Conceptual Framework for an Ice Load Model

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1. Introduction

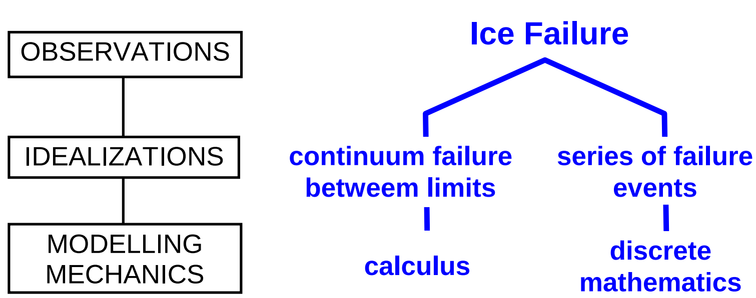
Floating ice is a significant hazard to offshore structures. The design of such structures requires a method for predicting the ice forces and pressures that they will experience. Adequate design further requires that the ice movement around the structure is estimated. Thus the design problem includes two problems. First the characteristics of the maximum loading must be assessed. This requires knowledge about load maxima, locally and globally, and also about the frequency content of the load, so that dynamic responses can be checked. Second, the ice movement around the structure must be clarified. Jamming between platform legs or rubble pile development will modify the effective size of the structure and possibly increase the loads. Ice ride-up may cause topside damage. For a structure to perform well, the ice must clear properly around it.

In the past, the problems of ice load and ice movement around a structure have mainly been treated separately. The concept model in this report will show that both these aspects follow from a simulation of ice action on a structure.

The earliest published method for predicting ice loads was developed by Korzhavin in the 1940's to predict ice forces on bridge piers. Korzhavin developed an empirical formula which related the average ice pressure to the ice strength, the geometry of the interaction and the speed of the interaction. Korzhavin's method was a recipe for calculating the design ice load for vertical structures, and not a description of the mechanics of the ice failure process. In this sense, Korzhavin's formula is not an ice load model. This applies to most of the design load formulations. Naturally the design formulations should be based on the physics of load through a theoretical model to evaluate the load. Here we will define ice load models as descriptions of the ice failure process.

At present all ice load models are derived from observations of ice-structure interaction and ice failure. The models idealize the ice failure process in some way and use the methods of the mechanics of materials to describe the failure process. It is important to recognize and distinguish among these three aspects, shown in Figure 1. Many ice load models share common observations (and measurements), but differ in either the idealizations or in the mechanical and mathematical methods employed.

Figure 1.
Ice Load Models



Continuum Mechanics   Event Mechanics

The concept model that is presented here represents a new approach to ice load models. The model views ice failure as a process comprised mainly of a series of discrete events, rather than as a continuous process. This approach naturally requires a different set of idealizations and methods. Figure 2 illustrates the four major scales that are considered. The new approach was formally applied to the local contact problem by Daley (1991) in order to provide a plausible description for the experimental data of Joensuu and Riska (1988).

Chapter 2 will discuss the general principles of the conceptual model. The model is based on a hierarchy of failure types. Chapter 3 will discuss local failure process. A number of continuum models will be described in Section 3.1. Section 3.2 will describe various discrete stochastic ice load models. Section 3.3 will describe various early discrete ice load models, the Daley (1991) model and subsequent additional developments. Chapter 4 will describe the mechanics of floe-structure interaction. Chapter 5 will describe global load models which include pack ice-structure interactions. Chapter 6 will describe variations on the new conceptual model, showing how it applies to different engineering problems. Chapter 7 will discuss why the new approach is worthwhile with reference to ice failure observations and measurements. All models make idealizations, so that no model can ever describe all the details that take place when ice fails. Our contention is simply that a discrete event model is best able to describe the important aspects of ice failure. Chapters 8,9 and 10 present recommendations, conclusions and references.

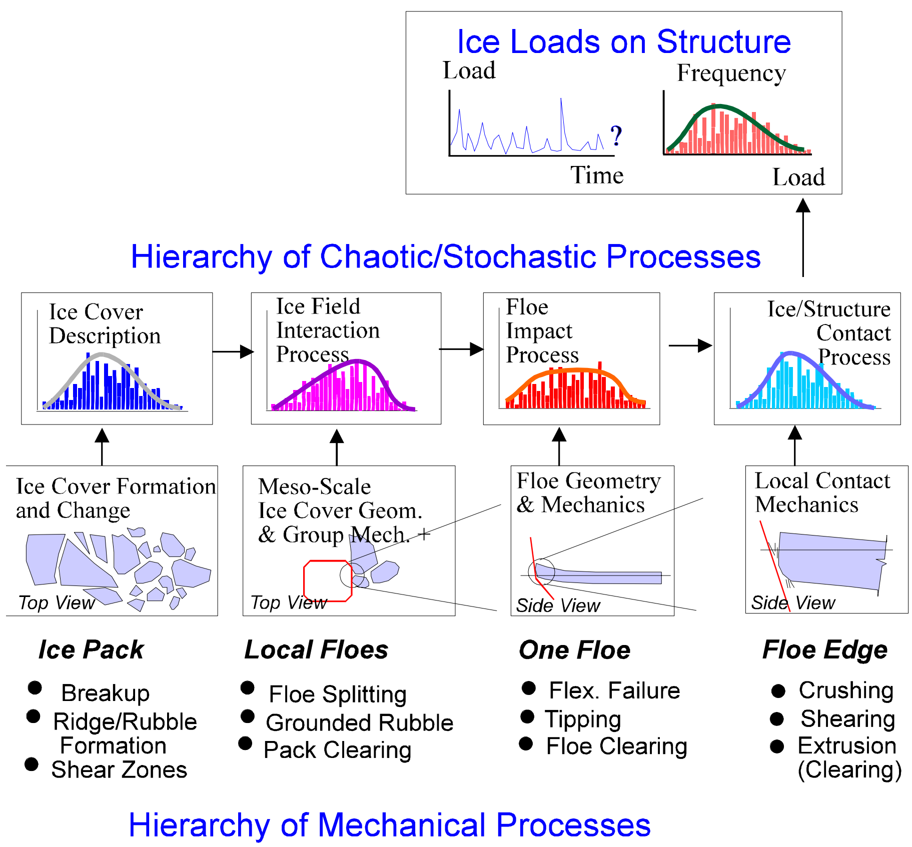


Figure 2. Scales of Ice-Structure Interaction Mechanics

2. Ice Load Conceptual Model

The conceptual model presented here is based on observations of ice failure. The model is a generalization of recent developments in the mechanics of ice loads. The idea of load limiting mechanisms is well known in ice mechanics. In the ice load model by Daley (1992) the ice edge failure was treated as a hierarchy of failures, each being superseded by "lower level" failure (i.e. failure of the supporting mechanism). In Daley's model, all the failure events were various types of shear cracks. The present model extends this to any type of lower level failure. The hierarchy of failure might be, for instance, micro-cracking until spalling and extrusion, continuing until flexural failure, continuing until rubbling and grounding, continuing until floe splitting and clearing.

The concept of a hierarchy of mechanisms is more general than the idea of load limits, because it contains the idea that the limits interact and that repeating limits are created. One failure event on any given level is preceded by a process of failure events on a smaller scale level. But every failure event is itself part of a process loading to failure at a larger scale. This concept is sketched in Figure 3.

Nested Failure Events

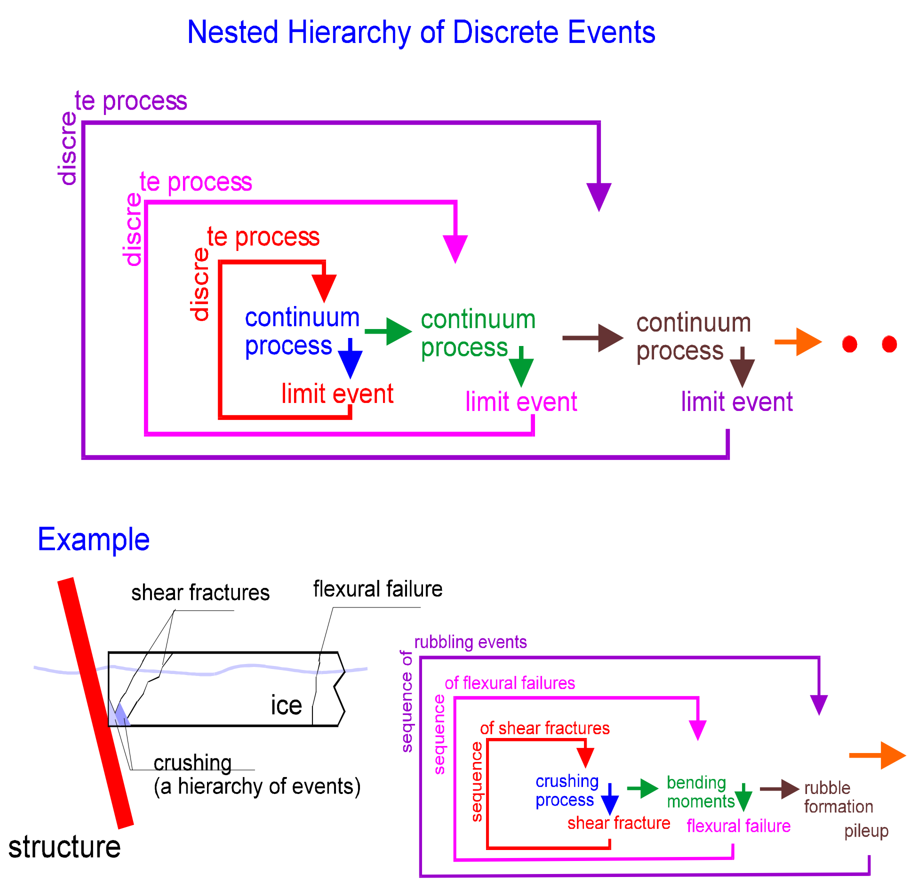


Figure 3. Nested Hierarchy of Failure Events.

The process as described in Figure 3 appears to be a straight chain of events. This would be the case for a one dimensional view of ice failure. In two and three dimensions there can be branching in the hierarchy. An example of this on a large scale is how rafting would occur in thin ice, with ridging occurring in thicker ice, both acting on the same floe. On a local scale, both extrusion and fracture are affecting the local loads. In all cases the ice load is defined by the interaction of the various processes. Chapter 6 will examine specific issues and describe conceptual ice load models for various situations.

One might ask why it is important to view ice failure as a linked hierarchy of failure events. Why not just model the 'design' failure event. The reason is that each failure event is a local limit and defines the value of the local load peak. It also sets the conditions for the next step in the process, leading to the next load peak. Only by considering this set of events can the correct global maximum be properly modeled.

The present conceptual approach encompasses all scales of ice failure. For this reason the following chapters will discuss existing models of local and global ice loads. This will allow the new approach to be seen in context, and in some cases provide the components of the new approach.

3. Local Ice Load Models

The aim of this chapter is to provide an overview of local ice load models. As this is an area of continuing research and development, a full treatment is beyond the scope of this presentation. The main issues and approaches used in continuum and discrete ice load models will be described, in order to see the proposed concepts in a proper context. The proposed concept model is seen as an extension and generalization of existing approaches.

3.1 Continuum Ice Load Models

The common feature of all continuum ice load models is that the behavior of the solid is idealized as being continuous. Statistical mechanics assumptions are employed to "smooth over" the minor failure events. The major failure events are seen as limits, between which the continuum solution is valid.

Kheisin's extrusion model

The first true ice load model was developed by Kheisin and others (Kheisin, Kurdyumov and Likhomanov, 1975). The model was based on data from experiments of dropping steel balls on an ice cover. The geometric idealization of the process is shown in Figure 4. A crushed layer of thickness t is presumed to form between the sphere and the intact ice. The crushed ice is assumed to be a viscous medium. The outer surface of the crushed layer is called the breakup surface. The crushed layer thickness is assumed proportional to the local pressure. Using these assumptions the authors are able to derive equations for the layer thickness, the pressure distribution, the force time history and the velocity of the sphere.

For example, the maximum force during the contact is;

$$F\max = 1.18 \cdot V^{\frac{11}{9}} \cdot M^{\frac{5}{9}} \cdot \left(3\mu k_p\right)^{\frac{1}{9}} \cdot (2R)^{\frac{5}{9}} \qquad (1)$$

where M is the mass and V is the velocity of the sphere. The material properties of the ice are expressed by the terms μ, the viscosity of crushed ice and k_p , a term representing the breakup behavior.

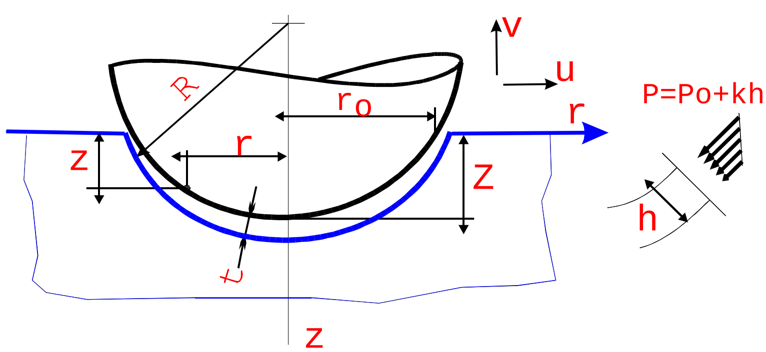


Figure 4. Sphere striking an ice surface. (Kheisin and others, 1975)

Varsta's wet/dry model

Varsta (1983) studied the ship-ice contact process. Ice crushing tests were performed in which water was sprayed into the contact zone during some of the tests. These "wet" tests were found to produce higher pressures and led to the search for an improved model of ice loads. Varsta introduced the idea that the contact was comprised of both direct contact with hard ice and contact through a layer of crushed ice. The two types of contact were assumed to be superimposed, and together would explain the differences between wet and dry contact pressures. The wet contact was essentially the same as the Kheisin model, while the dry contact required a new model. Varsta used finite element analysis coupled with the Tsai-Wu failure criteria to model the dry contact. Figure 5 illustrates Varsta's approach.

An important aspect of Varsta's model is the idea that part of the contact is direct contact with solid ice. The direct contact was seen as responsible for the very high localized pressures that were observed with small pressure transducers.

Varsta used the finite element method to calculate the stresses in the ice edge that resulted from two types of loading. In the first case, termed 'wet', the pressure was uniformly distributed on the ice edge. This was an idealization of the pressures that would result from viscous extrusion, along the lines of the Kheisin model. In the second case, termed 'dry' contact, the load was applied as a uniform displacement of the ice edge, as would happen when a rigid steel plate would contact a flat elastic ice edge. (see Fig. 4). In the case of dry contact the stresses are lower in the center. If the ice stresses are compared for the two cases, the ice was seen to be capable of sustaining higher loads in the 'wet' contact case, as was observed experimentally.

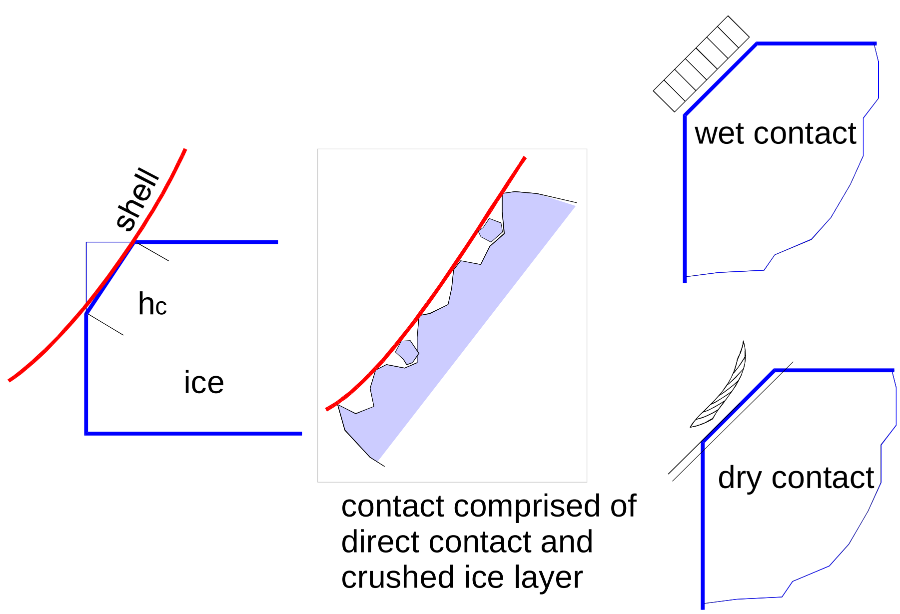


Figure 5. Wet-dry contact model (Varsta, 1983).

Riska's nominal pressure model

The developments by Varsta were continued by one of his colleagues (Riska, 1987). Riska introduced a new method of calculating ice loads based on the idea of nominal ice pressures. The contact geometry was modeled with finite elements and used to calculate the state of stress at each point in the ice, due to a unit pressure applied in the contact zone. Riska calculated the "failure number" for each point. The failure number was the fraction of failure that each stress state represented. The Tsai-Wu form was used to define the failure surface. Figure 6 illustrates a case in which λ_{max} = 0.088. This means that the worst state of stress in the ice is 8.8% of failure for an applied pressure of 1 MPa. Inverting this results in a nominal failure pressure p_{nom} = 11.3 MPa.

Riska was able to show good agreement between his p_{nom} values and measured small area pressures for both laboratory ice crushing tests and the ice ramming trials of a large vessel. The p_{nom} values depend on both the 3D failure properties of ice and the interaction geometry. The drawback with this approach is that it offered no explanation of the pressure-area effects.

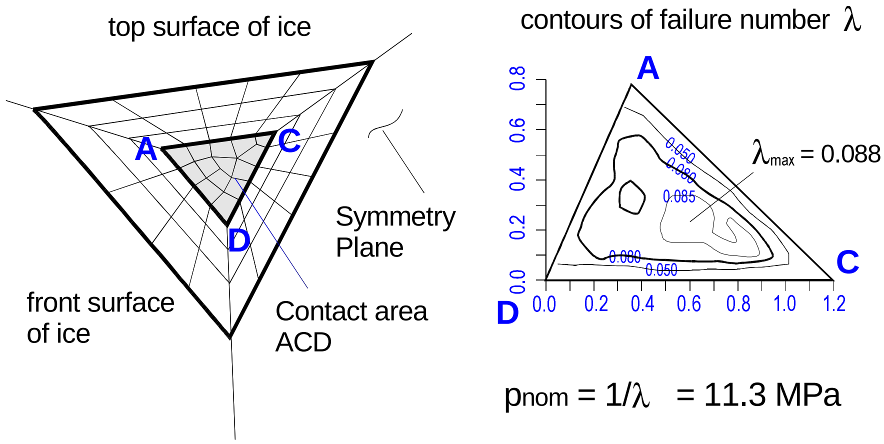


Figure 6. Nominal pressure ice load model (Riska, 1987).

Timco's damage/ pulverization model

The next significant development was based on experimental results by Timco (1986). During level ice indentation tests Timco noticed a region in front of the indentor that appeared to be undergoing damage, and then suddenly pulverized. Figure 7, a composite taken from Sanderson (1988) and Timco and Jordaan (1987) illustrates this idea. As the ice approaches the structure (1) the stress in the ice builds and causes damage. At a critical state of damage (2) the ice pulverizes (almost instantly), causing a rapid drop in the force. The ice is then extruded as the gap is closed and the pressure rises. At (4) the cycle starts again. To model this process mathematically, one must describe how the damage grows with time and stress, the critical damage level that will cause pulverization and the flow (extrusion) properties of the granular media. This model is an extension of the Kheisin model, with the added features of damage growth and instantaneous pulverization. This model has gained much support and has resulted in a growing interest in damage mechanics and the properties of crushed ice. Note that this model has two aspects that are treated with continuum mechanics; the growth of damage and the extrusion of a granular medium.

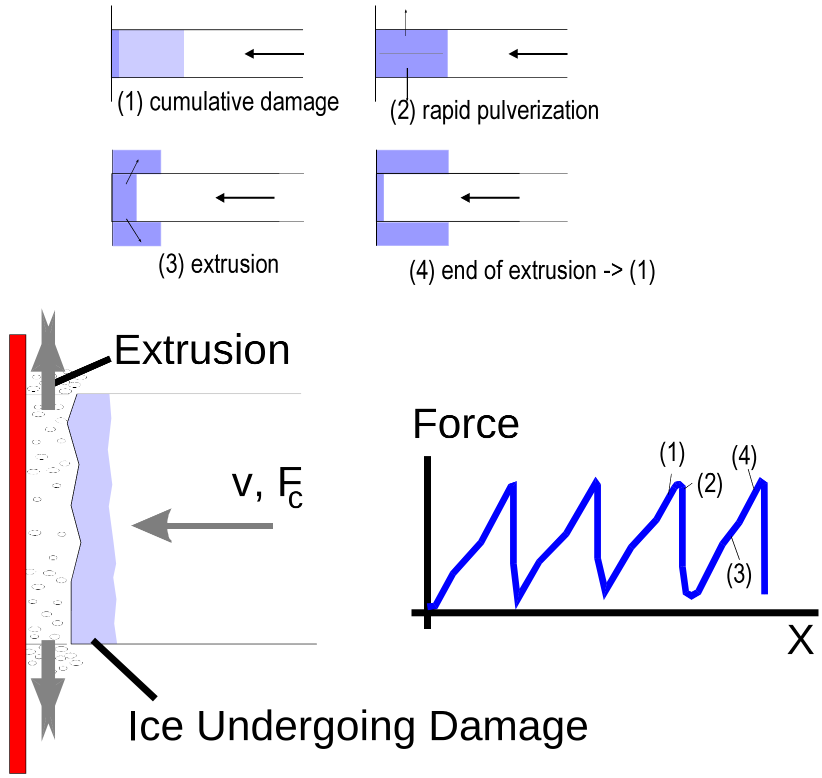


Figure 7. Finite depth pulverization and extrusion.

Kärnä's hybrid pulverization model

Figure 8 shows a variation on the finite pulverization model (Kärnä 1993), based on an extensive set of level ice indentation tests performed at VTT, the Technical Research Centre of Finland in conjunction with TKK, the Helsinki University of Technology (Muhonen et.al. 1992). This model combines the continuum damage approach with large shear cracking events.

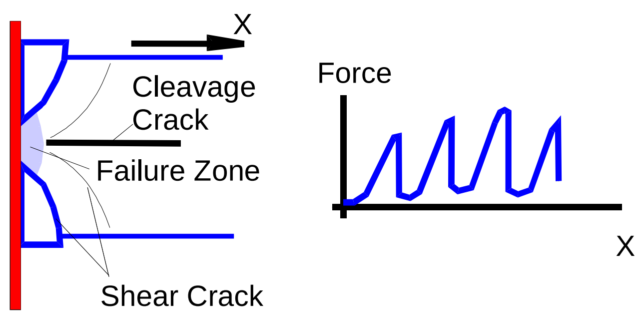


Figure 8. Finite pulverization coupled with major shear flakes.

Kärnä's model is strongly reminiscent of an concept proposed by Saeki and Ozaki (1979), as shown in Figure 9, in which a range of behaviors from crushing (with damage) to flaking is proposed. The type of behavior was said to depend on the speed of indentation.

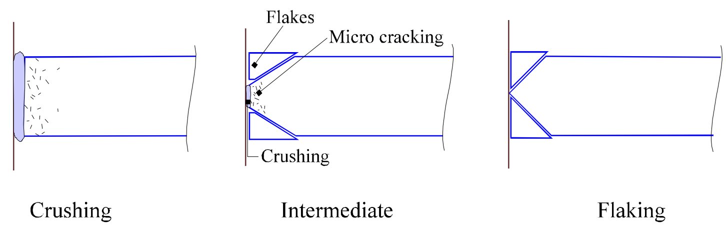


Figure 9. Failure Modes depending on speed of Indentation (Saeki and Ozaki, 1979)

Timco's Failure Map

The idea of various failure modes was again discussed by Timco (1986). He observed that failure mechanisms were dependent on the indentation rate and aspect ratio of the contact zone. Figure 10 shows the failure mechanisms that Timco observed. Figure 11 shows the failure modes plotted as a function of indentation rate and aspect ratio. The strength of Timco's map is the recognition of the multiple failure mechanisms that can take place. The drawback of this kind of map is that the various modes are seen as dependent on fixed external parameters (aspect ratio and indentation rate). The present concept model views the various failure modes as being part of an interdependent hierarchy.

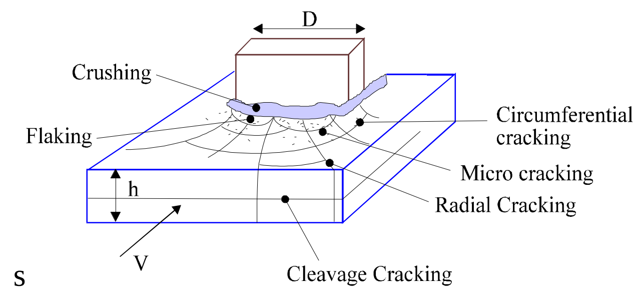


Figure 10. Ice Failure Modes on a Vertical Structure (from Sodhi (1992) and Timco (1986))

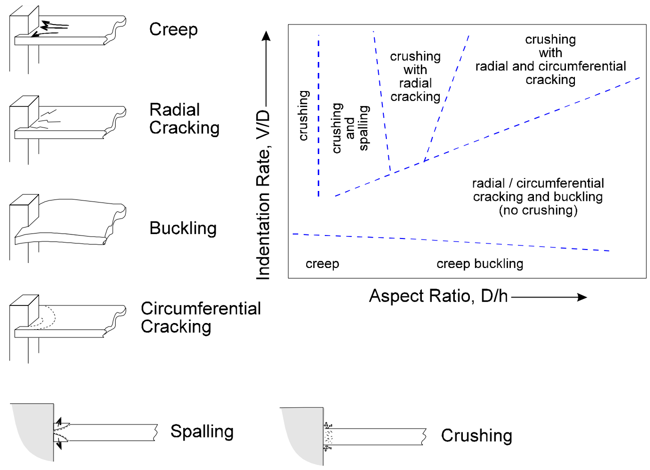


Figure 11. Failure Mechanisms and Failure Mode Map (Timco, 1986)

Continuum ice damage models

The idea of continuum damage in ice is illustrated in Figure 12. The ice is viewed as containing numerous small cracks which serve to lower the effective stiffness of the ice. Figure 12 illustrates the definition of the true local stress σ on an infinitesimally small element of uncracked ice, and the nominal stress $\tilde{\sigma}$ on a small by finite element of ice containing numerous cracks. These stress tensors assume that the cracks are small enough that their effects can be spatially averaged.

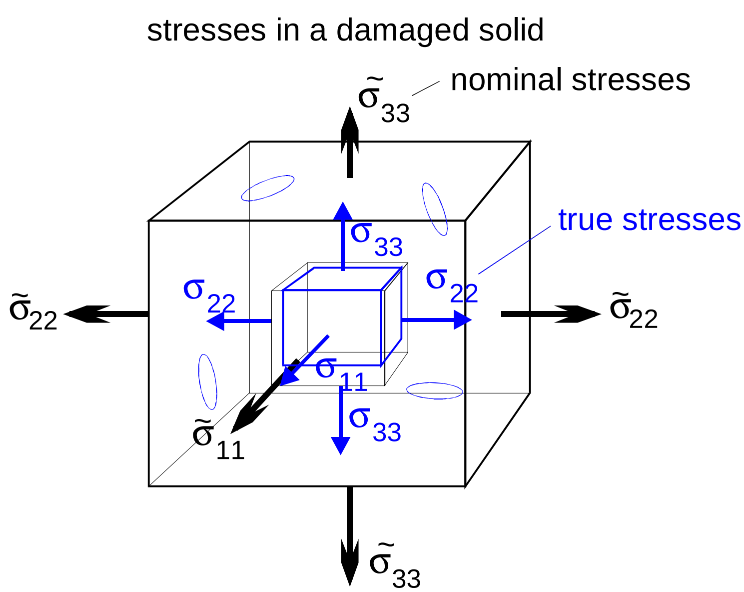


Figure 12. Definition of true and nominal stresses in a damaging solid.

Models based on damage mechanics and extrusion have been reported by MeKenna, Jordaan and Xiao (1991) and Selvadurai et.al. (1992). The damage based pulverization models have continued to focus research on the micromechanics of damage and fracture. Topics of active research include:

- grain boundary sliding, diffusion
- dislocation pileup

- elasticity
- delayed elasticity
- intra-sub-granular microstructural damage (slip bands +)
- micro-cracks
- creep

Summary of Section 2.1

The main continuum mechanics methods for ice load calculation have been presented. The continuum methods employ calculus to solve the mathematics of the problem. This requires that all parameters be locally smooth (differentiable), between spatial or temporal boundaries. The strength of these methods is based on the rich and powerful mathematical and numerical tools that have been developed. Damage mechanics, fracture mechanics, the finite element method, and fluid mechanics are all available to treat ice failure. The only drawback with these methods is that they must be applied to smooth systems. If any function or property changes suddenly, the problem must be broken into piece-wise smooth parts. This can work fine for a few sudden changes, but becomes unmanageable if too many sudden changes take place.

The next sections will describe a different approach to ice failure, in which the process is viewed as a series of discrete events. Only recently have suitable mathematical and numerical tools been developed which allow these approaches to be vigorously pursued. The major change in approach is that the process is viewed as a series of related events. The individual events can be based on continuum mechanics, but the overall behavior is solved with the use of discrete mechanics and discrete mathematics. The strength of this method is that it can naturally handle a large set of interrelated sudden changes in the system. It is our view that this is a more appropriate way to view ice failure.

3.2 Stochastic Ice Load Models

Several discrete ice failure models have been developed to examine the stochastic aspects of ice forces. These models assume that the ice failure is comprised of a series of random discrete failure events.

Independent failure zones

+ finite failure depth

Kry (1978) introduced the concept of independent failure zones as a way of understanding the loads on wide structures. Kry was primarily concerned with the statistical aspects of the ice load. Ashby et al. (1986) showed (see Figure 13) that the independent failure zone idea must necessarily be joined to the idea of a finite failure depth (L_i), in which the failure may be flexural, crushing or another failure mode.

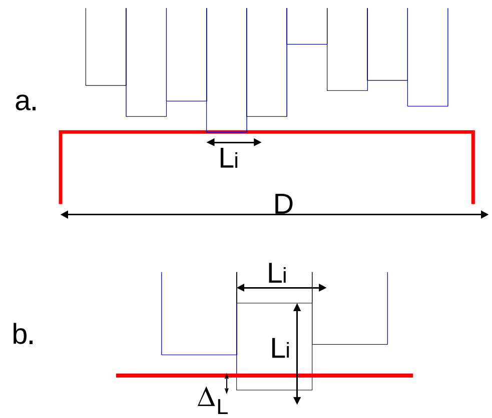


Figure 13. Non-simultaneous failure model (Ashby et.al., 1986)

multi-zonal failure model

The idea of non-simultaneous failure was extended by Sanderson (1988) to a multi-zonal failure model. While Kry and Ashby et.al. had tried to explain why ice load intensities would decrease on wider structures, Sanderson's model (Figure 14) also attempted to explain why the time histories of ice loads showed so much variation. With a random number of zones failing at any time, Sanderson's model showed highly varying forces.

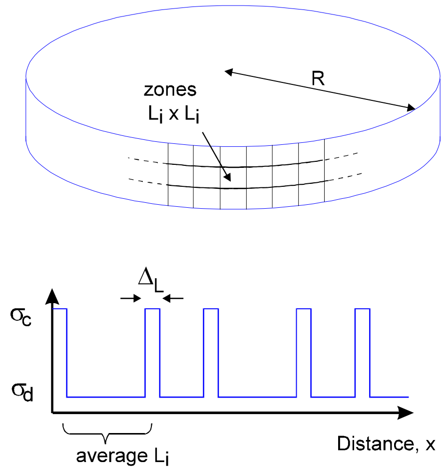


Figure 14. Multi-zonal stochastic ice edge failure model (Sanderson, 1988)

fractal failure zones

Two further extensions of the idea of non-simultaneous failure came with the emergence of fractal geometries (see for instance Pietgen et.al. 1992). Bhat (1990) proposed the idea of a fractal ice edge profile (Figure 15) in which the finite failures would generate fractal surfaces. The pressure-area effect is a natural outcome of such a contact geometry. The idea was extended to three dimensions by Palmer and Sanderson (1991) as shown in Figure 16. The fractal crushing volume will also naturally lead to a pressure-area effect in the loads on a structure.

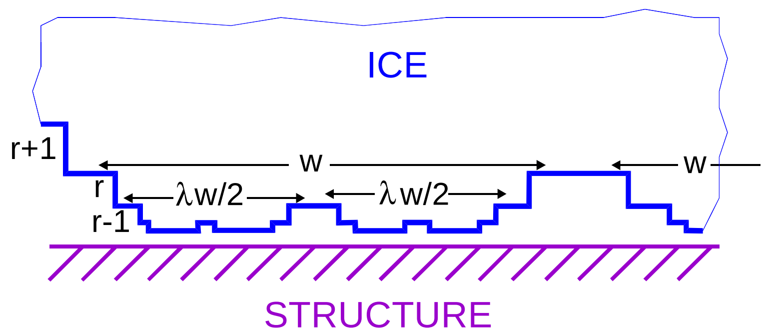


Figure 15. Fractal Contact Geometry (Bhat, 1990)

fractal crushing

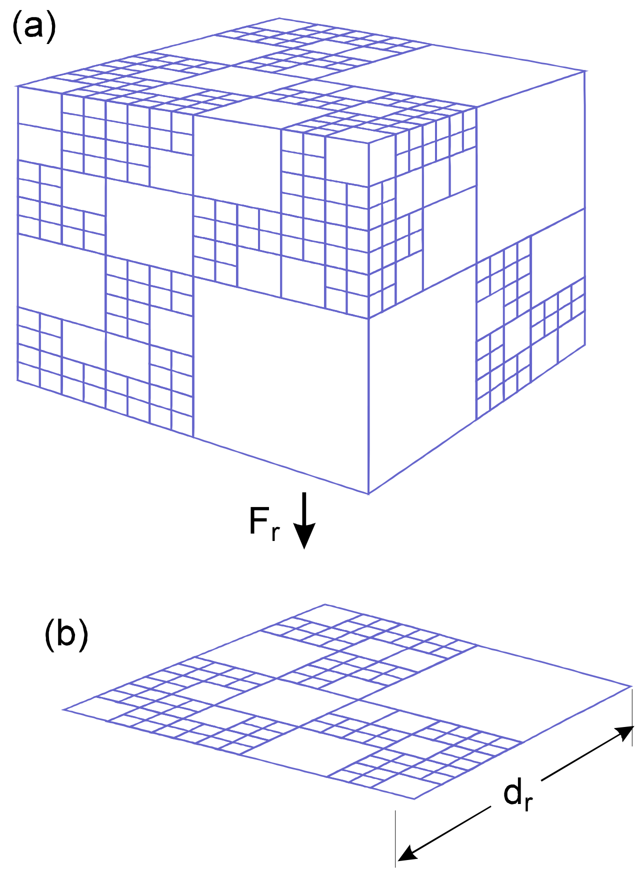


Figure 16. Three dimensional fractal crushing of a brittle solid (Palmer and Sanderson, 1991).

The authors emphasize that the crushing pattern in Figure 16 is not meant to be taken literally. It merely emphasizes the idea of a hierarchy of ice failure that leads to observable symptoms such as the pressure-area effect, the broad range of fragment sizes, and the highly variable nature of the force time history. The mechanical basis for Figure 16 is found in the investigations of Steacy and Sammis (1991), who showed that fracture stresses are highest when like-sized fragments are in contact. This tends to result in one of the pair of like-sized neighbors breaking. Figure 16 is a fractal pattern that minimizes the number of like-sized neighbors.

Summary of Section 2.2

The above statistical and fractal models view ice failure as a set of discrete events. These methods are all founded in observations of ice failure, and are plausible attempts to describe various empirical observations such as the pressure-area effect and the observed distribution of fragment sizes in crushed ice. Their shortcoming is that they assume an outcome of the ice failure process, rather than modeling the mechanics of the failure process.

3.3. Discrete Ice Load Models

It is important to note that while the earliest quantitative models of ice failure were based on continuum mechanics, many early qualitative ice failure models and descriptions were clearly of discrete processes. The following sections will describe several discrete event ice load models, from simple load limit models to the latest multi-level chaotic ice load models.

Matlock's discrete failure models

Ice strengthened oil production platforms were constructed in Cook Inlet Alaska in the 1960's. Peyton (1966) observed the ice failure on the legs of the platforms. Matlock, Dawkins and Panak (1969, 1971) took these observations and developed a process model of the ice load. The ice behavior was idealized as a series of brittle fractures as shown in Figure 17. The brittle flaking events were idealized as a series of independent cantilever failures, as shown on the right of the figure. The model was able to explain why the structure was dynamically excited at certain ice velocities. Unfortunately, Matlock's model lacked the ability to effectively predict load levels, and so was not developed further. It can now be seen that the Matlock model was on the right track, but lacked one important feature. The ice failures were assumed to be independent. This was most likely assumed for pragmatic reasons. The numerical and mathematical tools to handle a system comprised of interdependent discrete events were not available in the late 1960's.

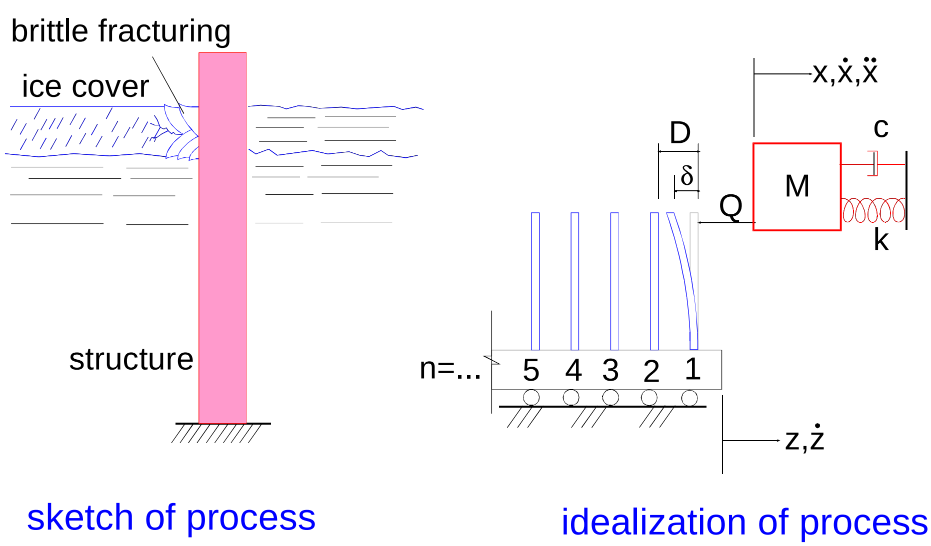


Figure 17. Discrete ice load model (Matlock et. al. 1971).

ship-ice edge failure model

Another early qualitative description of ice edge failure was presented by Varsta and Riska (1977) (see Figure 18). The failure was viewed as a combination of crushing, shearing and flexural failure. The model was used to qualitatively illustrate why the ice loads contained several local peaks and lasted longer than a Kheisin type of model would predict.

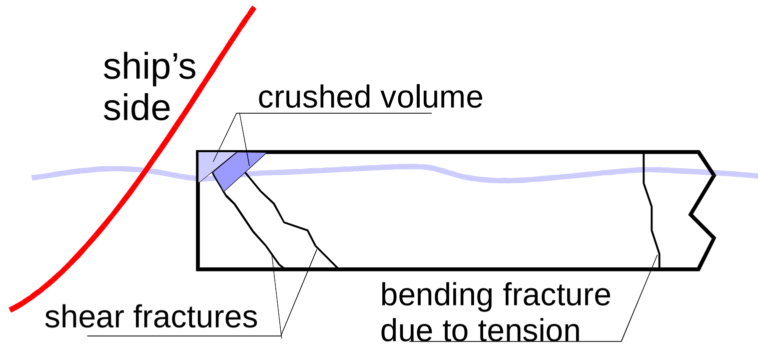


Figure 18. Ice edge failure process on the side of a ship (Varsta and Riska, 1977)

Croasdale's plastic limit failure models

Croasdale, Morgenstern and Nuttall (1977) presented an ice load model based on an upper-bound plasticity solution. Slip planes are assumed to form based on a Tresca yield criterion. Croasdale (1980) again presented the model and added the diagram shown in Figure 19. A sequence of parallel shear failure planes was postulated. Croasdale only showed a load calculation for the first failure. This approach is based on intact material properties and only seeks a maximum quasi-static value for use in design. It does, however, lead one to think of a process model of the type described by Daley (1991).

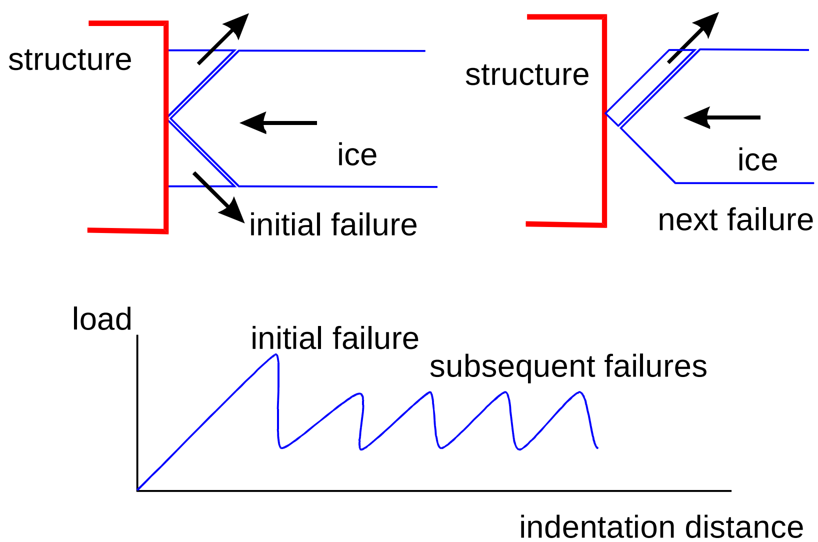


Figure 19. Croasdale's Upper-bound plasticity ice load model (Croasdale, 1980).

The early models of discrete failure (Figs 18, 19) viewed the discrete failure as a single limit which would define the peak load. While these early models obviously recognized that a sequential failure process was at work, no mechanics based model of the process was developed. The following paragraphs will describe the development of discrete ice failure process models.

A set of laboratory experiments were conducted in 1988 (Joensuu and Riska, 1989). Some ice blocks were crushed with a plate window and filmed. Others were crushed with a steel plate affixed with small PVDF pressure sensors. Figure 20 illustrates the experimental arrangement. One of the measured force time series is also shown in Figure 20. The startling feature about this plot is the regularity of the pattern, which is characterized by triangular ramps within ramps. This plot is prototypic of all the force time series, albeit the most regular. An even more startling observation came from the video records filmed through the window. The ice-plate contact was noted to be a line, a thin wavering line at the watershed of the contact.

Joensuu-Riska tests

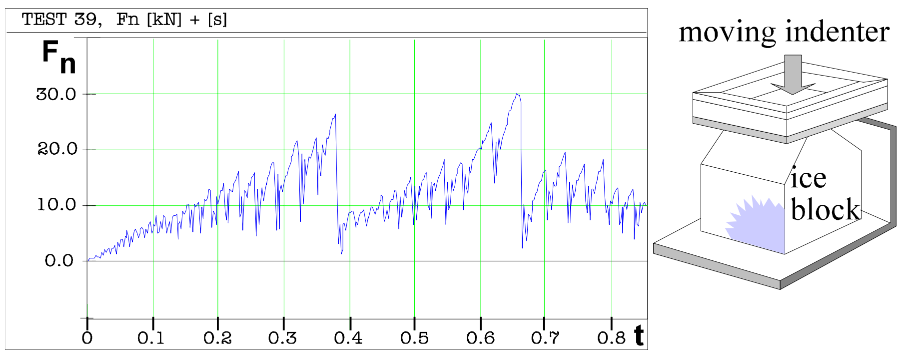


Figure 20. Block crushing tests (test 39, Joensuu and Riska, 1988)

Daley's contact model

The search for an explanation of the Joensuu-Riska tests lead to the development of a discrete event ice failure model (Daley, 1990, 1991). The sharp drops in force led to the assumption that the load time history was being governed by sudden fracture events. Extensive review of the video records of the tests led Daley to hypothesize that the sudden fracture events were flaking.

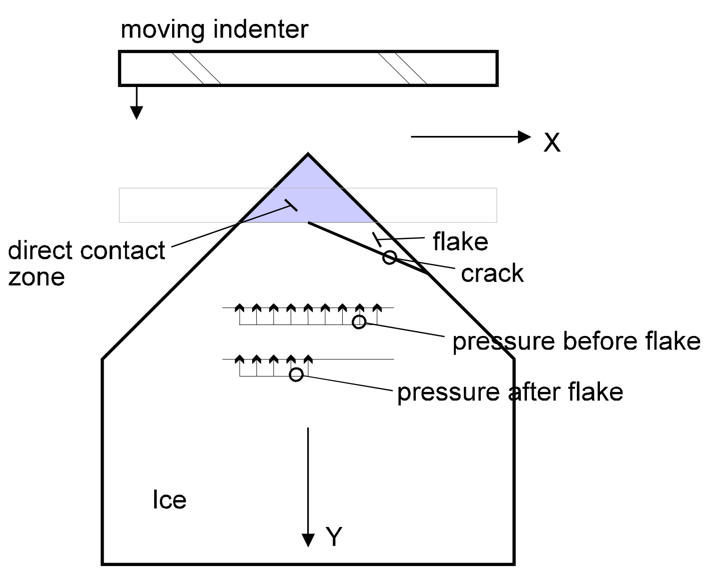


Figure 21. Mechanics of Contact Model (Daley 1990)

A very simple model of the failure mechanics was proposed as shown in Figure 21. The indentor comes into direct contact with the ice block. As the penetration increases the stresses in the block increase until a flake is formed. The force required to cause the flake becomes a local force peak. In the first implementation of this idea, all possible failure planes were iteratively checked for each small step movement of the indenter. Whenever the stresses on any plane exceeded a simple coulomb failure criteria, a flake was formed and removed. This modified the ice geometry and the interaction continued. The sequence of flakes and force peaks is shown in Figure 22.

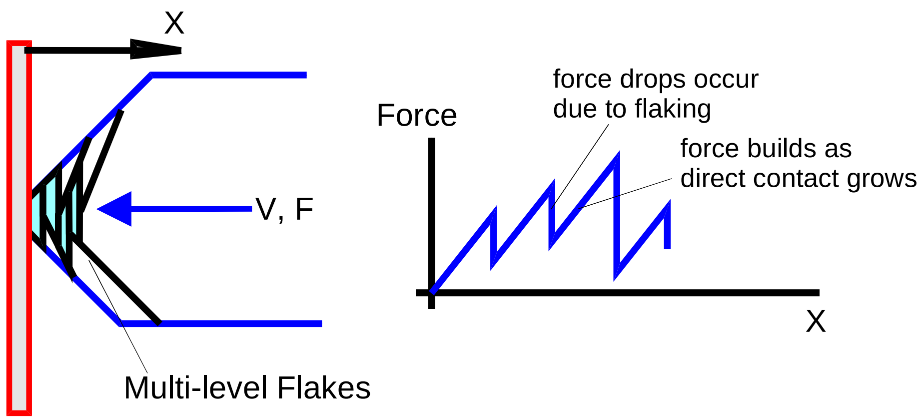


Figure 22. Flaking sequence and force time history.

Comparisons with the measurements and observations showed excellent agreement. Figure 23 shows a comparison of forces. In addition the contact model was able to explain the pressure-area relationship, the thin line of direct contact, the piece size distribution, and the source of randomness in the load measurements.

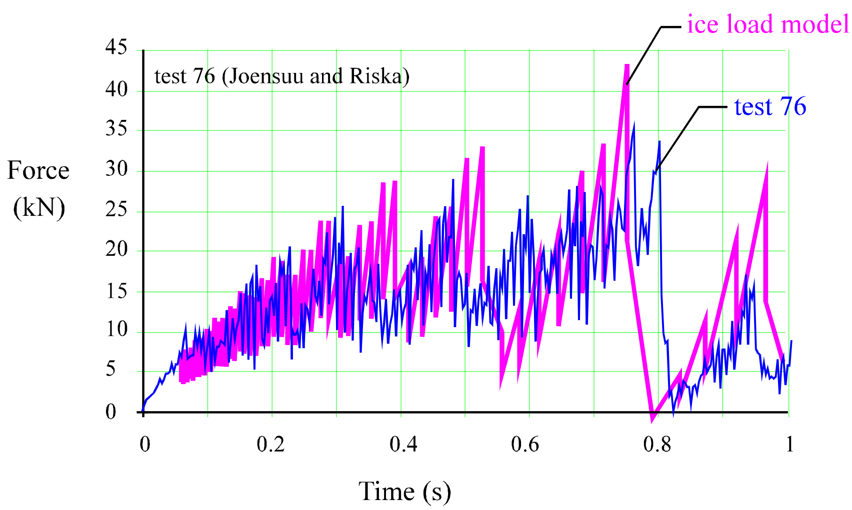


Figure 23. Comparison of Joensuu-Riska test with ice load model (Daley 1991)

The full details and behavior of the Daley contact model are found in the referenced material. The rest of this chapter will review enhancements that have been developed in other investigations.

Enhanced contact models

Fransson et al (1991) reported on tests similar to the Joensuu-Riska tests . They were independently coming to the same approach, and after reading Daley (1990) they developed several enhancements. The first was to consider a sloped indentor, which changed the breaking pattern somewhat. They also reported the possibility of cracks running normal to the indentor that would split the ice block, referring to the work of Kendall (1978). A similar type of crack was reported by Muhonen et al (1992).

Kujala (1993 a, b) developed a chaotic flaking model to investigate the loads on the side of a ship in pressured ice, which is essentially the same as the loads on a wide vertical structure. Kujala added several features to Daley's model. Kujala's model included treatment of the rubble pile above and below the ice, and flexural failure of the ice. The flexural failure was caused by a combination of the unbalanced rubble (mass of ice on top was approximately equal to ice below, with net force being strongly downward) and eccentricity of the in-plane load.

Kujala also added the idea of curved flakes as illustrated in Figure 24. By assuming, quite reasonably, that the principle stress direction was radially oriented, and that the crack would run at a constant angle to the local principle stress, Kujala derived a spiral flake shape. While the calculations proved more difficult than for straight cracks, the new model was generally similar to straight flake models.

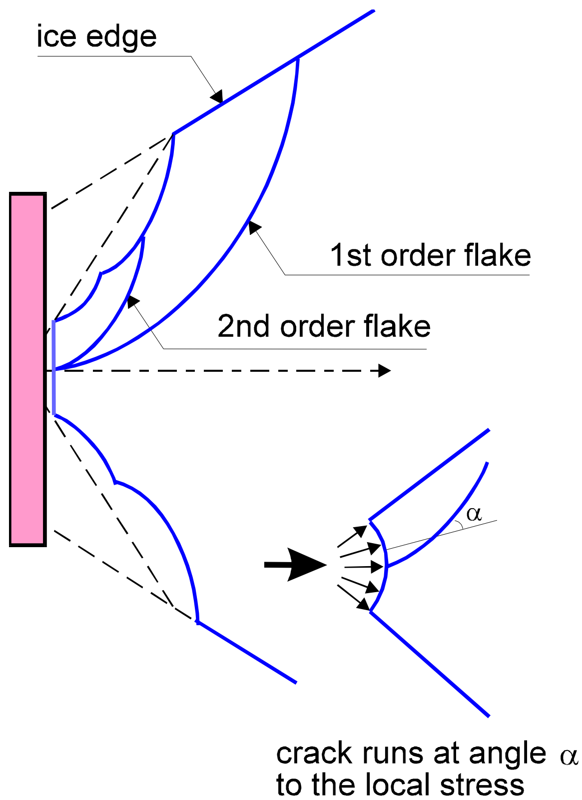


Figure 24. Curved Flake Ice Load Model (Kujala, 1993 b)

A number of additional applications of discrete ice failure processes in the areas of ice cover evolution and icebreaking resistance of ships can be found in Daley (1992). The preceding presentation covers the main developments to date concerning both continuum and discrete ice load models. The aim of the concept model is to examine the broader possibilities of discrete process ice load models.

Summary of Section 2.3

The main features which contrast discrete ice load models with continuum approaches are:

- Continuum models often include one or two discrete events, which are treated as limits on the continuum solution. The effort is focused on the behavior between such events, albeit with the aim of predicting the event.
- Discrete models include some simple continuum behavior, but focus on the discrete failure events as a process.
- Both approaches can be chaotic; continuum normally are not, discrete normally are.
- Discrete ice load process models contain a hierarchy of failures, each being superseded by “lower level” failure (i.e. failure of the supporting mechanism)

3.4 Discussion of Local Ice Load Models

The above sections have described a variety of approaches to local ice failure, as the basis of local ice load models. All are concerned with the behavior of ice at the ice-structure interface. All represent descriptions of how the ice is broken into pieces.

Pulverization Mechanisms

In the figures above, all pulverization is called 'crushing' as if there were only one mechanism, and as if crushing was not connected to spalling and faulting. The figures below will summarize the various ways that solid ice can become pulverized ice. Each of these mechanisms has a basis in observations, although there is active debate as to the importance of the various mechanisms.

1. Damage Mechanics. Growth of a field of microcracks occurs under applied load until critical explosive growth occurs, causing near instant creation of pulverized ice. Figure 25 shows pulverization as the result of microcracking. This is the idea behind damage mechanics models of ice load. Observations of this type of damage and a discussion of the mechanics of microcracking can be found in Sinha (1989, 1991).

Micro Cracking

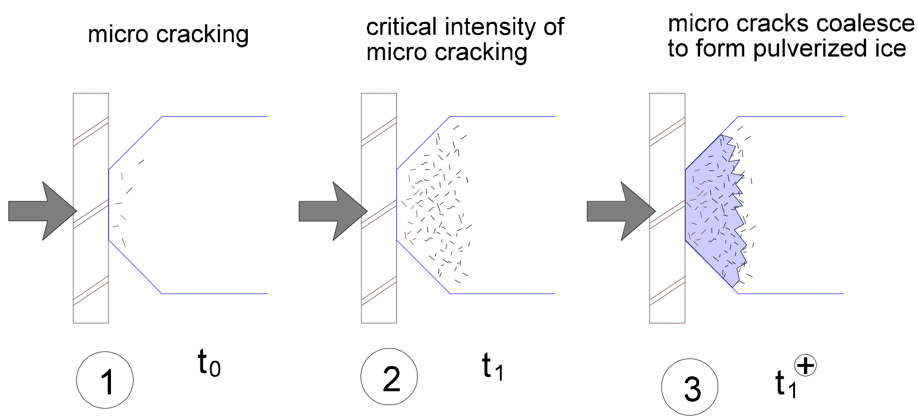


Figure 25. Pulverization due to microcracking.

2. Explosion of Flakes. Upon release of stress when flakes are formed, microcracks grow rapidly, producing pulverized ice. Figure 26 illustrates the mechanism.

Flake Explosion

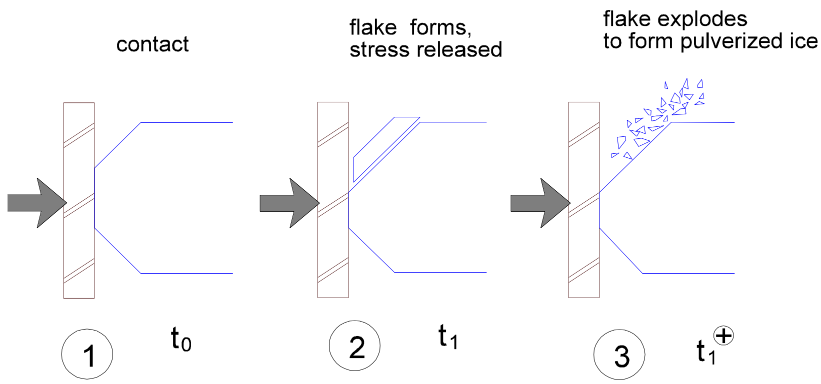


Figure 26. Pulverization due to flake explosion upon stress release.

Comminution

3. Macrocracks + Comminution. Macrocracks release pieces of ice into the extruding stream. In the stream they are repeatedly broken as they interact and are extruded. Figure 27 illustrates the mechanism. The macro crack occurs almost instantly, but the pulverization takes time to occur.

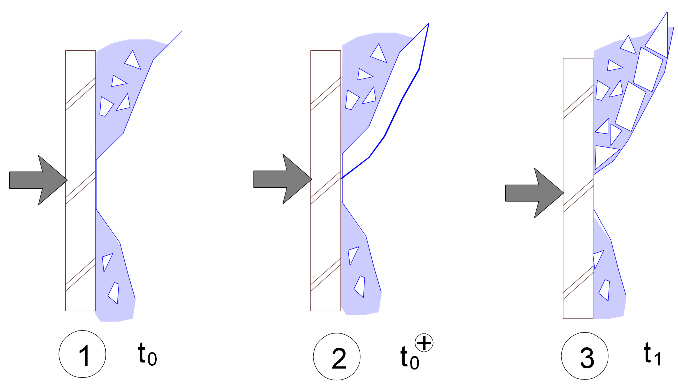


Figure 27. Pulverization due to comminution of flakes created by macro cracks.

Cascade of Cracks

4. Rapid Cascade of Macrocracks. Ice evolves into a state of stress that causes a macrocrack. The first macrocrack does not restore the ice to a temporary balance, so that another macrocrack will immediately occur. A cascade of macrocracks occurs very rapidly, until the material is sufficiently pulverized to flow enough to relieve the stresses. Figure 28 illustrates the mechanism. In this concept the macro cracks will form depending on the stress state in the cracked body, which is rapidly changing as the cracks occur. Once a critical level of breakage has occurred, the assemblage of pieces will not be able to sustain the load and will rapidly relax and extrude.

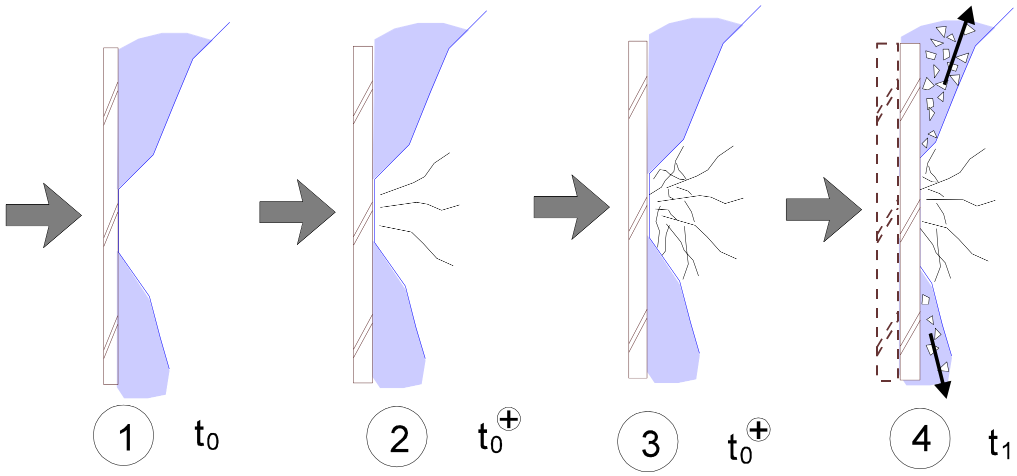


Figure 28. Pulverization due to a rapid cascade and coalescence of macro cracks

The above mechanisms will not likely occur in isolation. Figure 29 illustrates the case of many or all of them occurring together. Figure 29 also shows a flexural crack, to illustrate the point that a hierarchy of cracking occurs.

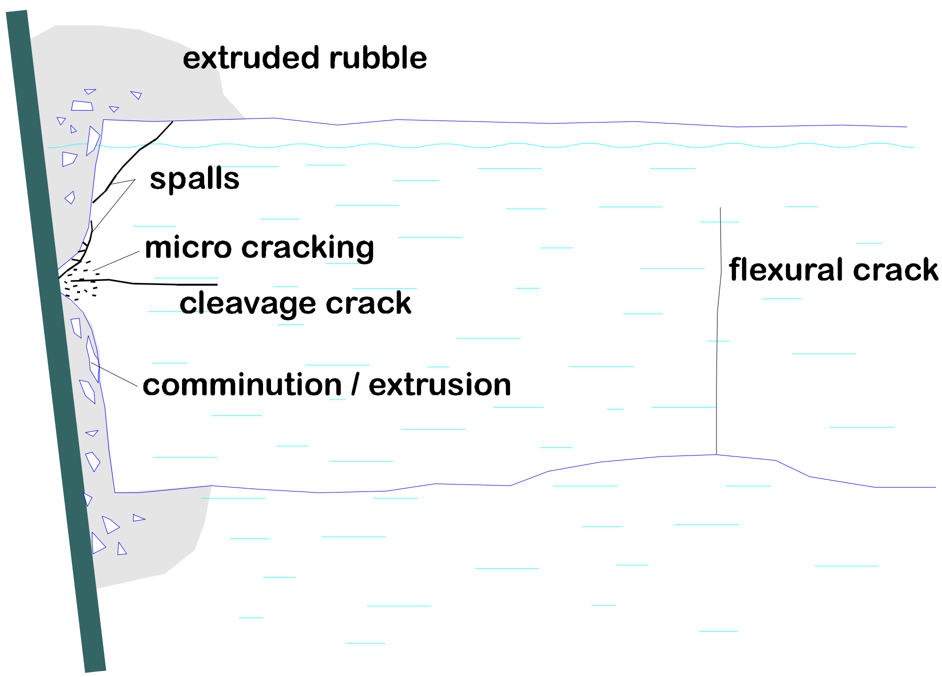


Figure 29. Ice-structure contact showing various and simultaneous breakage mechanisms.

4. Floe-Structure Interaction

While chapter 3 focused on the local contact between the ice and structure, this chapter will look at the larger scale of floe-structure interaction. Refer again to Figure 2 to see the hierarchy of the problems. In floe-structure interaction models, there must still be a contact model, in addition to a model of the floe motions, flexural failure, tipping and rubble formation.

The interaction of Daley's ice load model with a dynamic structure was investigated by Riska et.al. (1993). The modeled system is illustrated in Figure 30. The ice floe, local and global structure were included in the system. The ice failure was two dimensional, and naturally chaotic. The structural response was mildly chaotic, as is shown by the phase diagram of response in Figure 31. The response dynamics are affected by the chaotic ice failure, but dominated by the regular and linear dynamic properties of the structure. This model builds upon the local contact model by adding larger scale ice response and failure mechanisms

Dynamic Chaotic Loads

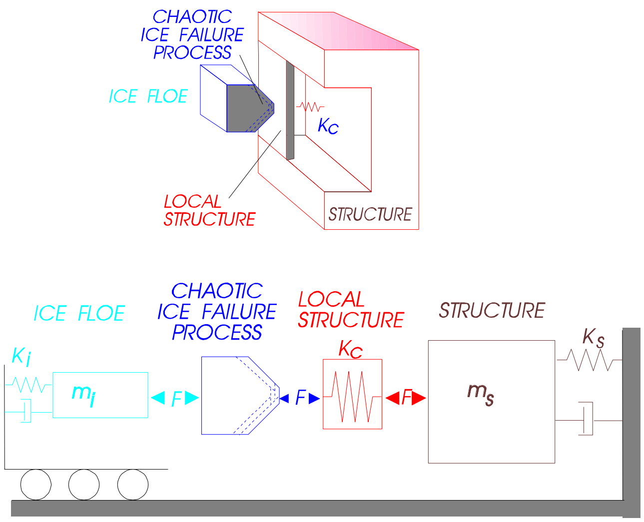


Figure 30. Single Zone Dynamic Ice Load Model (Riska et. al. 1993).

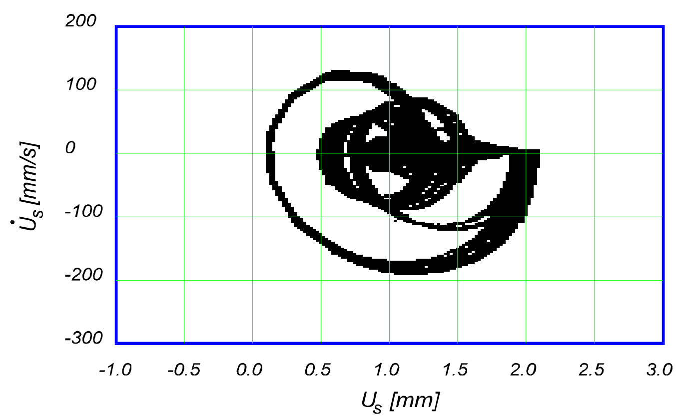


Figure 31. Response to Chaotic Ice Failure, in Phase Space, (from Riska et.al. 1993)

Figure 32 (from Sanderson, 1988) illustrates some of the issues to be considered in this case. If no rubble is formed around the structure, then flexural failure of the floe edge is the only mechanism to consider. If the ice does not clear, then ice rubble may form and may ground. In this case the effective size, mass and shape of the structure changes. Crushing failure may occur at the outer edge of frozen rubble.

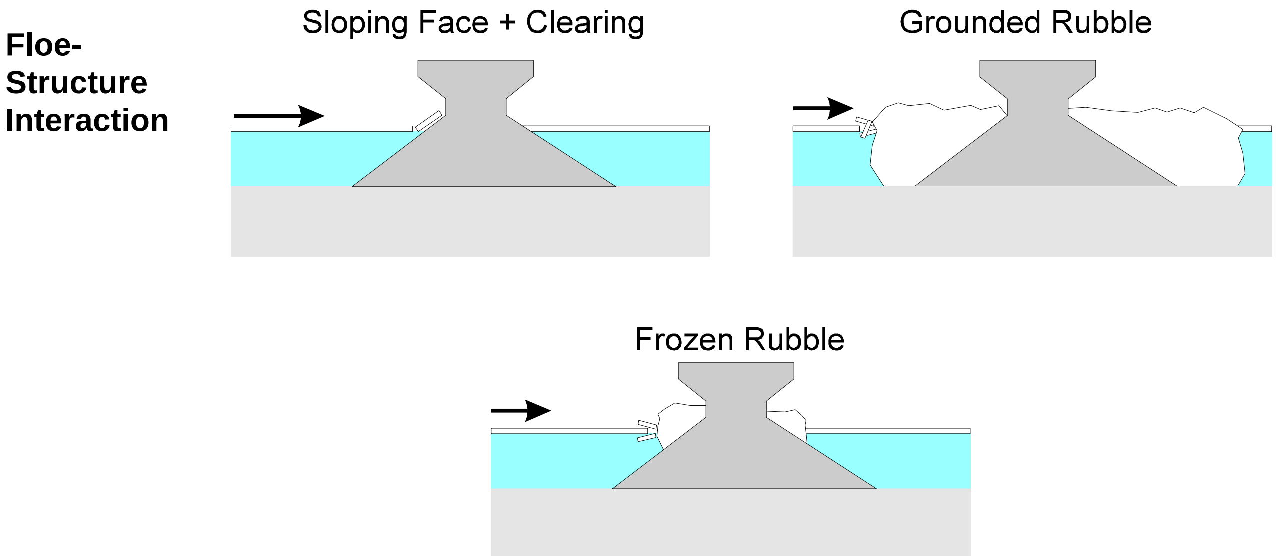


Figure 32. Floe-Structure Interaction with and without Rubble Formation (Sanderson, 1988)

This brings us to an illustration of the hierarchy of limits that occurs when a floe strikes a structure, as shown in Figure 33. When the floe edge first strikes the structure, a local shear failure occurs. As the ice penetrates further, the force required to cause further shear failures increases. In part (a) of Fig. 33, a series of shear failures have occurred. In (b) a flexural failure has occurred and a second is imminent. The force required to cause a flexural failure grows with each failure, because the previous pieces brace the free edge. The third limit is described as the rubbling limit and refers to the force required to build a rubble pile. The force increases as the size of the rubble pile grows. After each bending failure there are further shear failures. Each higher level failure causes a new limit line to be formed on the lower level. These repeating limits are an essential aspect of these hierarchical limits. Figure 33 is only a conceptual sketch. Nevertheless it illustrates the overall concept that is being presented here. This hierarchy of limits is just concerned with intermediate scale failure processes. A similar hierarchy is present on smaller and larger scales.

Hierarchy of Limits

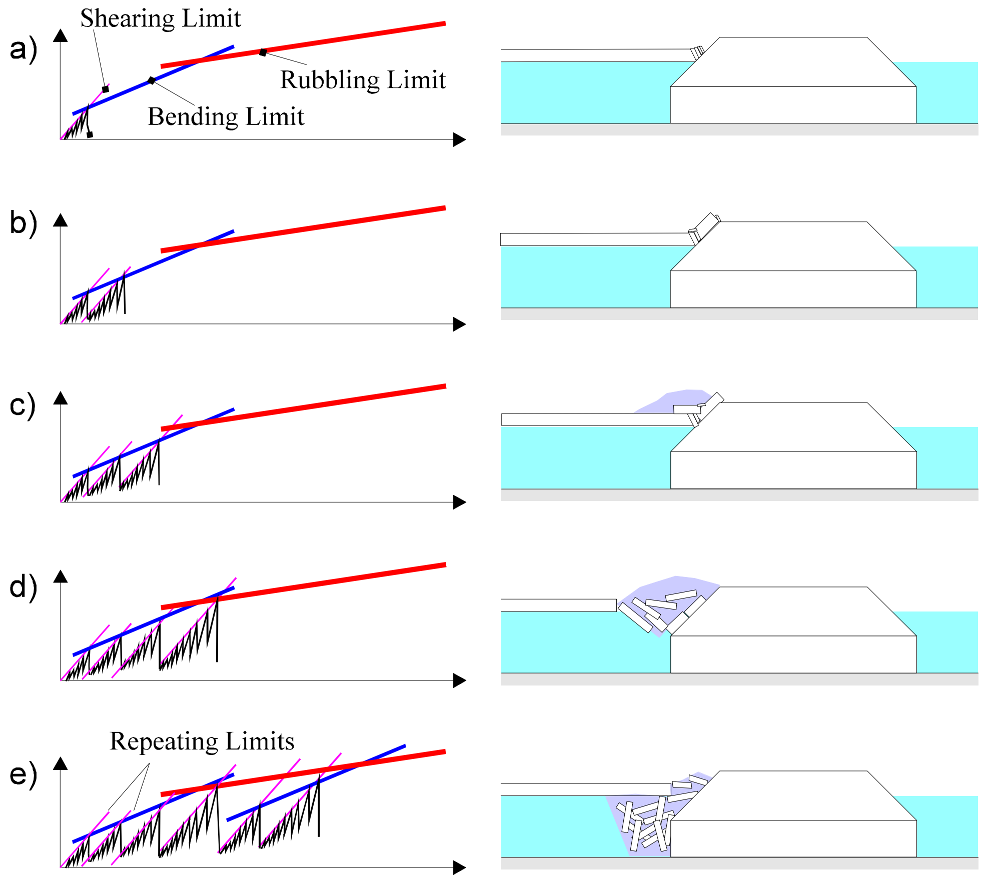


Figure 33. Floe-Structure Interaction showing Hierarchy of Limits

5. Global Ice-Structure Interaction

Global ice structure interactions are concerned with the large scale mechanics (see Fig. 2). Very large floes or floe groups are of concern at this scale. The concept of limits is well established at this scale. Figure 34 (based on Croasdale 1984) illustrates the multiple mechanisms that can be expected to limit the ice load.

There are two thing lacking in the concept shown in Figure 34. The first is that these limits are part of an interconnected hierarchy of limits. The other is that Figure 34 fails to recognize the even larger scale mechanics that creates the pack ice. This largest scale process (pack ice mechanics) must be examined in order to know what floe sizes and energies are to be applied to the ice-structure interaction. And again the pack ice mechanics is a hierarchy of failure processes.

Limit Loads

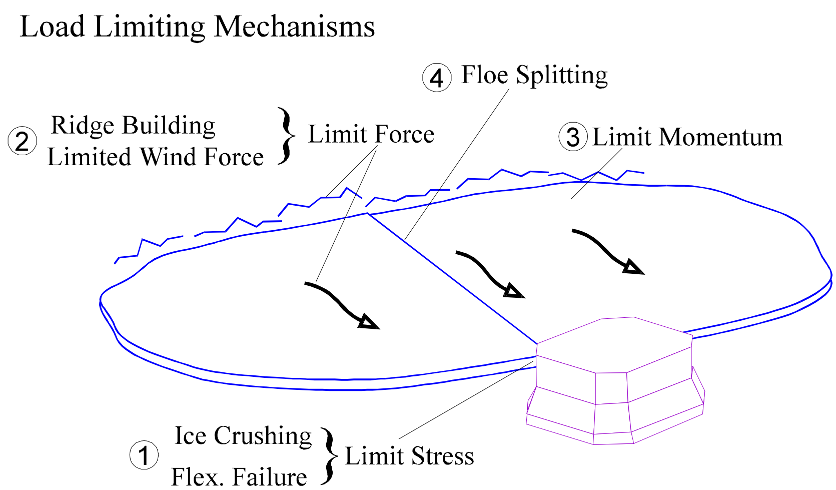


Figure 34. Load Limits in Global Ice-Structure Interactions (modified from Croasdale 1984)

The issues of pack ice mechanics and the state of stress in the pack is a complex issue. The following items will only give some ideas of how the pack ice could be treated in a manner consistent with the general approach of a hierarchy of limits.

First consider Figure 35, which illustrates the geometry of a ridge field (Daley, 1992). This figure was created by outlining and separating level ice from ridged and rubbled ice. The geometry of the rubble is fractal, showing a pattern that is typical of percolation. The authors believe that such patterns arise because of a hierarchy of failures, much as occurs in local ice failure. In this case, flexural failure and rubble building are the main mechanisms. A discussion of the behavior of complex systems, and how they tend to evolve to a critical state can be found in (Ruthen, 1993).

The breakup of pack ice under shear was examined by Erlingsson (1988). Erlingsson noted that the patterns in the pack ice could be explained as a repetitive sequence of shear failures, each resulting from one of four basic stress conditions (Figure 36). This idea is very similar to the concept of a hierarchy of limits.

The idea that similar fracture processes may be occurring on small and large scales was put forward by Schulson and Hibler (1991). They argued that the wing crack mechanism could be observed in satellite imagery of sea ice, as well as in small sample of ice in the lab. This may be further evidence of the broad similarity of failure processes on a wide range of scales.

Much more work is clearly required to understand global ice failure and global loads. An approach based on a hierarchy of failure events provides a framework for this work.

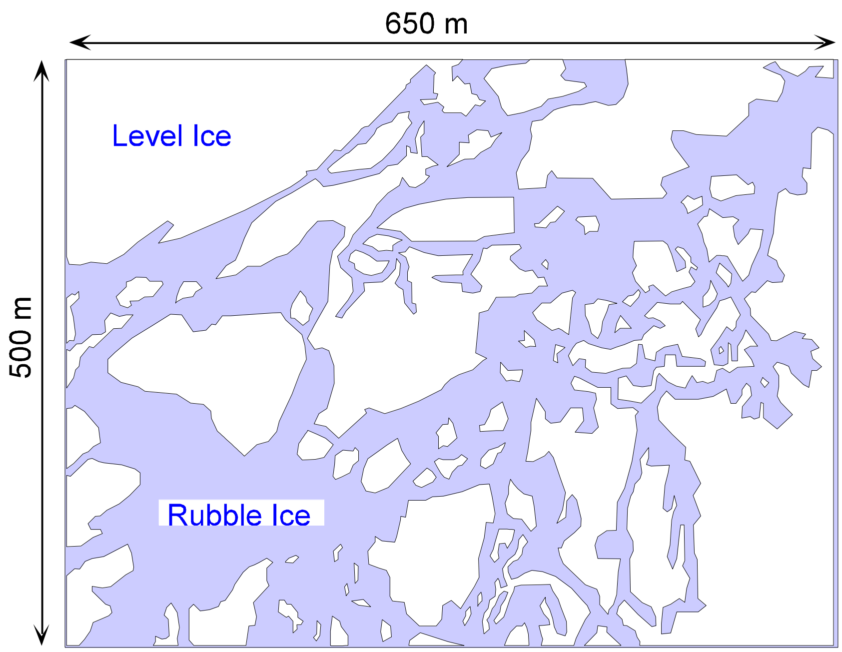


Figure 35. Baltic Ridge Field (Daley, 1992).

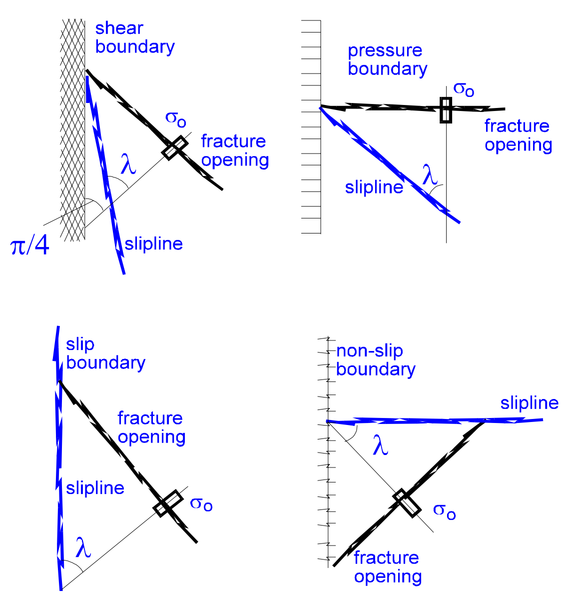


Figure 36. Fracture Mechanics of an Ice Field (Erlingsson, 1988)

6. Applications of the Conceptual Framework

The general concept presented in this report describes ice loads as the outcome of a multi-level ice breakup process. Each level of breakup repeats until a failure occurs on a lower level. The specific ice loading process will depend on the ice and structural parameters. The following figures will show examples of the types of processes that occur in local and global ice failure processes.

Figure 37 shows the elements of a local ice load model. The various aspects of the failure process will occur repetitively. The actual sequence of mechanisms will depend on the specific conditions.

Local Loads

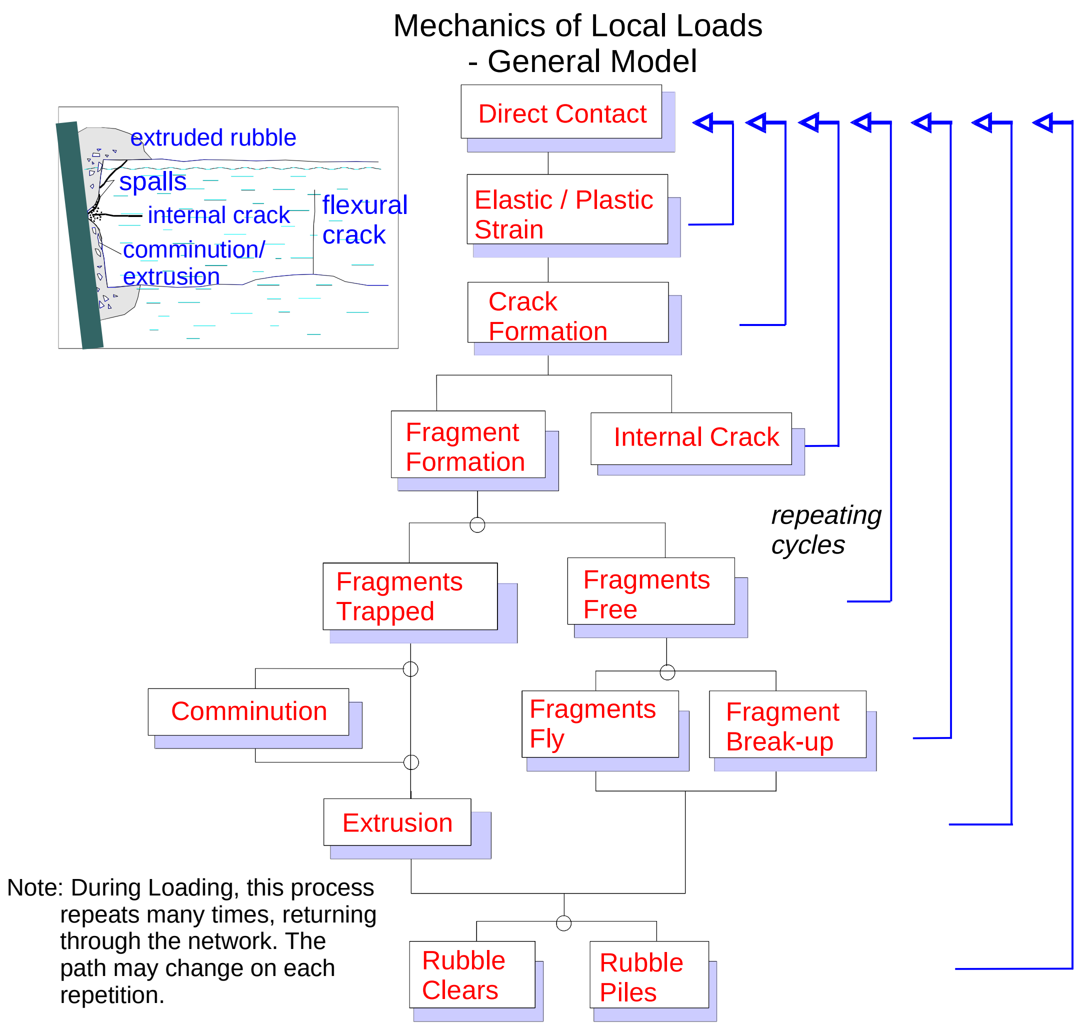


Figure 37. Conceptual Elements of Local Ice Loads

Figure 38 shows the elements of a floe-structure ice load model. The box titled local crushing will be the model shown in Figure 37. As the crushing continues the forces which cause flexural failure and rubbling will build. So too will the forces which will split the floe. During this time the momentum of the floe is decreasing, and in the absence of sufficient additional wind forces, the floe may come to rest. The specific properties of the floe will determine which limit will occur first. Further failures will occur as further limits are reached. The interaction will continue until the floe clears the structure, by some movement of the floe, either as a single floe or in parts.

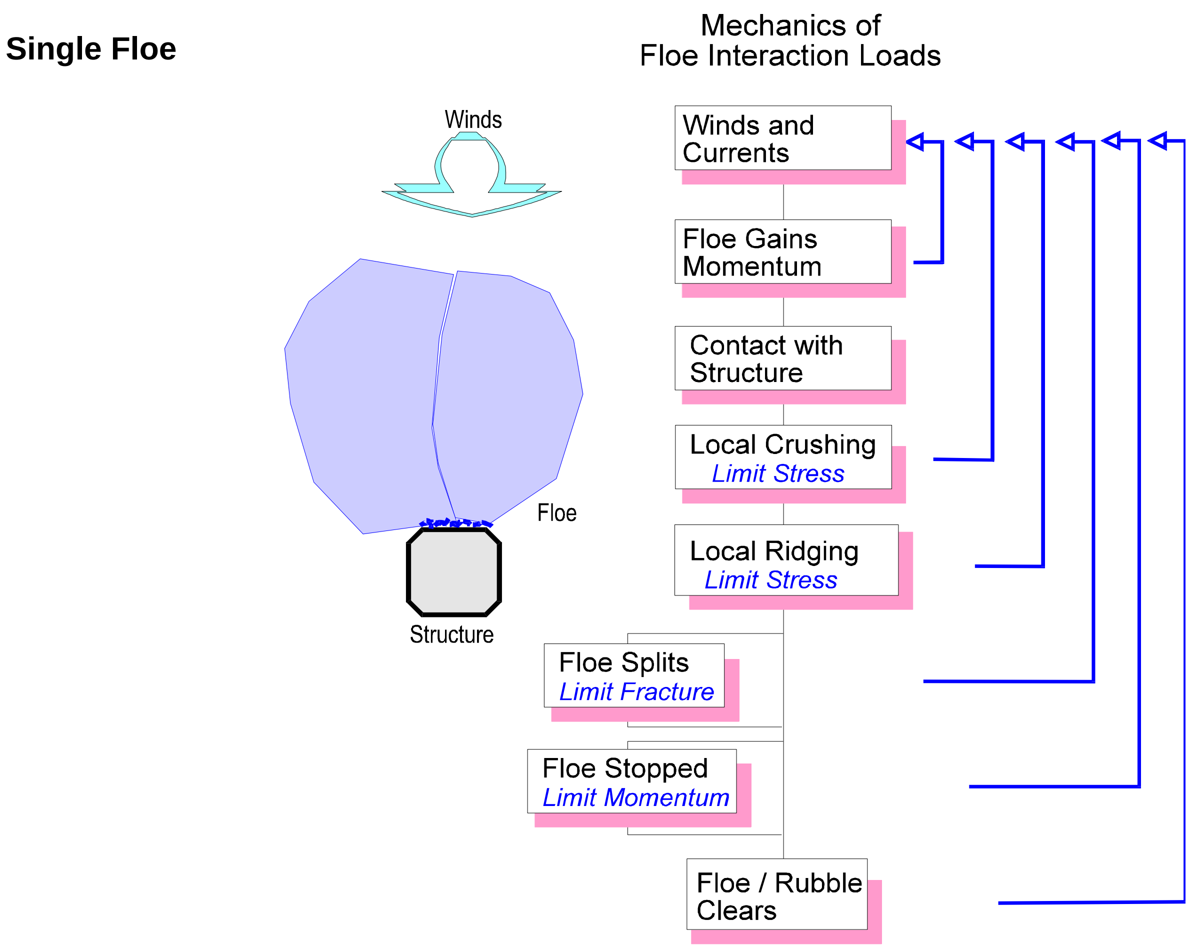


Figure 38. Conceptual Elements of Floe-Structure Interaction

Figure 39 shows the elements of a pack ice-structure ice load model. Again the box titled local crushing will be the model shown in Figure 37. This is a somewhat more complex version of the model shown in Figure 38. The additional complexity comes from the direct ice-ice forces applied by the surrounding pack ice.

Pack Ice

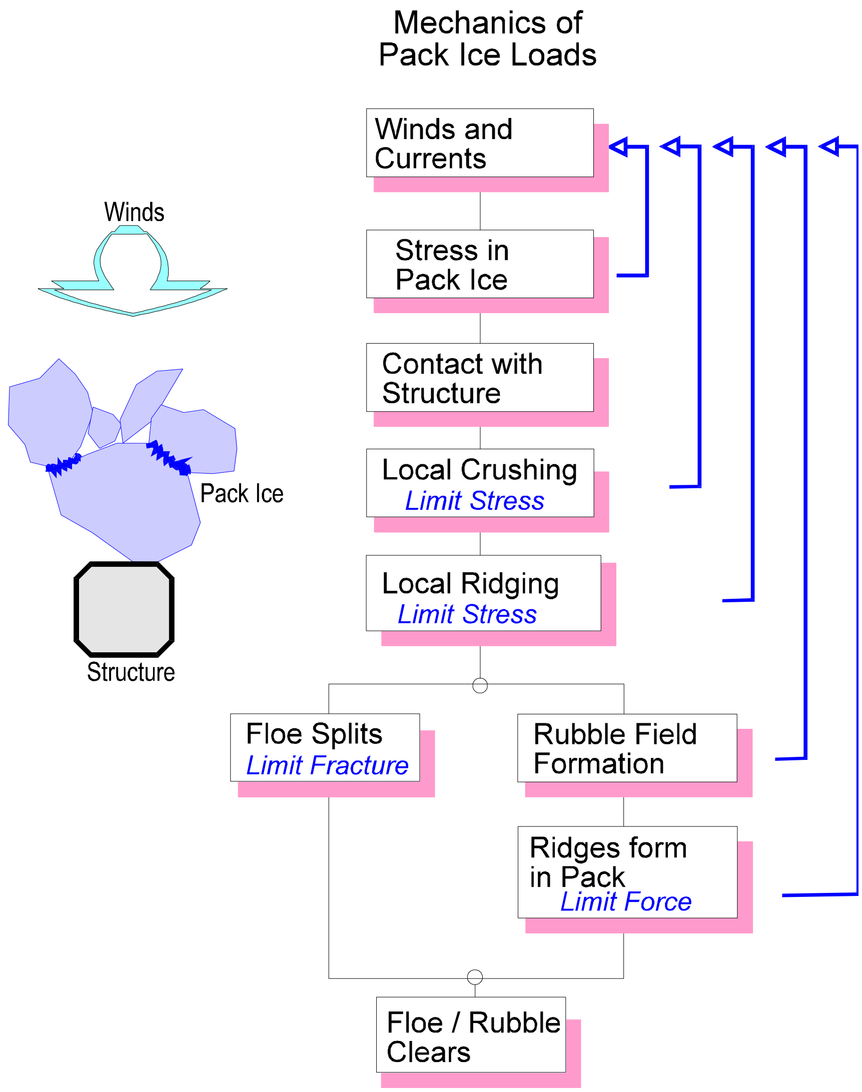


Figure 39. Conceptual Elements of Pack Ice-Structure Interaction

7. Discussion of Experimental Evidence

Observations of ice loads in nature are hampered by the difficulty of measuring the true loads and breakup events. Experiments in the lab or the field are performed so that natural phenomena can be observed in a controlled and complete manner. Experiments serve as a way of isolating natural physics so that theoretical models can be developed. The following section will describe a number of ice mechanics experiments, with the aim of showing how the experimental evidence supports the conceptual model that is presented here. The reader should refer to the original reports for a full description of the experiments.

The experiment by Joensuu and Riska (1988) were described earlier (see Figure 20). The high speed of data acquisition in those tests was part of the reason that the discrete sequence of failure events were observed. Full scale test (Riska, Rantala and Joensuu, 1990) in on the Icebreaker Sampoo showed many similar phenomena, including the multiple saw-tooth nature of the pressures, and the thin line of direct contact that results from repeated flaking.

Tests by Fransson et. al. (1991) confirmed the results by Riska and Joensuu and provided further evidence of the appropriateness of Daley's mathematical model (Daley, 1990, 1991).

A series of field experiments were conducted in the Canadian Arctic. These tests are called the Medium Scale Indentor Tests (MSI Tests) (see Frederking et. al, 1990, Jordaan and McKenna, 1991, Muhonen, 1991, Sandwell, 1990, 1992 and 1993). These tests are the largest scale controlled ice load tests that have been performed. A review of the test results can be found in Daley (1994). In terms of the present concept model there are two points of significance. The first is illustrated by Figure 40, which shows two pressure-area records from two tests with a wedge indentor. There is a remarkable similarity in the form and magnitude of the pressure peaks. If ice failure were dependent on random flaws and random failures in the ice, then such similarity between two tests would be highly unlikely. The tests support the idea that ice failure is primarily a deterministic breakup process. The differences between the plots can be attributed to randomness, but are more likely due to the chaotic nature of the process. Chaos will cause some variations in the process, but will leave the major trends unchanged, as seems to be the case here.

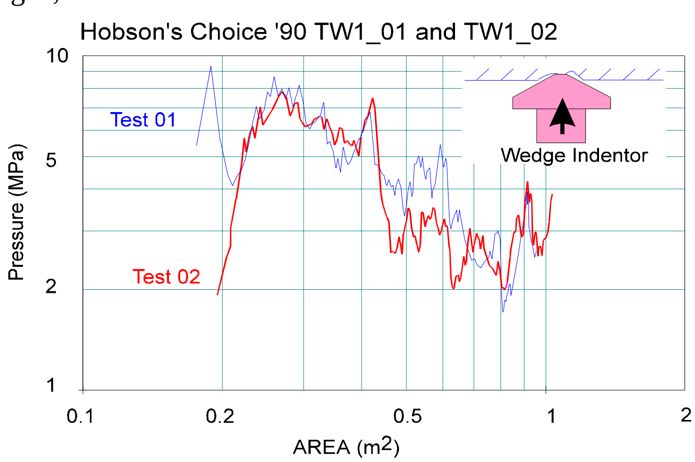


Figure 40. Two Pressure-Area Records from MSI Tests (Daley, 1994)

The second point is shown in Figure 41, showing results for test TFR02. The test was one in which a flat rigid indentor was forced into a shallow pyramid of ice. The plot shows a repeating series of events. The pressures steadily rise during a sequence of minor failure events, until a major failure occurs. The local trend is one of rising pressures, but the larger trend due to the limits imposed by the major failures is one of falling pressures. The reason for the two trends seems to be due to the interaction of extrusion and flaking processes, with flaking acting repeatedly to limit the buildup of loads caused by extrusion over a growing area.

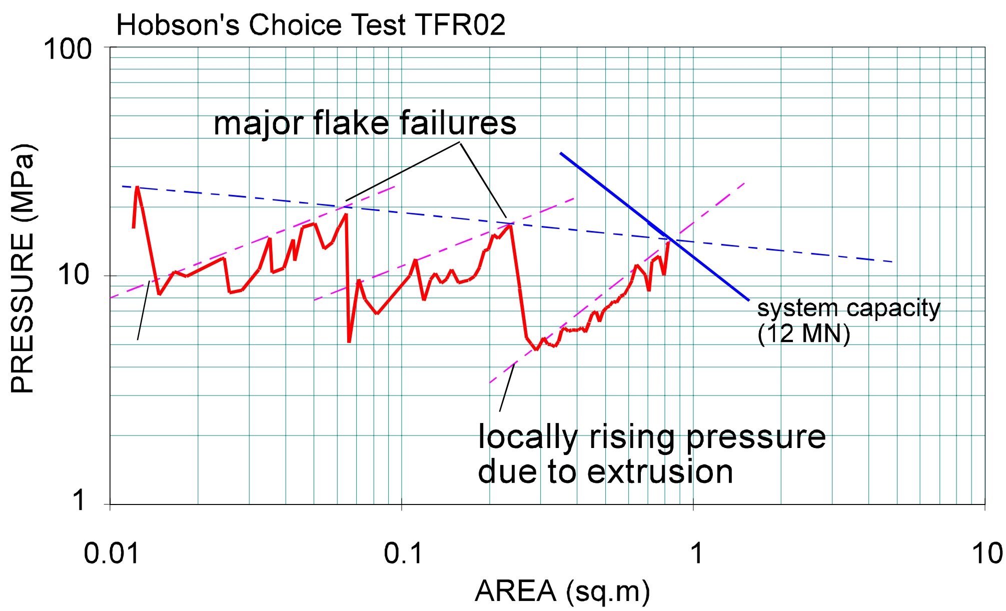


Figure 41. Hobson’s Choice Data Showing Discrete Failures (Daley, 1994)

Figure 42 is a sketch that explains how extrusion may affect the flaking mechanics and thus the overall ice load, as was seen in Figure 41. Figure 42 shows four views, with and without extrusion and local and global flakes. In Fig 42 (a), local ice flakes form without the influence of crushed and extruding ice. The ice pressure is highly concentrated at the center of contact, and the flaking cracks are free to run. In Fig 42 (b) we consider the addition of crushed and extruding ice which extends out to the edge of the nominal contact area. The extrusion has two important effects. The first is that the ice force is spread laterally. Both the direct contact and the contact with crushed ice contribute to the total force. The more important effect is that the pressure in the extruding ice will tend to add a confining stress on the solid ice and thus raise the force level required to propagate the crack. The extrusion pressure will grow exponentially as the width of the nominal contact grows. Both the direct contact and extrusion pressures will grow together. Now consider the situation of the global cracks and flakes as shown in Fig. 42 (c) &(d). The crack paths are outside the local contact zone and are unaffected by the presence or absence of the extrusion process. The total force is all that matters to drive the global crack.

This argument may explain why Fig.41 shows growing pressures between major cracks, but major peaks that do not increase. The question of what governs the major cracks and why the major peaks exhibit a decreasing trend is not yet answered.

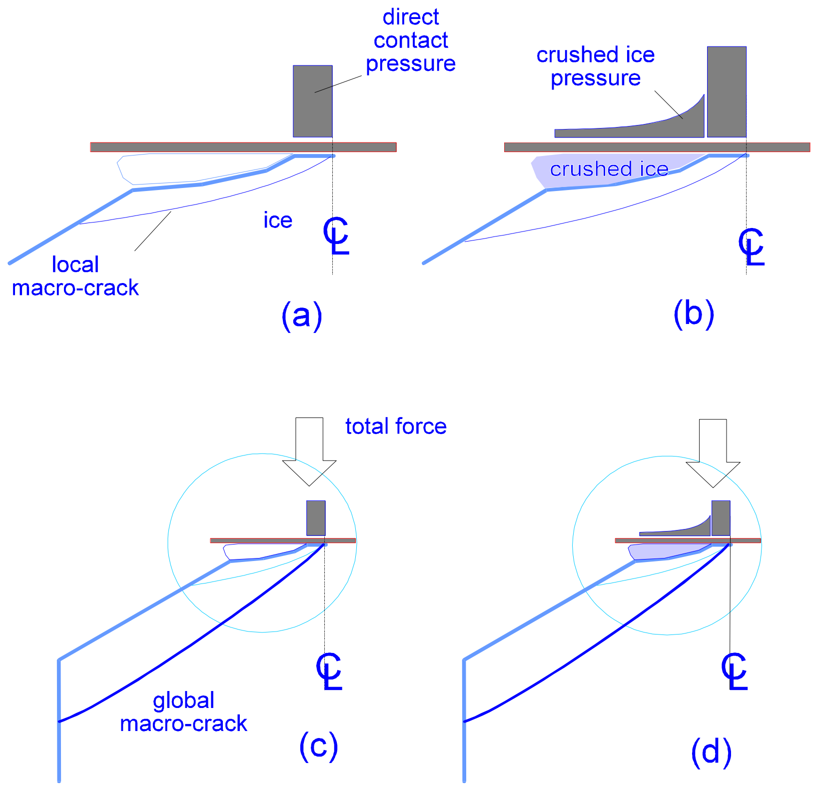


Figure 42. Extrusion with Local and Global Flaking (Daley, 1992).

Dynamic indentor tests (Muhonen et.al 1992) were conducted in Finland at the Technical Research Centre of Finland. Figure 7 shows one of the observations of those tests. The tests showed a combination of splitting, flaking and possible microstructural damage.

Extrusion of ice was experimentally examined by Tuhkuri (1990). The tests showed an exponential growth in pressures as the pulverized ice layer thickness decreased, and as the distance to a free edge increased. In a further series of ice load experiments, Tuhkuri (1993) found evidence for a combination of flaking and extrusion processes happening together. Further, Tuhkuri’s tests provide evidence of the type of fragmentation described in Figures 27 and 28. Further discussion of fragmentation is given in Tuhkuri (1994).

A final set of data that is very interesting are the results of Veitch (1993, 1994) in which an ice block was indented by a set of cutting tools with various cutting angles. These tests give a lot of insight into the process of flaking and crushing. As the conceptual model presented here is developed into an engineering tool, it will be important to compare and contrast results will all available experimental data.

8. Recommended Research

The conceptual approach presented in this report raises many questions. Many of the issues are already the subject of active research, although the concept of a hierarchy of failure processes puts the issues in a new light and emphasizes some issues as being of particular importance. The topics listed below are the key issues that require work. The first topics concern local loads and are fundamental issues in the science and mechanics of breakup. The second group are larger scale issues, which tend to be less well defined scientifically at this stage. The last issues are ones of implementing engineering tools. All are important.

Concerning local loads the topics that require study are;

- flaking mechanics
  - 3D flaking
  - curved flakes
  - ductile flakes
  - effect of extrusion on flaking
- pulverization mechanics
  - microcrack expansion
  - comminution
  - internal macrocracks
  - macrocrack cascade
  - extrusion processes
- direct contact processes (pressure melting, ductility, damage)

Concerning the larger scale loading processes, topics for study include;

- interaction between processes
  (for instance between crushing and rubble building)
- pack ice formation mechanics and geometries including
  large scale fracture mechanics
- ridge field formation
- ice-structure interaction dynamics

And finally, related technical concerns include;

- discrete processes and chaos
- working software to implement a hierarchy of
  limits methodology for load calculation

9. Summary and Conclusion

A new conceptual model of ice load has been presented. Ice failure is modeled as a hierarchy of failure events. The hierarchy covers all scales from sub-crystalline to the whole ice cover, although practical cases cover scales from local crushing to floe splitting. Each type of failure event represents either the culmination of a process at a smaller scale and a step in a process leading to a failure event at a larger scale. For example, crushing occurs until a flexural failure happens, flexural failures are the component events in rubble formation. Crushing itself is a hierarchy of cracking and extrusion processes.

To put the new approach in perspective, the main ice load models and components were reviewed. The concept model was based on recent work on local ice failure, which was based on experiments and observations. The new approach generalizes those results.

The new approach relies on discrete mechanics, while many previous methods were based on continuum mechanics. One need only watch ice break up to see that it does not appear to be a smooth process. The methods of discrete mechanics and mathematics are not yet as well developed as continuum methods. This provides one of the challenges of the proposed approach.

To illustrate the general approach, three example cases of ice load models; ice edge crushing, single floe-structure impact and pack ice-structure interaction, are sketched as flow charts. These serve to illustrate the nature of the interaction of failure events. The local load model is more developed, as this topic is more mature than that of global loads.

A list of recommended research topics is given. The mechanics of cracking and flaking, the mechanics of pulverization and extrusion, the interaction of discrete processes and the implementation of software tools are seen as the key issues.

As with any new idea, this conceptual model is not fully developed and still quite brittle. The approach has many advantages, but many aspects still require development. The strength of the approach is that it naturally covers the whole scale of ice failure processes, and in a way that reflects the mechanics of the breakup of a solid.

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APPENDIX A - REPORT & FIGURES ON DISK

Disk 1 contains a folder ilcm_1, containing the text of this report and half of the figures in *.wmf (Windows MetaFile) format. Disk2 contains a folder ilcm_2, containing the rest of the figures. The report was prepared in Microsoft WORD v6. You may need to use the **Edit, Links...** feature in WORD to reestablish the links to the figures, which were not stored with the text. The figures were created with Corel*DRAW!* v3, but are stored in a .wmf format for easier retrieval and use by WORD. You may contact Claude Daley, Daley R&E @ (613) 825-4582 if you have any questions.