vessel, and can be expressed as;

$Me = M/Co$

where *Co* is the mass reduction coefficient. This approach was first developed by Popov (1972).

The inertial properties of the vessel are as follows;

Hull angles at point P:

α : waterline angle
β : frame angle
β' : normal frame angle
γ : sheer angle

The various angles are related as follows:

$tan(\beta) = tan(\alpha)\ tan(\gamma)$
$tan(\beta') = tan(\beta)\ cos(\alpha)$

Based on these angles, the direction cosines, l,m,n are

$l = sin(\alpha)\ cos(\beta')$
$m = cos(\alpha)\ cos(\beta')$
$n = sin(\beta')$

NOTE: for a symmetrical collision on the stem the direction cosines are
$l = cos(\gamma)$
$m = 0$
$n = sin(\gamma)$

and the moment arms are;
$\lambda 1 = ny\text{-}mz$ (roll moment arm)
$\mu 1 = lz\text{-}nx$ (pitch moment arm)
$\eta 1 = mx\text{-}ly$ (yaw moment arm)

The added mass terms are as follows (from Popov);

AMx = added mass factor in surge = 0
AMy = added mass factor in sway = $2\ T/B$
AMz = added mass factor in heave = $2/3\ (B\ Cwp^2)/(T(Cb(1+Cwp))$
AMrol = added mass factor in roll = 0.25
$AMpit$ = added mass factor in pitch = $B/((T(3-2Cwp)(3-Cwp))$
$AMyaw$ = added mass factor in yaw = $0.3 + 0.05\ L/B$

The mass radii of gyration (squared) are;

$rx^2 = Cwp\ B^2/(11.4\ Cm) + H^2/12$ (roll)
$ry^2 = 0.07\ Cwp\ L^2$ (pitch)
$rz^2 = L^2/16$ (yaw)

With the above quantities defined, the mass reduction coefficient is;

$Co = l^2/(1+AMx) + m^2/(1+AMy) + n^2/(1+AMz)$
$+ \lambda 1^2/(rx^2(1+AMrol) + \mu 1^2/(ry^2\ (1+AMpit)) + \eta 1^2/(rz^2\ (1+AMyaw))$