PERD/CHC Report 7-108

A Study of the Process-Spatial Link in Ice Pressure-Area Relationships

Prepared for
National Research Council of Canada

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March 20, 2004

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Symbols

A	Area	m^2
A_T	total contact area	m^2
C	Ice pressure constant	MPa at $1m^2$
e	Area exponent in the pressure	-
P	Ice Pressure	MPa
P_{AV}	average ice pressure in the contact area	MPa
F	Force	MN
p/a	pressure-area	

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Acknowledgement

The work was made possible with the support of funding from PERD (Panel on Energy Research and Development). The author wishes to thank Dr. Garry Timco of the NRC/CHC (Canadian Hydraulics Centre) for his support and patience.

1 Introduction

Ice is the dominant feature in artic waters for most or all of the year. In sub-artic regions, ice can be present in many forms for part of the year. The eastern coastal waters in Canada are prone to extensive ice coverage. Ice will often be the dominant load when considering the design of ships and offshore structures for many regions, including in the Gulf of St. Lawrence, Newfoundland waters and along the Labrador coast. Many see ice as the dominant design challenge, and in other cases, the primary impediment preventing the economic development of resources. Consequently, improving our understanding of ice loads is a topic of large practical significance.

Ice loads on structures occur over a specific, often quite small area. The area of contact is, more or less, the area of overlap of the ice edge and the structure. The earliest measurements and models were primarily concerned with the total ice contact force. Early ice load models [6] did include terms to show that the average ice pressure varied, but there was no representation of pressure variation within the contact. As an approximation, the pressure within the contact was assumed uniform.

The interest in ice loads grew significantly in the 1970s and 80s, as offshore oil and gas developments expanded. From about 1980 onwards, there have been many field trials and measurements in which ice loads have been measured [8,9,10,11,12,13,14,15]. These include ice loads on ships and offshore structures. Many of these experiments and trials were able to measure the distribution of pressure within the contact area. This lead to the realization that ice pressure is far from uniform. To a degree far greater than with wind and current loads, ice pressure measurements depended on the size of the sensor, especially for quite small sensors. To describe this effect, the pressure data was often plotted with area as the independent variable. Investigators began to see area as one of the dominant, if not the dominant, determinant of ice pressure. Parameters such as ice strength, thickness, and velocity, tended not to vary much in any data set and thus had less influence on pressure. On the other hand, pressures on small sensors (for example a few square centimeters) were observed to be orders of magnitude higher than pressures measured on large sensors (square meters). As more data became available, the pressure-area plot became the most common way to present ice pressure data [16].

Today, pressure-area (p/a) models are commonly used to determine both local and global ice loads on ships and structures. See [5] for an example of the use of pressure-area models to determine impact forces. There are two distinct types of pressure-area models [4,18]. The 'process' p/a distribution describes how the average pressure relates to the total contact area, and is used to calculate the collision force. The 'spatial' p/a model is a description of how local peak pressures relate to area for areas within the total contact. The 'spatial' model is used to determine design loads on local structure, such as plating and framing.

This report examines the link between the two pressure-area models. Evidence from both field measurements (e.g. Polar Sea [1]) and numerical models (e.g. NEB/PERD report [2]) appears to show that local (i.e. "spatial") pressures are correlated positively with total force, as well as being inversely related to area. This is in contrast with many code requirements (e.g. [3,17]) which do not directly include the correlation. Further, the evidence relating to the process p/a relationship is unclear. It is not certain that the average pressure declines as the total area increases, as is often assumed. This has a very significant impact on the calculated maximum loads. Coupled with the link to local (spatial) pressures, there is a question about the proper level of design pressures in situations involving large forces.

To clarify this last point, let us consider design of an offshore structure for iceberg impact. All available p/a panel data has been gathered in cases where the total force is less than about 20MN, with almost all data for cases below about 5MN. While there is some pressure data for very large forces, such data only gives the overall average pressure, not the pressure distribution, and comes from cases of very large aspect ratio. Such data is of little value when studying the general nature of ice loads. In the case of iceberg impact, calculated forces can easily be in the range of 50 MN and up to several hundred MN. Such predicted loads and pressures cannot be empirically validated directly. The values are an extrapolation of the data, and rely on the relationships inherent in the pressure-area models. If the average pressure rises with area, calculated pressures become significantly larger than for constant or declining pressures. Further, if local pressures are strongly correlated with total force, the local pressures in a large iceberg collision may be higher, even significantly higher, than any measured to date. The two effects combine to create an important question.

Thus, there are significant practical implications to a linkage between the two p/a curves. This report will examine the pressure-area data from the Polar Sea to study this linkage. As well, a numerical model of contact will be examined from this perspective to see if it can shed light on this issue.

2 Description of Ice Contact

2.1 General

Figure 1 shows an idealized sketch of the contact between a large ice feature and a structure. All items except the flexural crack will be present in every ice contact, though to varying extents. The ice exerts pressure on the structure both directly and through a layer of extruding crushed ice. The highest pressure will occur in the direct solid contact. The ice in the solid contact region may be damaged by internal cracks and material damage, but is quite confined and capable of sustaining very high pressures. Towards the edge, the structure is only in contact with crushed and extruded rubble and will exert quite low pressures. The pressures may well vary over many orders of magnitude within the contact region. Outside the contact, the pressures are effectively zero. The sketch may describe an event that is only centimeters across, or it may be meters across. This report examines the ice pressures that occur in situations such as this.

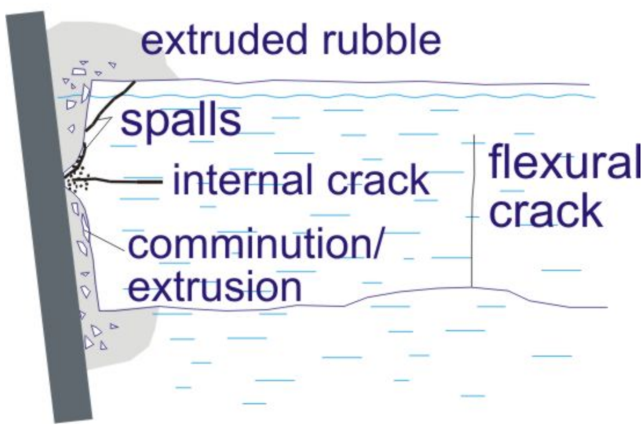


Figure 1. Sketch of ice contact with a structure.

2.2 Spatial Pressure Distribution

The spatial pressure distribution describes the variation of pressure in an ice contact at one instant in time. Figure 2 illustrates the idea. The pressure varies within the contact, forming one or more peaks. The highest pressure occurs on a small area at the peak. The average pressure within larger areas will necessarily be smaller than the peak pressure. Average pressures over progressively larger areas (each containing all the smaller area and more) well decline. Consequently, spatial pressure-area plots will always show an inverse relationship between pressure and area. Typically, such relationships take the form:

$$P=C\ A^{-e} \tag{1}$$

where C is a positive value, representing the average pressure at one unit of area, and e is in the range 0..1. C is typically in the range of 0.5 to 5 MPa and e is typically be in the range of –0.25 to –0.7 . The values vary from dataset to dataset.

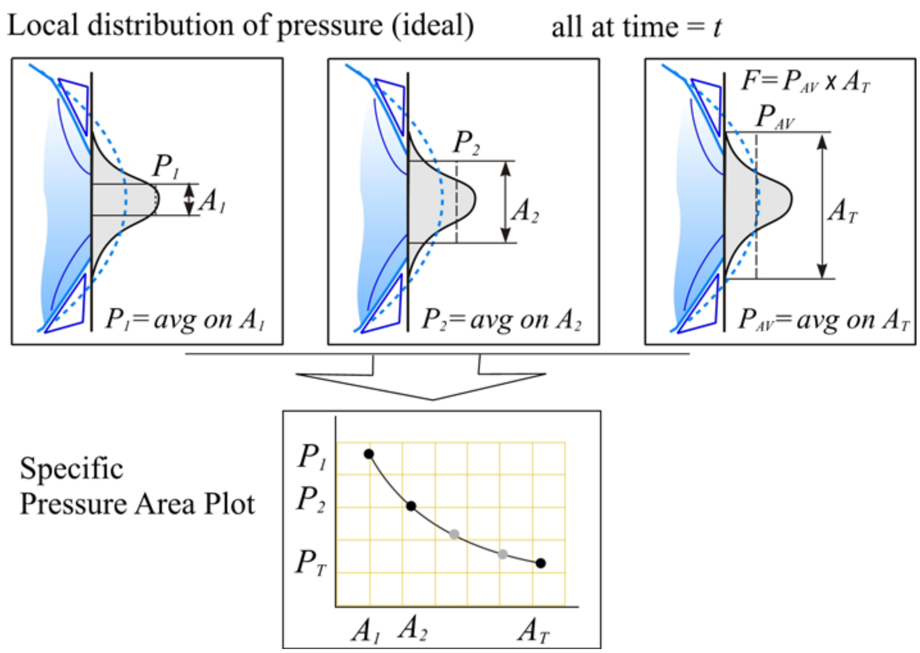


Figure 2 Sketch of ice pressure and the meaning of a specific pressure-area plot.

There are several ways to define both pressure and area, and then the meaning can be affected by the measurement procedure. We are rarely able to measure with both fine spatial resolution and large areal coverage. As a result, the data tends to be coarse and may obscure the real trends.

Figure 3 illustrates three variations of the meaning of the word pressure, and associated area. On the left, we define 'nominal pressure'. If we have independently measured the total force, and we have observed the overlap area (nominal area) of the ice and structure, we can divide one by the other and find the nominal pressure. This is a useful value, but gives no information on the local pressure distribution. In the central sketch, we postulate the true pressure distribution. We could presumably observe this if we had the ability to measure pressure contiguously over the entire surface with high spatial resolution. This type of data is practically non-existent. The right hand sketch shows the situation that we normally face. The pressure has been measured on a rather coarse array, and may be subject to noise and other forms of error. Consequently, the coarseness of the array and the data collection/reduction algorithms can influence the estimates of the local pressures. There are always some pressure and areal resolution limits to deal with. These points should be kept in mind when thinking about ice load data. Figure 4 shows the spatial pressure-area plat that would be derived from measured values.

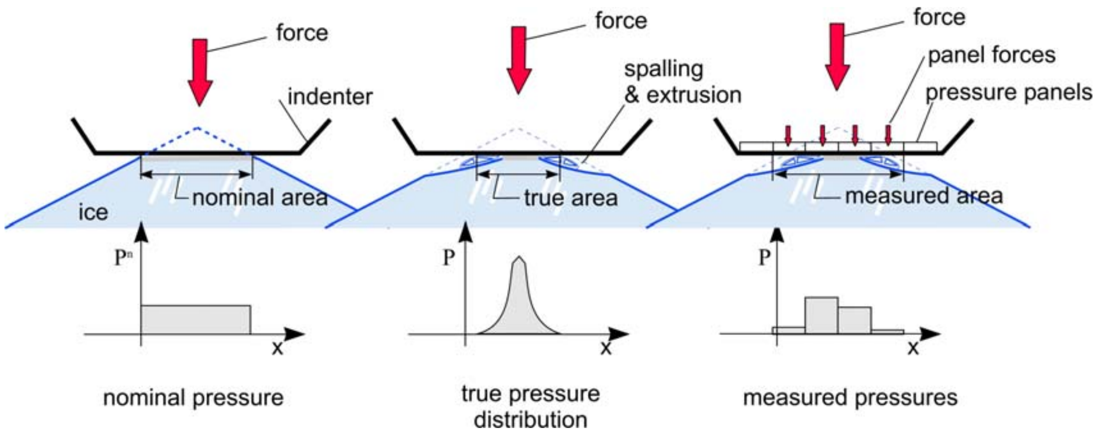


Figure 3. Types areas and pressures related to pressure-area data.

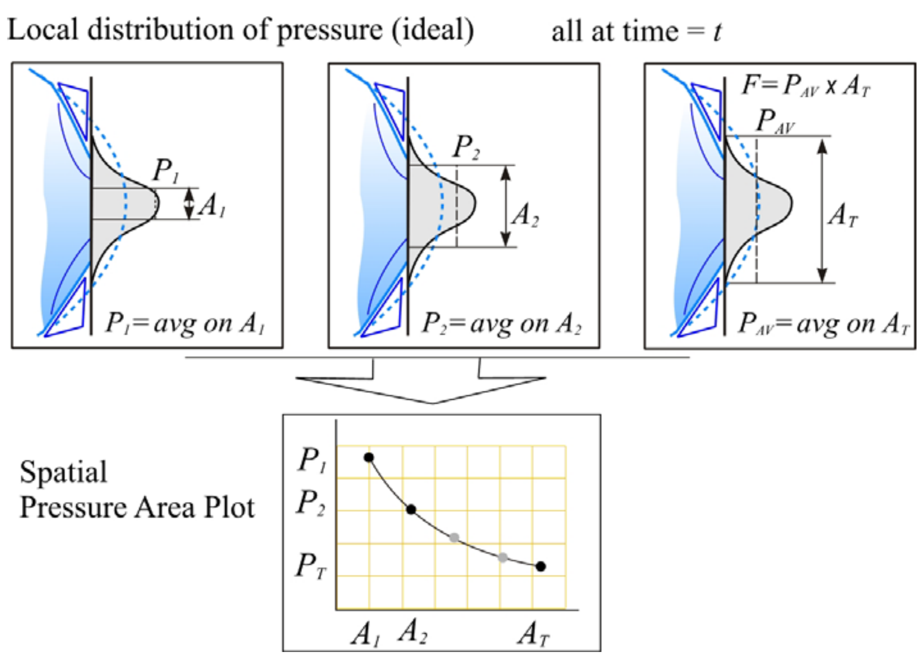


Figure 4. Sketch of measured ice pressure data and spatial pressure-area plots.

2.3 Process Pressure Distribution

At any point in time, there is a total area, and an average pressure. The product of these two values is the force. Figure 5 illustrates the process pressure-area plot, as would be derived from measured data using an array of pressure sensors.

Referring again to Figure 3 it is obvious that the measured average pressure and the measured total area is similar to the nominal pressure. In cases where there is no independent measure of both total force and nominal area, the values as would be estimated in Figure 5 are the only way to determine the nominal values. This is an important point. Nominal pressure area values are required for those cases in which the design load is estimated from an impact analysis. This is the case for iceberg-structure collision and many ship-ice collision loads. No field data has both complete coverage with pressure panels, and independent measurement of force and nominal contact. Such a data set would allow force to be determined by two independent measurements. All the extensive data from ships only contain pressure panel measurements. Consequently, we are left with the measured process pressure-area values as often the only estimate of the nominal pressure-area relationship. It is hoped that future ice load data collection programs will be able to gather both types of data.

Figure 5 illustrates another point about the process pressure-area relationship. Unlike the spatial pressure-area relationship, there is no a-priori reason for the pressure to fall with increasing area. Factors such as increasing confinement could well lead to increasing average pressures as the interaction proceeds. Most authors have suggested declining trends [16], yet others have suggested rising trends [7, 13].

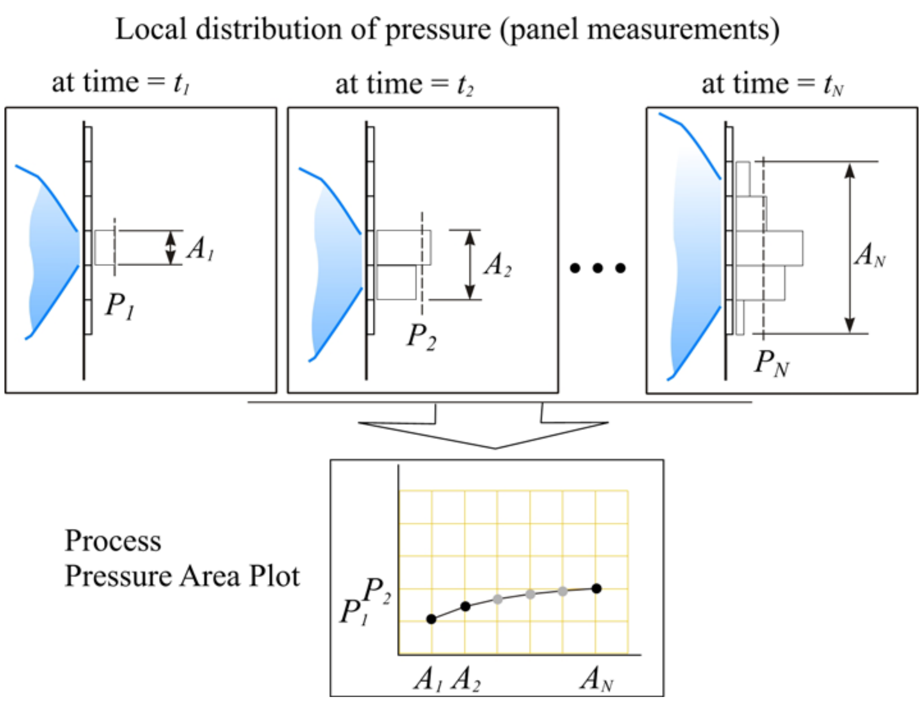


Figure 5. Sketch of measured ice pressure data and process pressure-area plots.

2.4 Link between Process and Spatial Distributions

The spatial and process pressure area plots are derived from the same data. The process values are just the average pressures over all the measured sensors (the non-zero pressures). Figure 6 shows both types of data on the same plot. This again illustrates how the spatial pressure area curve can be falling, even as the process curve is rising. Note that the connection between the two types also suggests that higher local pressures will tend to occur with greater total areas and total contact forces.

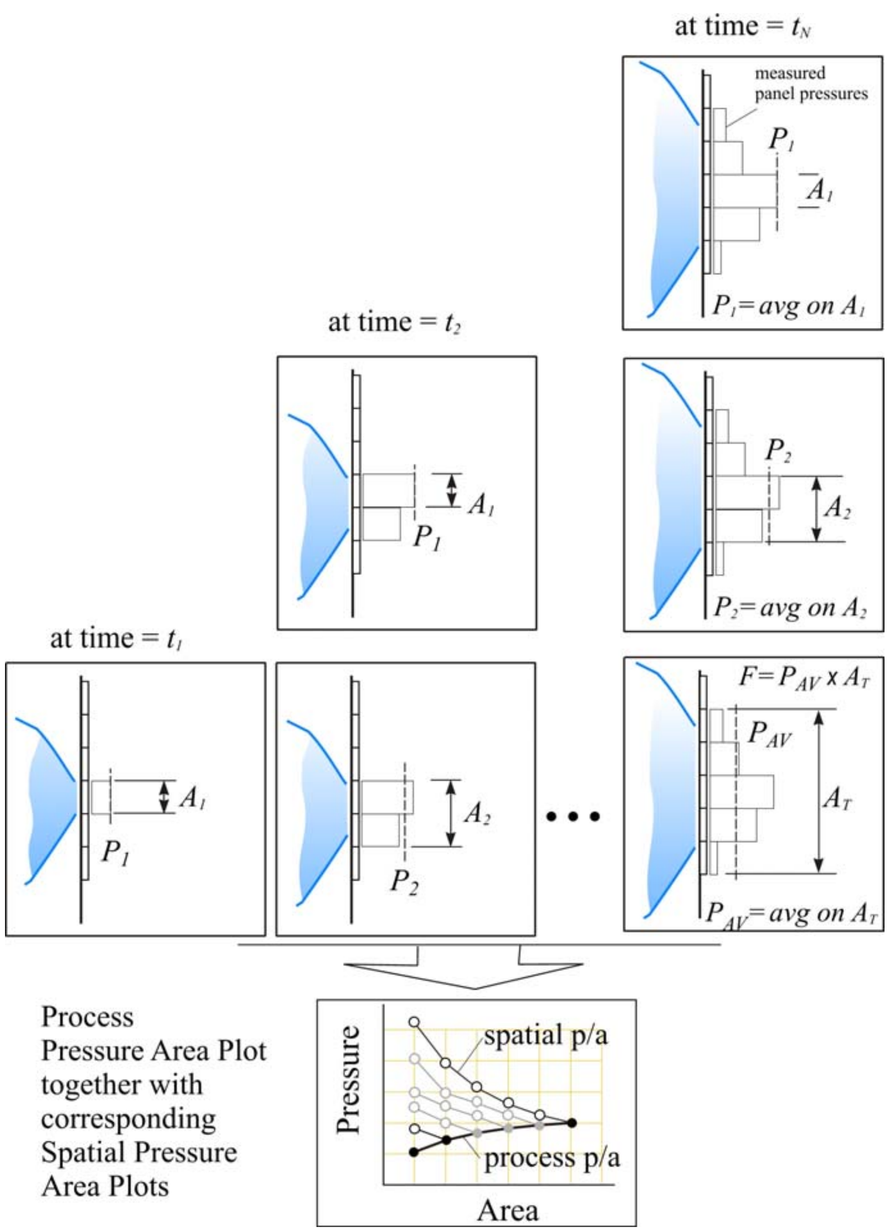
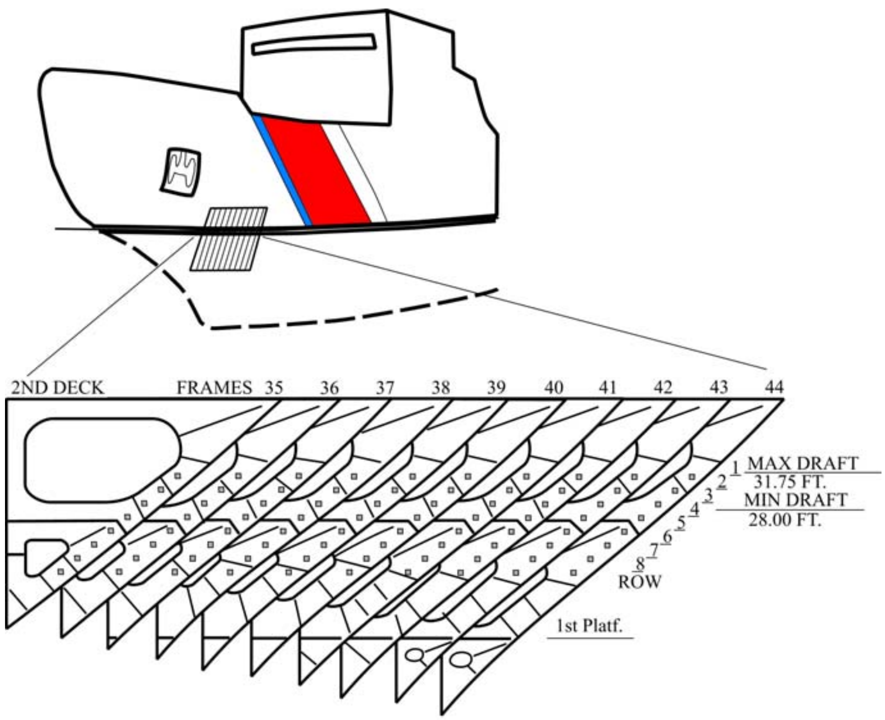


Figure 6. Combined spatial and process pressure-area data.

3 Polar Sea Data

3.1 Description of the pressure measurement system

Figure 7 shows a sketch of the instrumented portion of the bow of the Polar Sea [8]. An array of strain gauges was placed on 10 structural frames in the bow of the ship. The location was chosen to give the highest chance of large collisions. Each of ten frames was instrumented with 8 strain rosettes. The primary measurement was compressive strain normal to the shell. The gauges were placed in such a way give continuous coverage within the panel, and yet be is insensitive as possible to pressures outside the local region. Cross-over effects were removed by the use of a matrix of influence factors derived by finite element analysis. The measurement system was validated by means of a physical calibration.

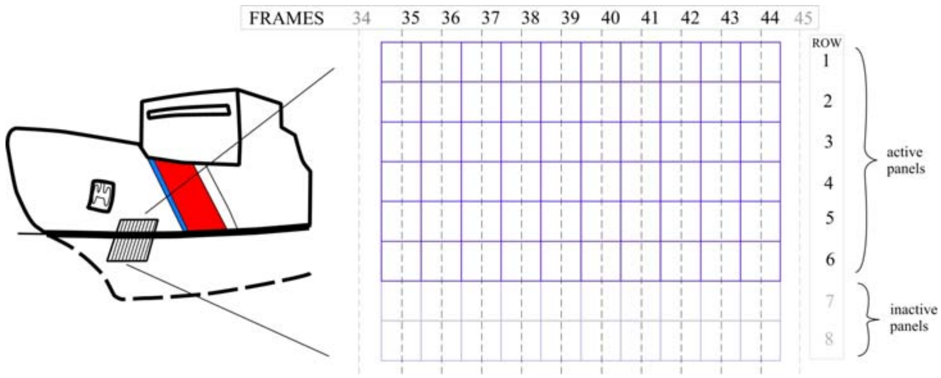


Overview of the instrumented panel on the bow of the USCGC Polar Sea. Strain gage locations are indicated on frame expansion.

Figure 7. Ice load panel as installed in the Polar Sea.

The actual panel layout is shown in Figure 8. Each sub-panel was 0.152 m^2, and the total panel covered 9.1 m^2. The strain gauges were sampled 32 times per second. Each event began when a threshold level of ice pressure was read. Once triggered, the event was sampled for the same amount of time. The event lasted from one second before the trigger to approx. 4 seconds after the trigger.[1]

[1] More precisely, the first trials in the Beaufort Sea in 1982 contained at record length of 200 samples (6.25 sec.), while all subsequent trials (e.g. Chukchi Sea 1983) contained record length of 158 samples (4.94 sec.). It was found that collision events ended within the shorter time, and those few that did not, lasted much longer (because they were static). Nevertheless, for the few very long events, the system would re-trigger.



Layout of the instrumented panel on the bow of the USCGC Polar Sea.
Each sub-panel is centered on a frame.

Figure 8. Ice load panel layout on the Polar Sea

Annex A and B show a partial summary of the events. The largest panel pressures and forces are listed for the 1982 and 1983 trials in Multi-year ice. Annex C shows an example of the pressure data in a single event file. The full records are available in electronic form.

3.2 Polar Sea Data Reduction

The Polar Sea data, after conversion, is a set of panel pressures. In its original form [21], these are in units of p.s.i., stored as an integer. Each time step contains 60 values, referring to the six rows on each of ten frames. There are either 158 or 200 time steps per event, and there are thousands of events.

The data was analyzed initially in a number of ways. The force on the panel was found by summing the sub-panel forces (product of pressure and area). Thus a force time history of the collision could be plotted. Peak force and peak pressures were tabulated for each event. Similar values were found for each row and frame (to be used to assess loads on transverse and longitudinal frames). For each of the events, a spatial pressure-area plot was calculated for two cases; the time of peak force, and the time of peak pressure. Figure 9 shows how the spatial pressure-area plots are calculated.

For the present project the data from the Beaufort Sea trials of 1982 and the Chukchi Sea trials of 1983 were re-analyzed. These trials contain mainly collisions with large multi-year ice floes. For a large selection of events the pressure records were re-analyzed to give a spatial pressure-area data for every time step. The highest force events were included. With the spatial pressure-area data at every time step, it was then possible to extract the process pressure-area data, which required the total area and average pressures. The process pressure-area data were divided into two parts. The first part was during rising force, which is presumably while ice penetration occurs. The second part was while the force declined, when presumably the penetration was over and rebound and slide-off occurred. To use the data as a basis for extrapolation to larger collisions, it is reasonable to separate these two types of data. This data is only plotted for that part of the event when the main activity (the main impact) occurred. This further helps to clarify the processes that occur during collision from the general ‘noise’ that occurs before and after.

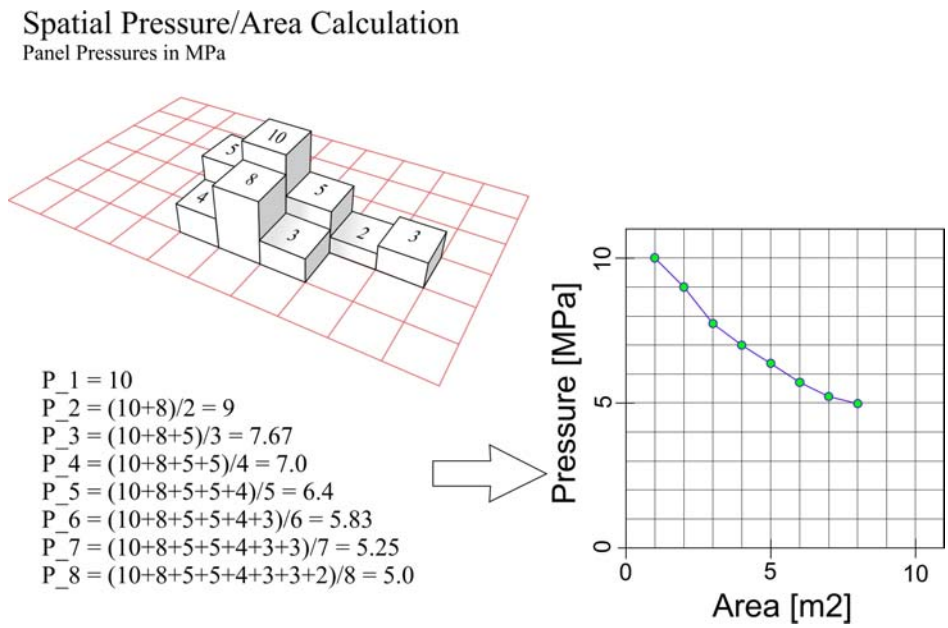


Figure 9. Illustrative example of the spatial pressure-area calculation with Polar Sea data.

3.3 Polar Sea Data Analysis

The Polar Sea data has been re-analyzed to extract both spatial and process pressure-area curves. Figure 10 illustrates the types of analysis plots for an example data set (which is similar to an actual data set, but with less noise). The top plot is a set of spatial pressure-area curves during ice penetration phase of the impact, along with the process pressure area curve for that event. As can be seen, the peak pressures (closest to the pressure axis) rise as the whole curve rises, and as the total area rises. The process pressure-area curve is found by joining the ends of the spatial p-a curves. The process p-a curve rises in both pressure and area. The total force at each step from the product of the values of the process curve (pressure x area). The second plot illustrates the case during the declining part of the impact. The declining plots will not be included for the actual data, as they serve little purpose here. The four smaller plots below show the various relationships between average pressure, total area, total force and peak pressure. In the example all four of these quantities rise together. The plots show both the rising and falling values.

In Figure 11, 12 and 13 re-analyzed data is shown from the three largest events from the 1982 Beaufort Sea trials. All show trends that are similar to the idealized example, with all values rising together.

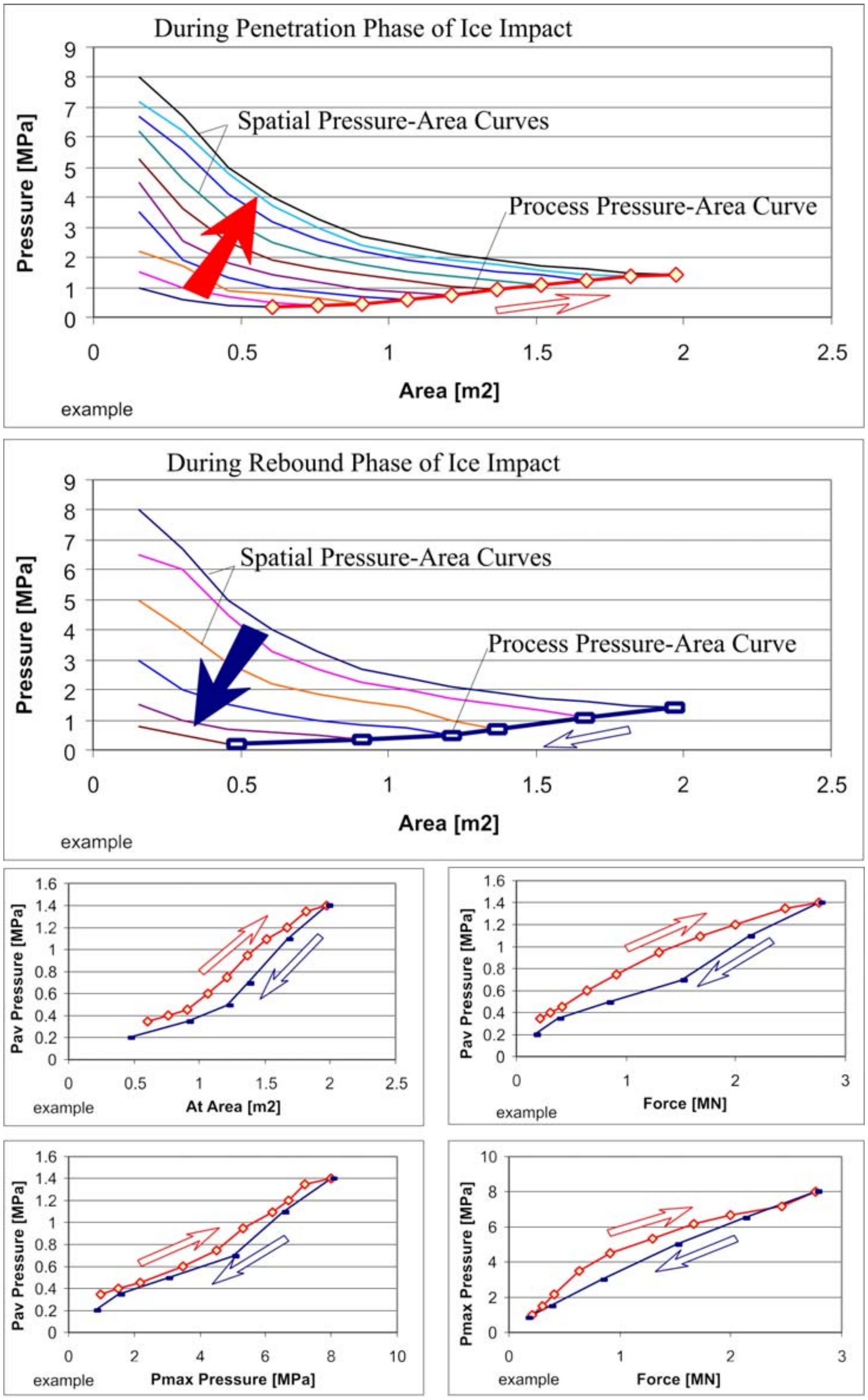


Figure 10. Example of analysis plots on one event from the Polar Sea trials (the data in this plot is artificial).

3.3.1 Data from the 1982 Trials

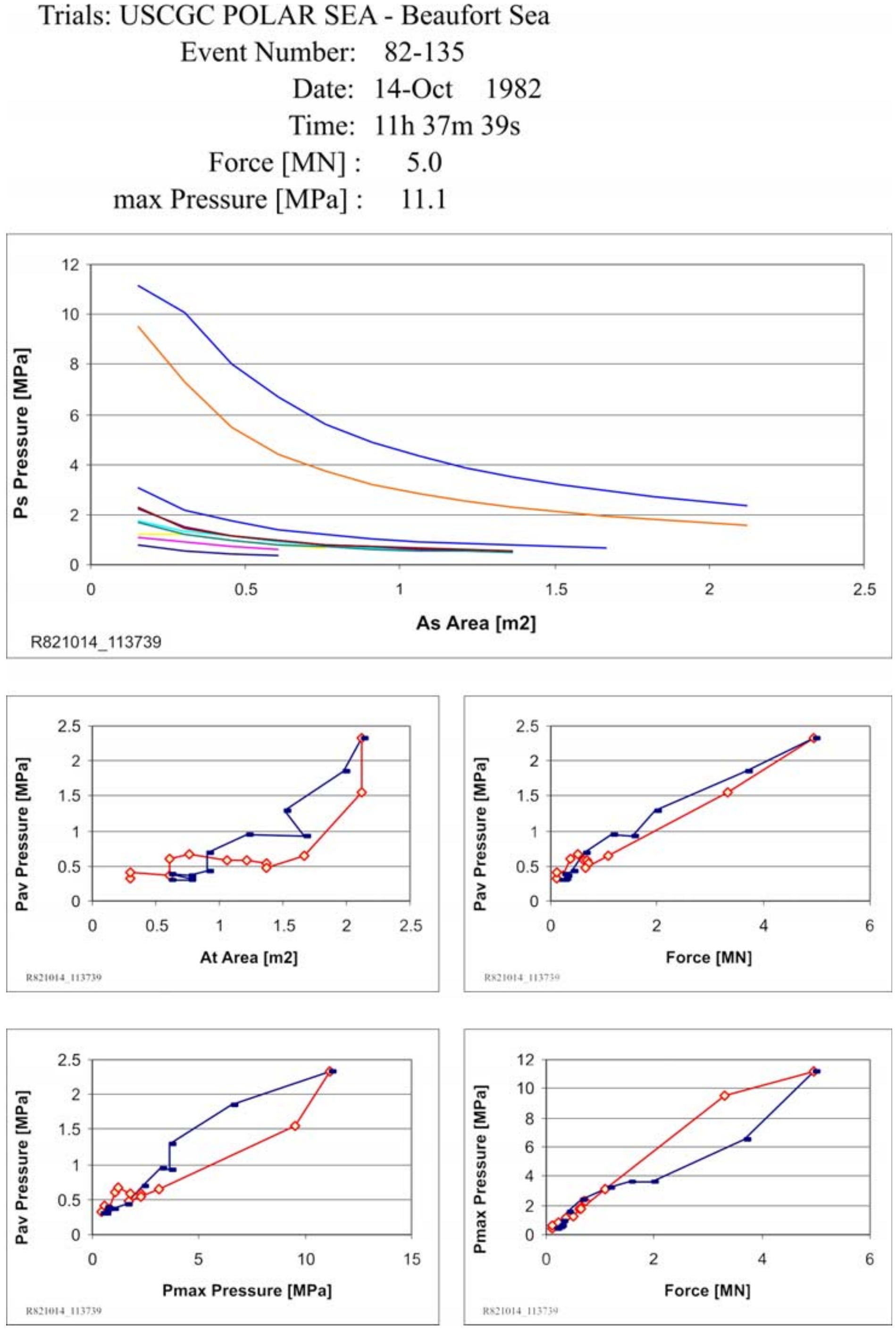


Figure 11. Re-analyzed data from event #135, from 1982. This was the highest force event in the '82 trials.

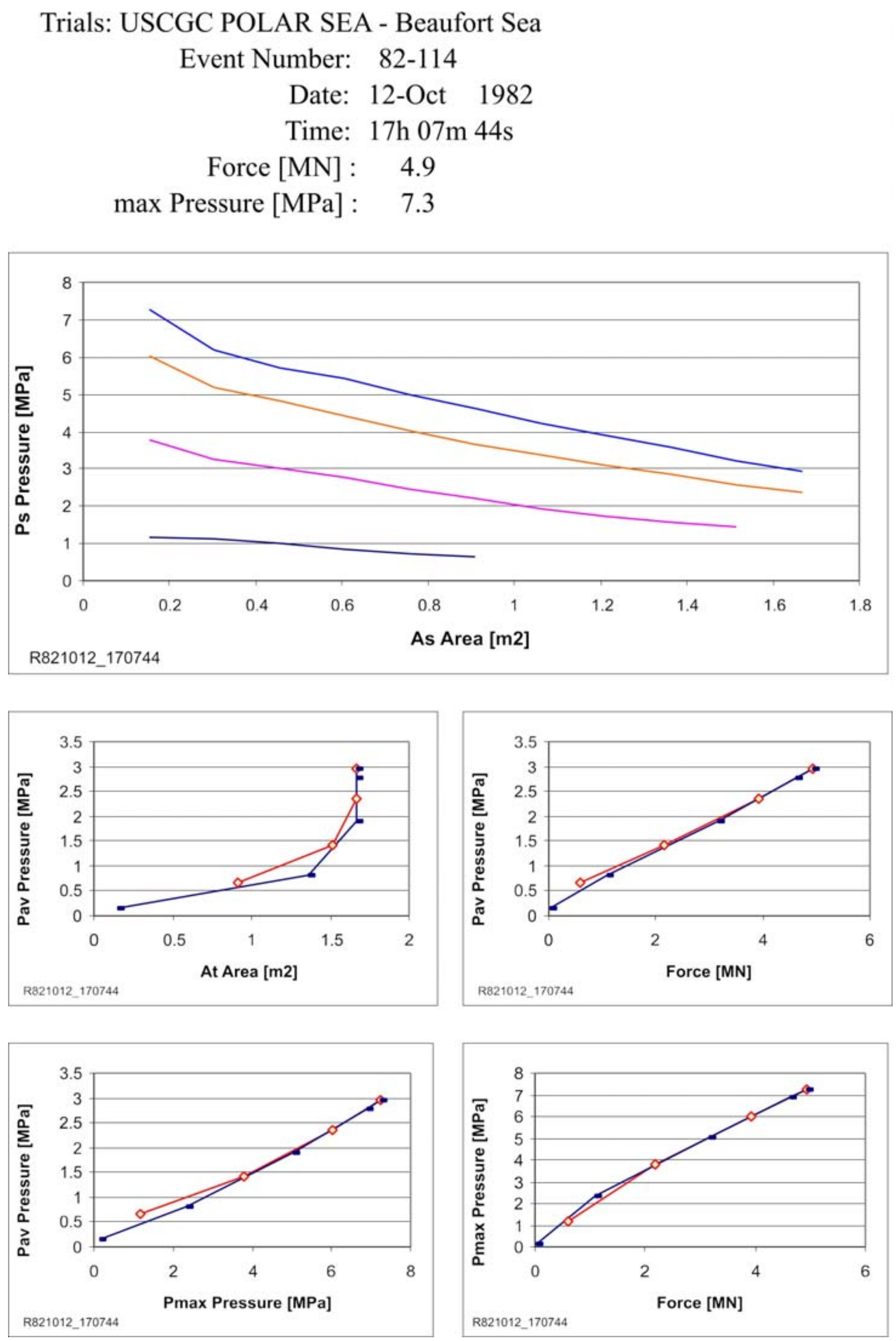


Figure 12. Re-analyzed data from event #114, from 1982. This was the 2nd highest force event in the '82 trials.

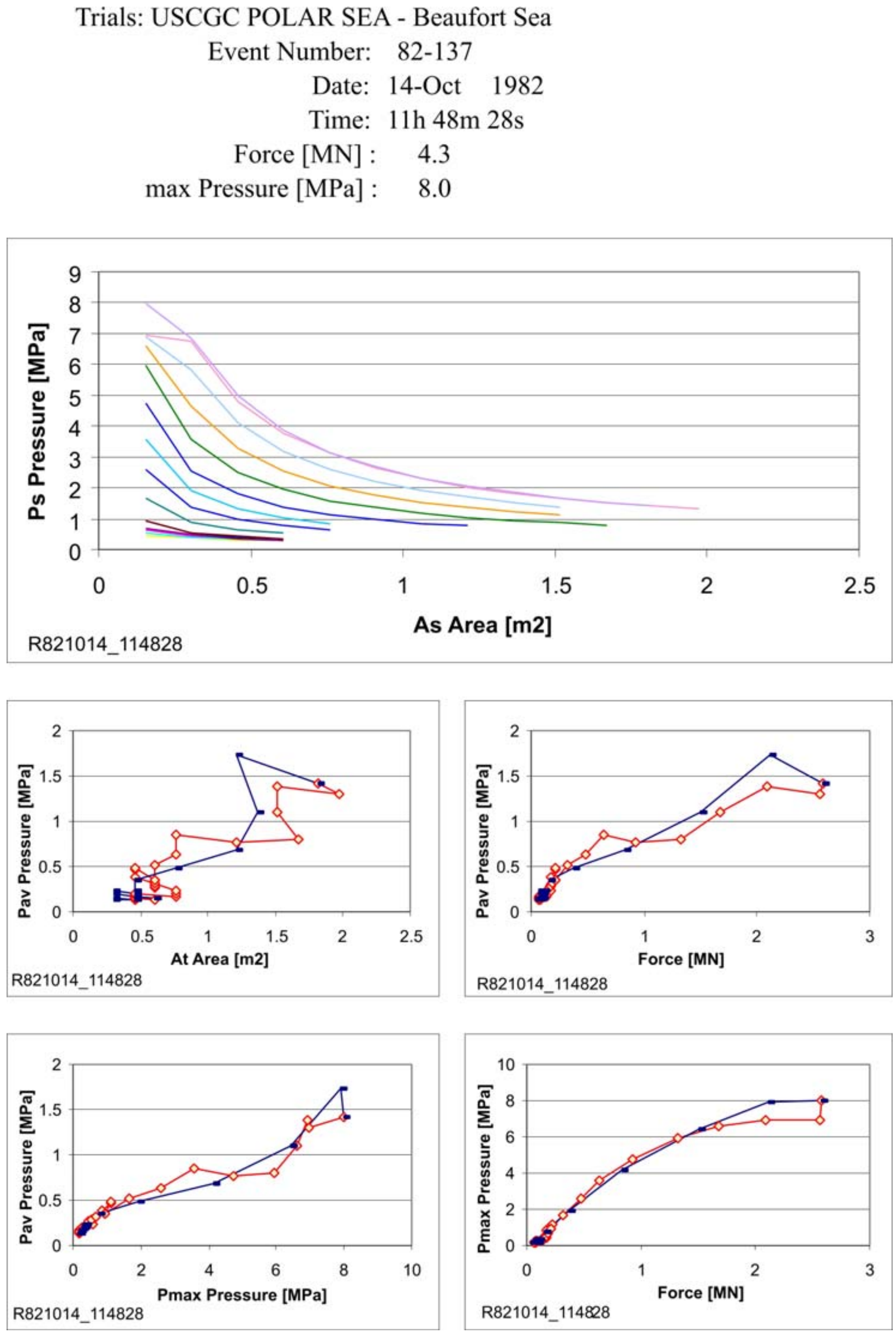


Figure 13. Re-analyzed data from event #137, from 1982. This was the 3rd highest force event in the '82 trials.

3.3.2 Data from the 1983 Trials

Trials: USCGC POLAR SEA - Beaufort Sea
Event Number: 83-410
Date: 24-Apr 1983
Time: 16h 11m 59s
Force [MN] : 4.9
max Pressure [MPa] : 7.9

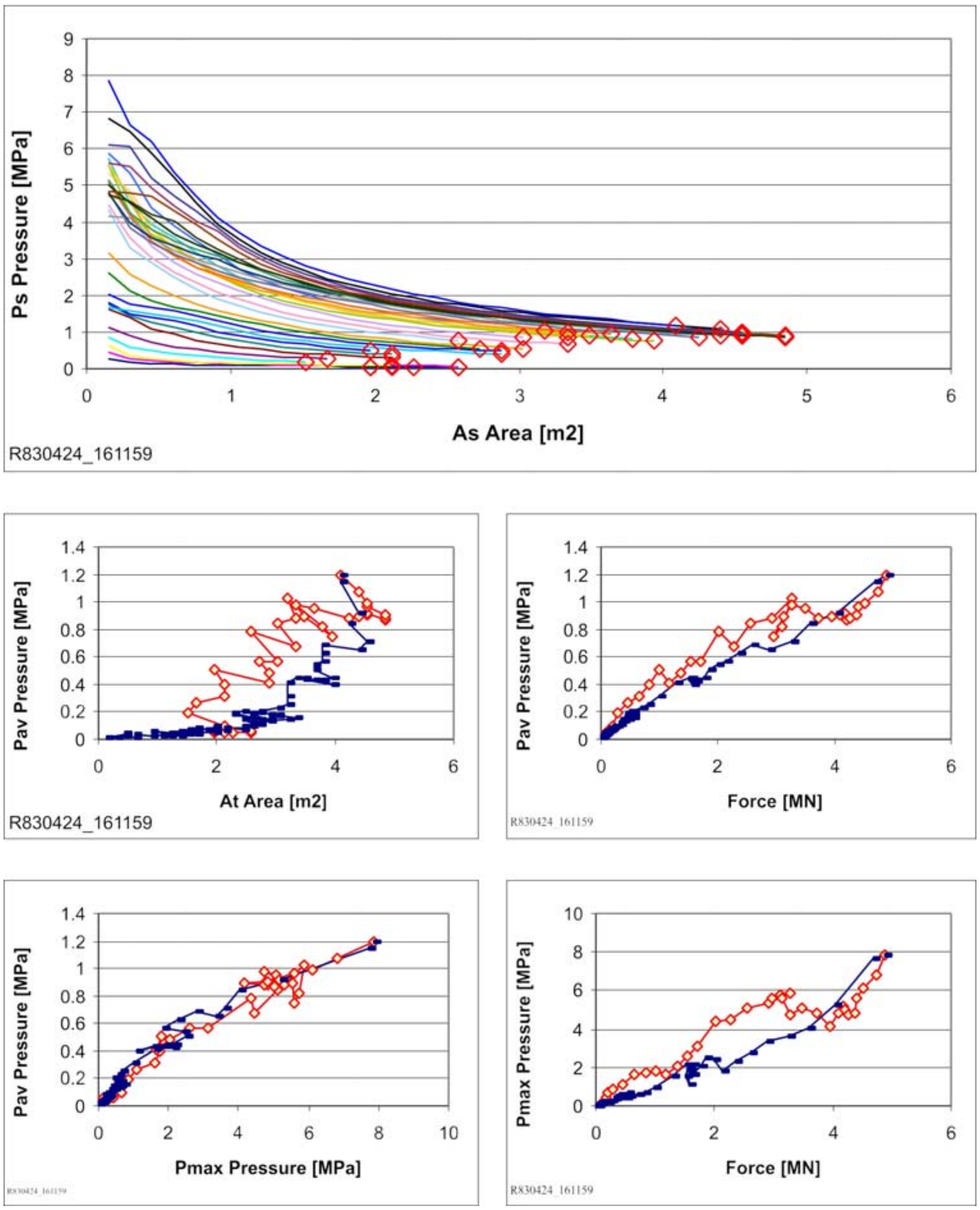


Figure 14. Re-analyzed data from event #410, from 1983. This was the highest force event in the ’83 trials.

Trials: USCGC POLAR SEA - Chuckchi Sea
Event Number: 83-366
Date: 20-Apr 1983
Time: 13h 06m 18s
Force [MN] : 4.4
max Pressure [MPa] : 9.1

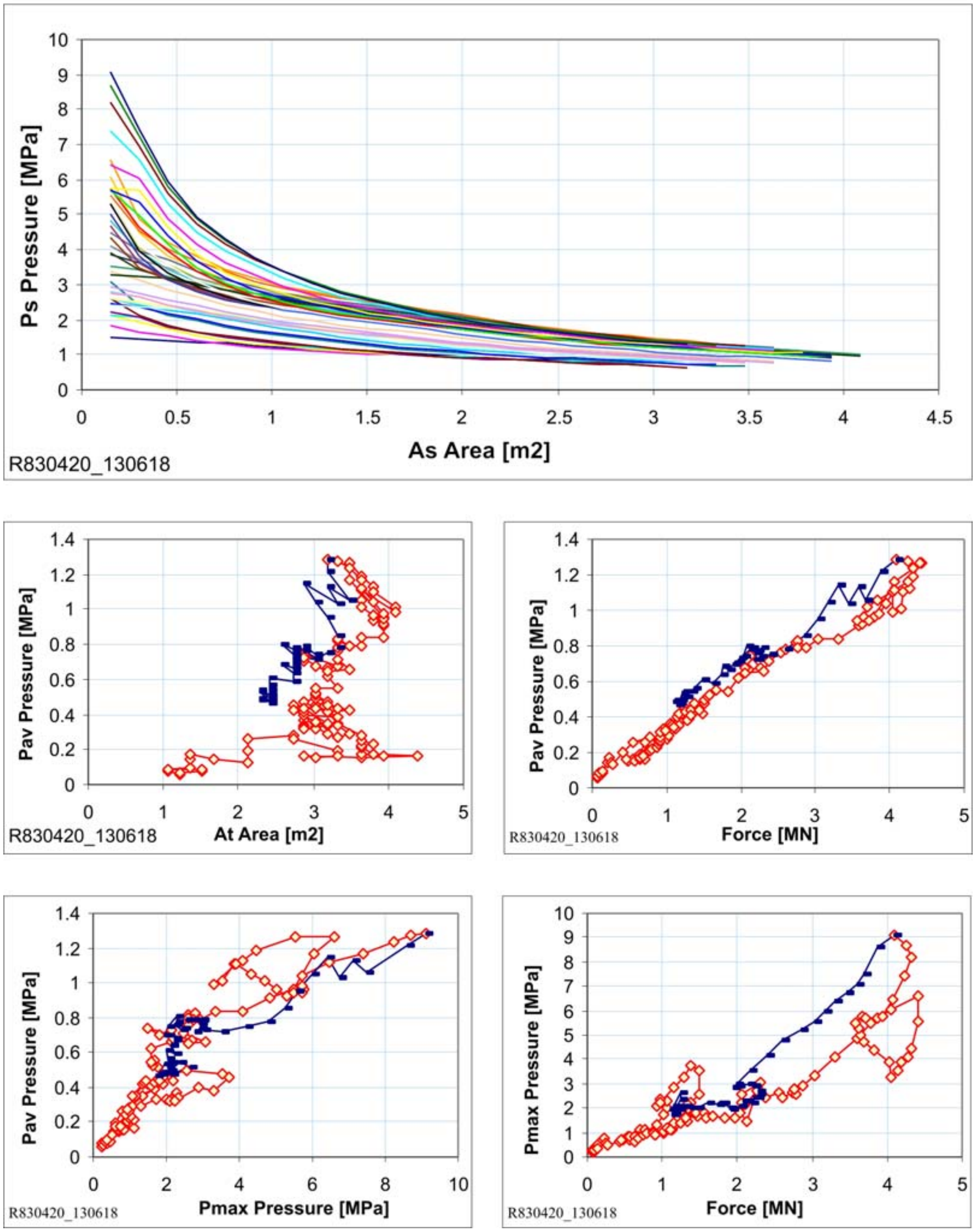


Figure 15. Re-analyzed data from event #366, from 1983. This was the 2nd highest force event in the ’83 trials.

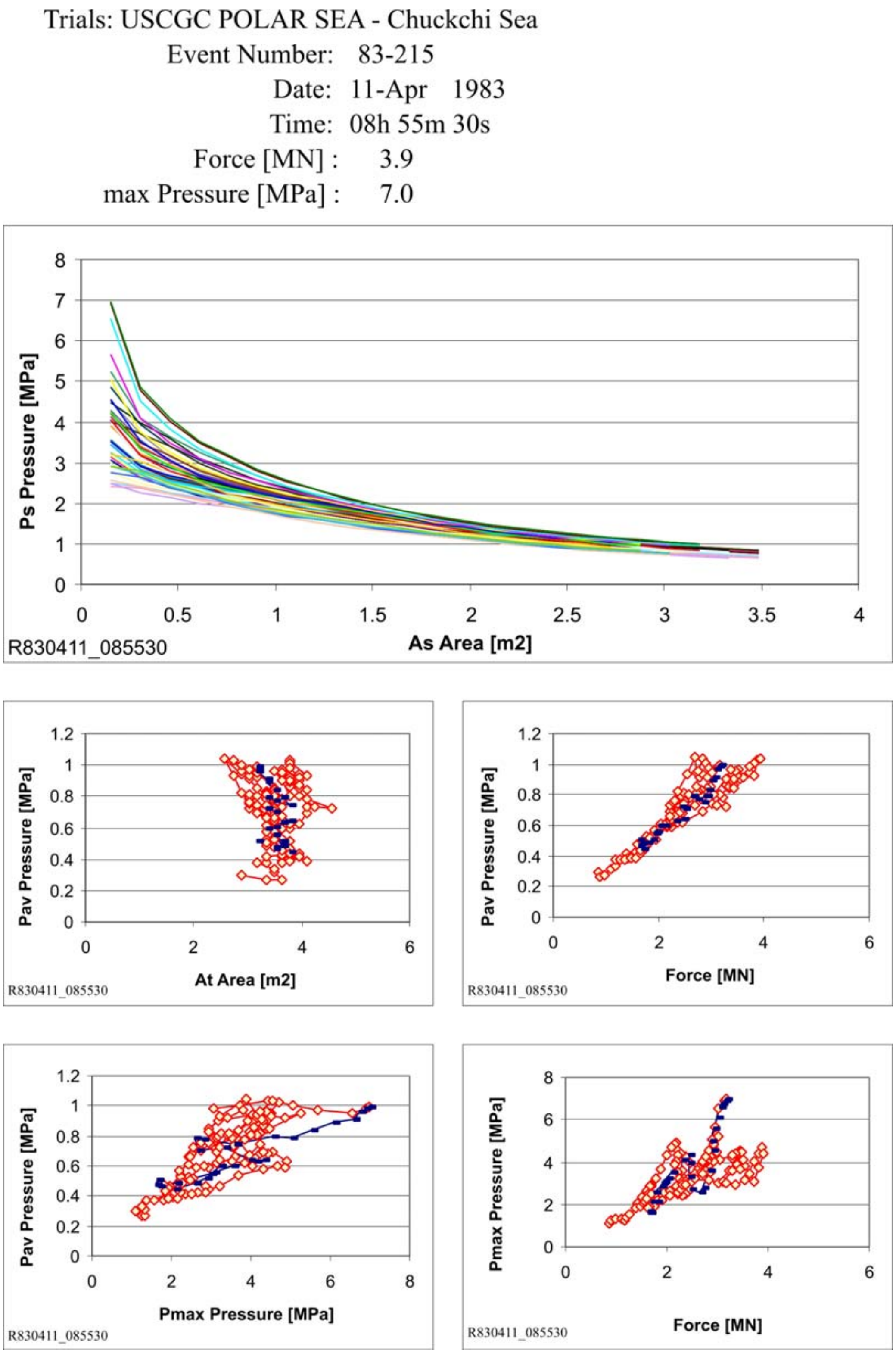


Figure 16. Re-analyzed data from event #215, from 1983. This was the 4th highest force event in the '83 trials.

3.3.3 Summary of ’82 Plots

The plots in Figure 17 show the relationships among average pressure, total area, force and peak pressure for the five largest events during the 1982 trials. All events show similar trends, the most important being that average pressure rises as area and force increase. Similarly, the peak pressure rises as well.

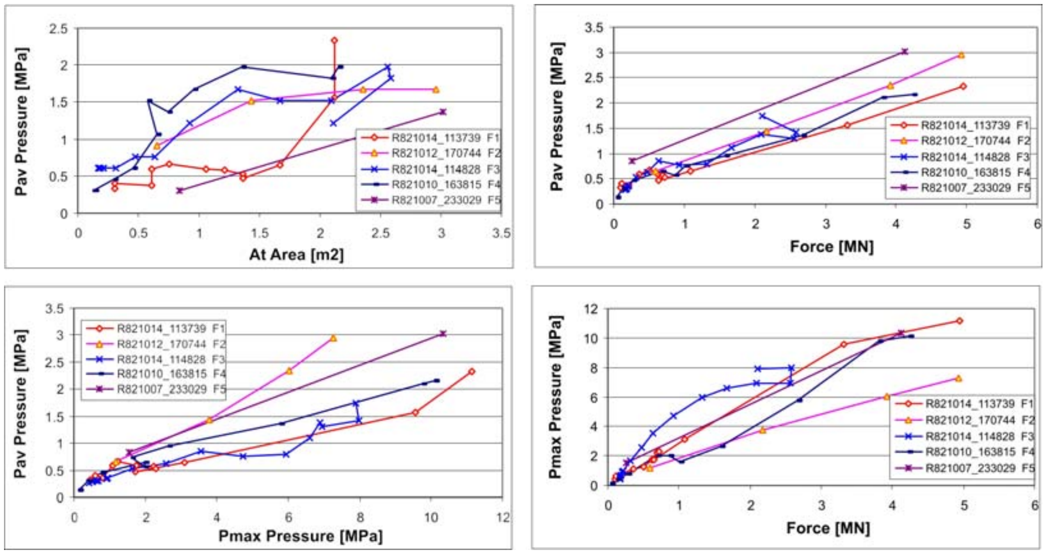


Figure 17. Compilation of pressure trend plots for 1982 Polar Sea data.

The plots in Figure 18 show the relationships among average pressure, total area, force and peak pressure for the five largest events during the 1983 trials. As before, all events show similar trends.

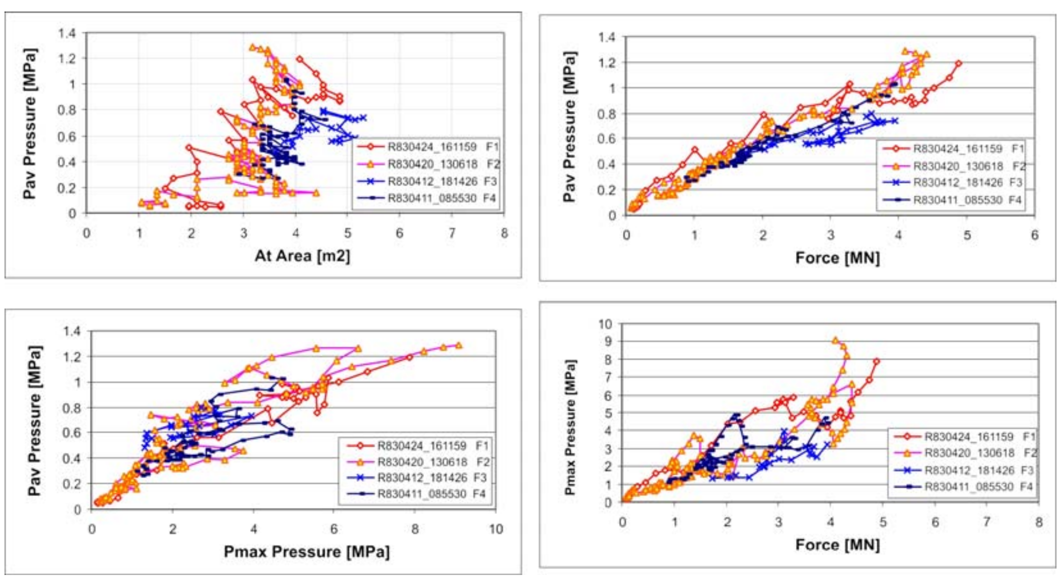


Figure 18. Compilation of pressure trend plots for 1983 Polar Sea data.

3.3.4 Summary of ’82 and ’83.

Figure 19 compares the 1982 and 1983 data sets. While following similar trends the two data sets do not match. The 1982 data shows higher average pressure. One can only speculate as to the cause of the difference. One obvious cause would be that the two data sets were from different times of the year (Oct. in 1982 and April in 1983). The only odd aspect of this is that the weather was warmer in October and so, presumably, was the ice. Could it be that warmer ice caused higher average pressures? This idea runs counter to the usual trend with ice strength. However, given that the colder ice may have been more brittle (and ‘dry’ as crushed ice was extruded), it may make sense that the warmer ice produced higher average pressures. This is a very curious result and deserves further attention.

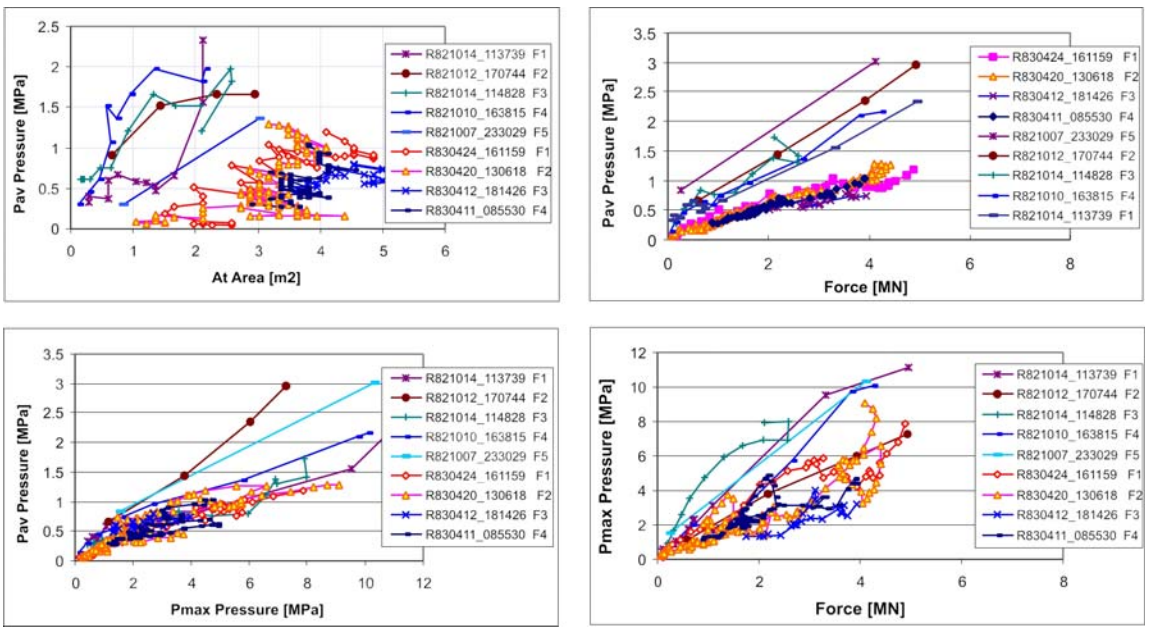


Figure 19 Comparison of pressure trend plots for 1982 and 1983 Polar Sea data.

3.4 Discussion of Polar Sea Data.

The Polar Sea data has shown some interesting and potentially important trends. When the data from experiments is all plotted together, the plot appears to show an inverse relationship between pressure and area. This is because the plot is primarily showing the spatial pressure area relationship. Further, when sets of data are grouped and plotted, an inverse pressure-area relationship is also often seen in the upper envelope of the data. Unfortunately, such a trend is only a reflection of the limited level of force in the various data sets. Pressure and area can never be truly independent variables, because pressure is really force per unit area. A line of constant force will appear as an inverse relationship between pressure and area.

Most of the empirical evidence we have for pressure-area trends comes from impact tests of quite limited force. Either the ram has limited force capacity, or the vessel has limited energy/momentum. Consequently, any assemblage of events from a given set of similar experiments will almost certainly be constrained by a level of force. When plotted, such data will necessarily show an envelope with an inverse relationship between pressure and area. Such a trend has absolutely no meaning when one is attempting to extrapolate to situations involving larger loads. In effect, our limited experience with large forces is falsely showing us that pressures (and forces) will stay small. Figure 20 [10] shows a plot of most available pressure area data. The cloud to the left of the plot tends to be from the kind of low aspect ratio impacts that are of interest here. An envelope line, often assumed to be highly conservative, is shown.

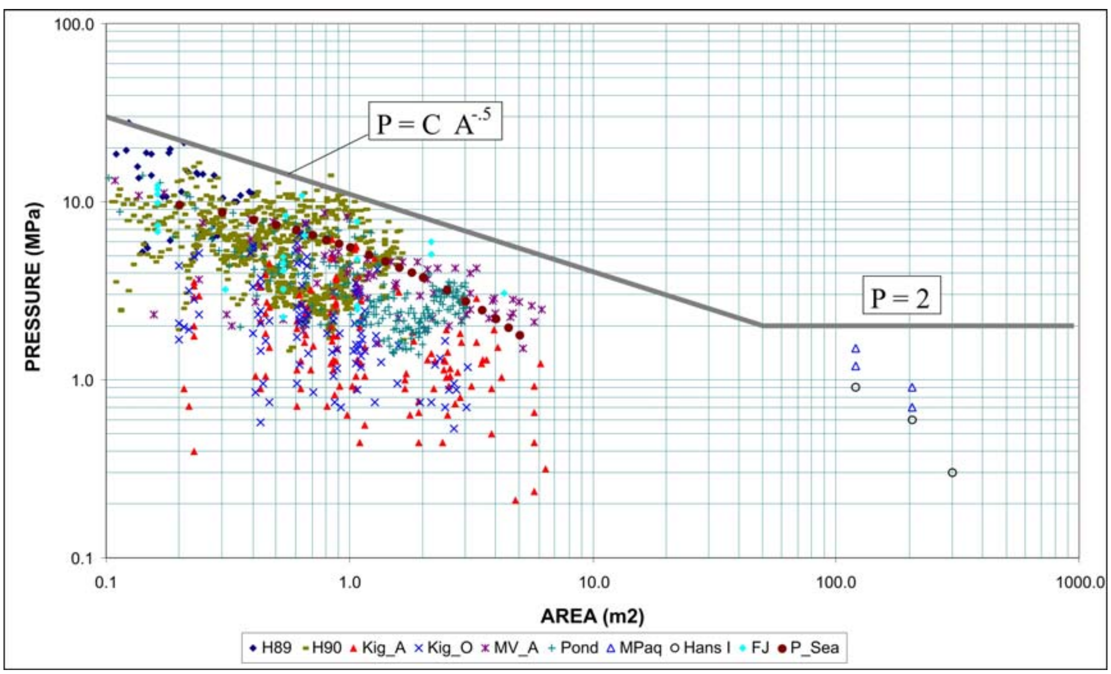


Figure 20. Assemblage of measured pressure area data.

The analyses presented above have attempted to determine the true pressure-area relationships as they occur when ice and structure are in contact. When the penetration phase of the collision is isolated for single impacts, we see a surprising trend. The average pressure rises as the force and contact area rise during the collision. This is to say that the process pressure-area curves rise, not fall. This is a very significant result. Figure 21 shows how some of the process pressure area curves from the Polar Sea compare to the general data. From this it would appear that ice pressure in high force collisions could be well above anything observed to date.

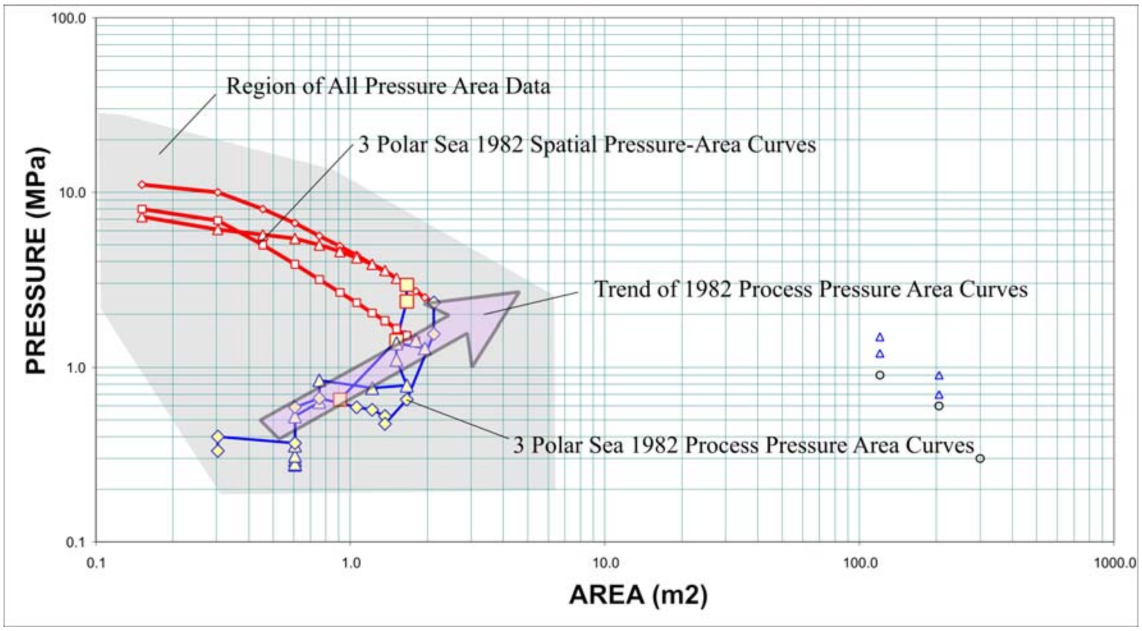


Figure 21. Assemblage of Pressure-area data with example spatial and process pressure-area curves from the 1982 Polar Sea trials.

By way of illustration, take the case of an ice mass of 80 kT striking a vertical wall at 2 m/s. Assuming the edge radius of 15 m, the force is readily calculated from energy considerations [5]. The calculation requires that the process pressure-area equation is known. Figure 22 shows the dramatic changes in estimated ice collision force that occur as e changes from –1 to 0 to +1. The constant C has a relatively minor effect by comparison. One might imply a value +0.5 for e, from the Polar Sea data. For the case shown below, this would result in a predicted force as almost 100 times greater that that obtained for a value of e=-0.7. Such a potential error is astounding. Of course, such an event might well be limited by another failure mechanism. Nevertheless, this possibility deserves further analysis.

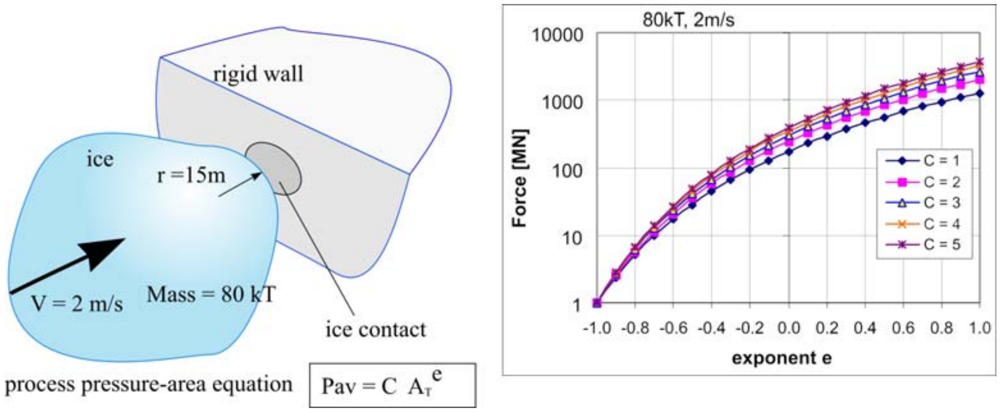


Figure 22. Influence of the e and C terms in the process pressure-area relationship.

4 Numerical Contact Model

This chapter will examine the process and spatial pressure-area curves from the perspective of a numerical model of ice edge contact. The model is a 2D model of contact, and this is a significant issue. The model was developed to help understand and explain certain laboratory experiments that were essentially 2D. The model may help to explore the issues, but can not be expected to model the general 3D impacts that occurred when the Polar Sea struck multi-year floes.

4.1 Model Development

The first version of the model used here was developed in [19] with only direct contact and flaking aspects considered. That model was well able to simulate and explain many observed phenomena in a set of lab experiments [20] involving crushing contact on large blocks of sea ice. Figure 23 shows the model as first developed. The model was able to explain the following key observations/phenomena in the Joensuu-Riska tests: multi-sawtooth force time-history, crushed particle piece size distribution, pressure-area (process) time history, local (spatial) pressures, observed direct contact geometry (line), and the variations in force records between successive experiments (i.e. the ‘randomness’, that was not actually random). This numerical simulation was able to explain the observed pressure-area curves without resorting to an assumption of random strength properties.

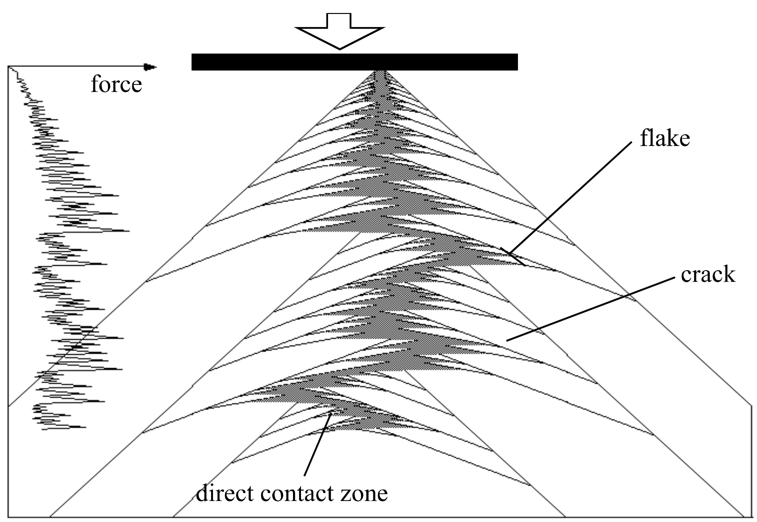


Figure 23. Contact Failure Process Model [19]. The contact process is modeled by a discrete sequence of through-body cracks.

A refinement of the above model was developed in [2] (see Annex E for listing), that added the consideration of an active extrusion rubble zone above and below the central contact point. In the refined model, the contact face is vertical, so that the ice is in the plane of the water. With that, the crushed ice forms two piles, one on top of the ice (in air) and one below the ice (in water). For the two piles, above and below the water, either gravity or buoyancy holds the piles in the contact zone and provides an initial pressure to constrain the extrusion. The extrusion zones (between the contact face and the ice) are a series of opening wedges, each with an outlet pressure and an inlet pressure. A simple model of extrusion of a granular material has been used to describe the pressures in the extrusion zones.

The most important result of the inclusion of the extruded material is not the loads carried directly by the crushed ice, but rather that influence of the confining effects of the crushed ice on the failure of the intact ice. The intact ice fails by shearing. Pressure in the extrusion zone has the effect of compressing the shear failures, strengthening them. This results in greater force transmitted through the direct contact, as well as force through the crushed ice.

The intent in this report is to explore the pressure-area effects with the model.

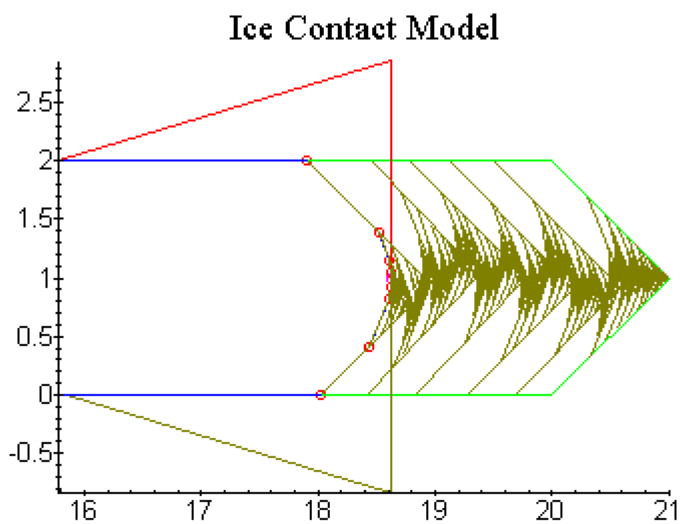


Figure 24. Contact/Extrusion model from [2]

4.2 Simulation Results

The model was exercised for the four conditions. Table 2 shows the key results for the relatively thin ice sheet (2m). Table 3 shows the key results for the relatively thick (20m) ice. Both shapes were run with and without consideration of extrusion. Extrusion can be eliminated by setting the various friction factors (ice-ice and ice-structure) to zero. The thick ice was included to examine the case where large 45 degree flakes do no take place. As such the ice thickness does not actually matter.

The other model parameters are as shown in Table 1. These were not chosen for their accuracy, but only to explore the sensitivities.

There are two very notable results. One is that the inclusion of extrusion effects can dramatically increase the average process pressures. The second is that the shape of the ice edge has an influence on the slope of the pressure-area curve, when extrusion is modeled. Note the pressure-area curve for run 4. This curve has peak values that form a near horizontal trend.

Table 1. Contact model constants

Item	Description [units]	Value(s)
Pc	direct contact pressure [MPa]	20
c	ice shear strength (Coulomb model) [MPa]	0.8
ϕ_1	ice friction angle (Coulomb - solid ice)	0º or 10º
ϕ_2	solid ice - structure contact friction angle	0º or 10º
ϕ_3	granular ice - structure friction angle	0º or 6º
ϕ_4	granular ice - ice contact friction angle	0º or 10º
sl_sail	slope of the extruded material above ice	0.8
sl_keel	slope of the extruded material below ice	0.8
rho	rubble mass density [kg/m^3]	560

Table 2. Contact/Extrusion model runs for 2m sheet.

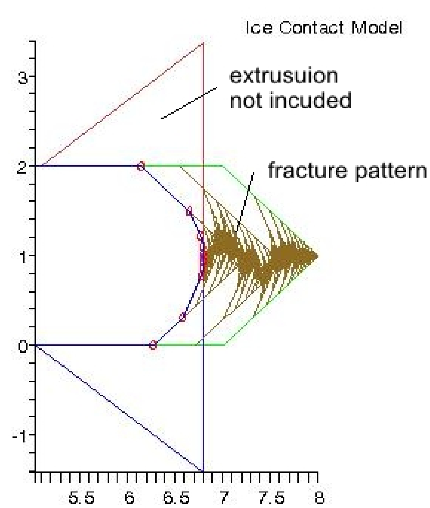
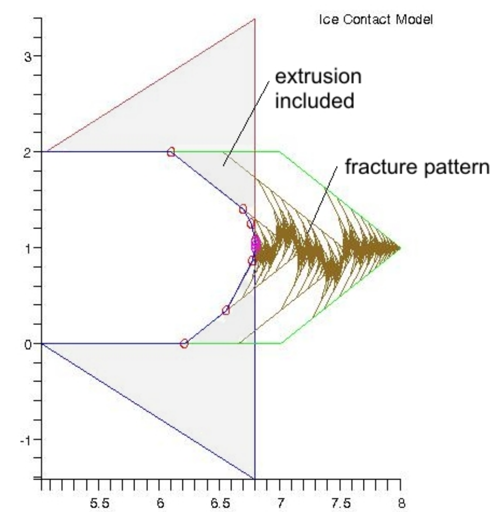
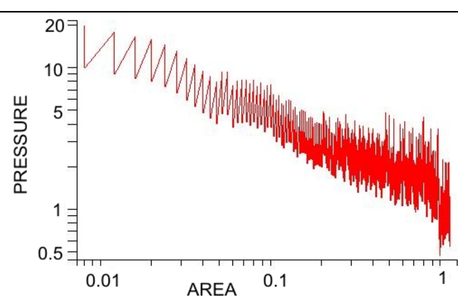
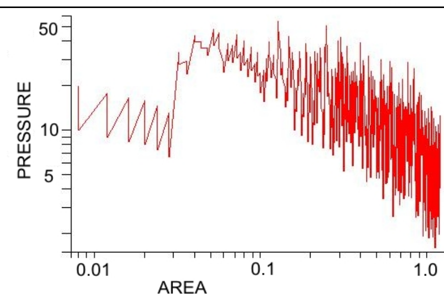
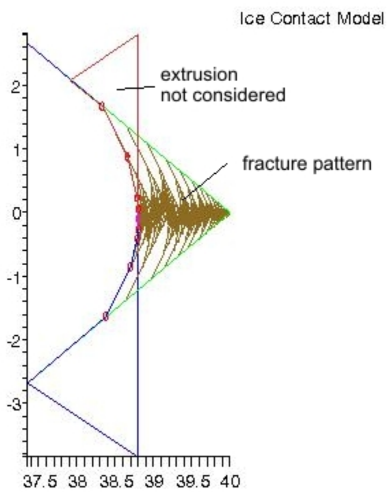
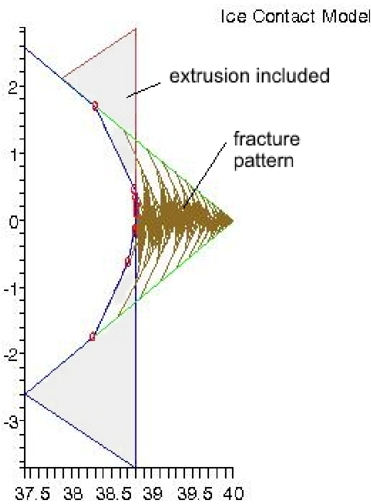
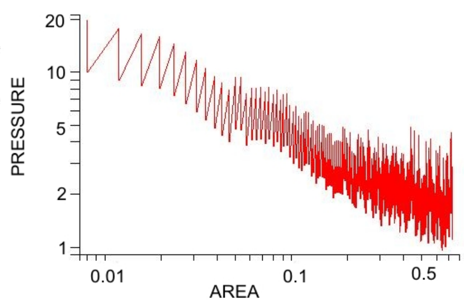
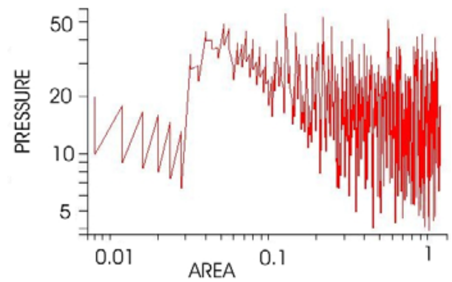
	Run 1	Run 2
Geometry	2m sheet with 45º edge	2m sheet with 45º edge
Extrusion included in calculations ?	no	yes
Fracture pattern		
Resulting Process Pressure-Area Plot		

Table 3. Contact/Extrusion model runs for 20m sheet.

	Run 3	Run 4
Geometry	20m sheet with 45º edge	20m sheet with 45º edge
Extrusion included in calculations ?	no	yes
Fracture pattern		
Resulting Process Pressure-Area Plot		

4.3 Discussion

The contact model has shown some interesting results. The influence of the extrusion mechanics is significant. It is clear that the slope of the process pressure-area curve is dependant on both extrusion processes and ice edge shape. This, at least in a small way, helps to explain the Polar Sea results.

Unfortunately, there are still many problems with the software that make it difficult to examine a variety of extrusion parameters and ice edge shapes. For wide variations in edge shape from those shown, the program does not run. The same is true for general changes in the ice parameters. Only with further development will this model be able to handle an arbitrary 2D ice edge. One might also expect that only with a full 3D model will the effects such as observed in the Polar Sea data be replicated and fully understood.

5 Conclusions

The re-analysis of past measurements has shown some surprising results.

The pressure-area relationship in ice should not be viewed as a single phenomenon. Instead it should be separated into spatial and process pressure-area relationships. These are not equivalent to local and global pressure-area trends, but are rather spatial and temporal trends of pressure. The spatial p/a curve is needed for local structural design, while the process p/a curve is used to determine impact forces. It has been indicated here that these two effects are strongly linked, but are essentially different.

The evidence suggests that the process p/a curve follows a rising trend in certain cases. This is in some ways opposite of the spatial p/a curve and the usual understanding of this data. This suggests that we may be drastically underestimating ice forces in the case of large collisions, and consequently underestimating the local maximum pressures.

We have little of no empirical evidence for the large force impacts for which some of our offshore structures are designed. This must be of significant concern.

6 Recommendations

1. Further analysis and re-analysis should be performed on any available pressure-area data, specifically to look for trends in the process pressure-area relationship.
2. Other ice-mechanics based models should be used or developed to help understand what parameters influence the process pressure-area curve, and its relationship to the spatial curve.
3. Large field programs are needed to provide at least some empirical evidence from large force ice impacts. These should be conducted with the highest quality data collection systems possible, to gain a better understanding of the true nature of the contact process. A high resolution array of surface pressure sensors is needed, along with ice shape, and relative position throughout the impact. Independent measurement of the ice load is highly desirable.

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Annex A – Polar Sea 1982 Event Summary

The table below is an extract from the summary report of the 1882 Beaufort Sea trials. Listed below are the peak values of force and panel pressure for the largest events ranked by force. (see accompanying CD data disk for the full summary)

Event Number	Date	Time	Maximum Single Sub-panel Pressure (psi)	Time of Peak Pressure	Peak Pressure Location Frame	Row	Total Panel Force (LT)
135	10/14/1982	11:37:39	1617	33	42	5	495
114	10/12/1982	17:07:44	1053	34	39	7	489
137	10/14/1982	11:48:28	1156	37	35	5	434
91	10/10/1982	16:38:15	1464	34	44	7	433
81	10/7/1982	23:30:29	1499	32	44	5	413
122	10/13/1982	19:17:49	597	35	43	7	388
84	10/10/1982	15:44:41	980	40	43	5	384
116	10/12/1982	18:58:17	1015	33	36	3	357
37	10/2/1982	20:43:21	656	36	35	5	319
131	10/14/1982	8:31:00	638	89	36	6	317
103	10/8/1982	9:54:07	877	33	36	3	306
73	10/8/1982	7:43:40	763	33	43	5	290
61	10/3/1982	20:47:00	551	43	38	4	287
139	10/14/1982	12:46:54	782	38	37	5	286
7	10/1/1982	12:21:26	1453	22	37	3	275
78	10/7/1982	19:42:23	794	33	43	4	270
89	10/10/1982	16:36:14	1010	34	42	5	254
110	10/12/1982	16:24:33	889	63	38	7	252
83	10/10/1982	15:23:02	901	37	35	8	251
72	10/8/1982	7:31:42	817	37	42	7	247
63	10/3/1982	20:48:16	702	42	42	5	235
20	10/1/1982	21:23:29	392	83	41	3	234
10	10/1/1982	12:23:07	663	137	36	3	232
127	10/13/1982	20:14:41	1115	35	44	3	232
97	10/10/1982	18:20:07	1093	35	38	4	226
9	10/1/1982	12:22:39	472	33	41	4	225
130	10/14/1982	8:17:38	481	32	37	8	217
32	10/2/1982	20:10:03	1013	44	42	8	214
133	10/14/1982	11:30:12	1206	33	36	8	214
80	10/7/1982	23:02:29	808	65	37	8	211
120	10/12/1982	20:05:43	584	33	41	5	211
100	10/10/1982	18:43:51	569	35	44	8	210
115	10/12/1982	17:16:31	573	40	40	7	207
158	10/7/1982	18:26:30	923	33	42	3	204
35	10/2/1982	20:30:09	439	40	35	4	203
126	10/13/1982	20:10:27	642	34	42	7	203
166	10/11/1982	0:19:37	544	47	42	7	203
152	10/7/1982	17:59:31	920	146	37	7	199
6	10/1/1982	12:20:58	951	196	39	3	195
99	10/10/1982	18:41:16	1109	32	37	4	193
74	10/7/1982	18:48:11	1030	121	35	8	187

Annex B – Polar Sea 1983 Event Summary

The table below is an extract from the summary report of the 1983 Chukchi Sea trials. Listed below are the peak values of force and panel pressure for the largest events ranked by force. (see accompanying CD data disk for the full summary)

Event Number	Date	Time	Ship Speed (kt)	Maximum Single Sub-panel Pressure (psi)	Time Step of Peak Pressure	Frame Number of Max Pressure Time of Peak Pres	Row Number of Max Pressure Time of Peak Pres	Total Panel Force (LT)
410	4/24/1983	16:11:59	7.79675	1141	56	42	8	491
366	4/20/1983	13:06:18	3.19728	1319	113	44	8	443
283	4/12/1983	18:14:26	7.00652	576	139	38	5	435
215	4/11/1983	8:55:30	5.57339	1011	132	39	8	393
70	4/3/1983	2:56:29	0	264	33	37	7	387
86	4/3/1983	3:07:55	0	303	33	40	5	383
276	4/12/1983	17:47:51	6.45813	680	35	43	4	377
72	4/3/1983	2:59:31	0	280	33	36	7	374
74	4/3/1983	3:00:02	0	312	35	39	8	371
85	4/3/1983	3:06:15	0	270	33	36	7	368
77	4/3/1983	3:01:07	0	295	33	42	6	362
82	4/3/1983	3:04:12	0	303	33	42	6	357
67	4/3/1983	2:52:04	0	349	34	36	7	355
87	4/3/1983	3:09:16	0	297	33	43	7	355
61	4/3/1983	2:39:27	0	322	33	36	7	353
280	4/12/1983	18:11:09	4.5471	541	65	40	5	351
135	4/8/1983	17:22:32	-0.26371	1235	101	39	3	347
83	4/3/1983	3:04:20	0	291	33	42	8	344
63	4/3/1983	2:43:54	0	267	34	36	7	341
90	4/3/1983	3:24:05	0	313	34	38	8	341
80	4/3/1983	3:03:29	0	275	33	36	8	339
69	4/3/1983	2:53:07	0	370	33	35	7	337
73	4/3/1983	2:59:50	0	321	33	36	7	336
75	4/3/1983	3:00:34	0	272	33	37	7	336
279	4/12/1983	18:10:47	6.24968	443	35	40	5	335
91	4/3/1983	3:27:02	0	268	33	38	8	334
92	4/3/1983	3:28:16	0	288	33	44	6	334
64	4/3/1983	2:49:08	0	338	33	35	5	333
65	4/3/1983	2:50:33	0	391	34	42	6	330
71	4/3/1983	2:59:23	0	278	33	39	8	330
78	4/3/1983	3:01:17	0	300	33	39	8	330
350	4/19/1983	13:05:56	4.87599	741	38	37	4	329
286	4/12/1983	18:15:08	3.39335	393	41	40	5	328
197	4/9/1983	10:42:54	4.98072	749	39	43	4	325
76	4/3/1983	3:00:41	0	276	34	38	8	324
282	4/12/1983	18:11:49	2.19214	432	42	42	6	324
81	4/3/1983	3:03:58	0	365	33	38	5	323
66	4/3/1983	2:51:53	0	417	90	44	6	321
116	4/8/1983	7:43:39	6.3199	516	37	36	7	321
68	4/3/1983	2:52:12	0	284	33	37	7	320
79	4/3/1983	3:03:21	0	294	31	38	5	320

Annex C – Example of Polar Sea Event Data Files

Shown below is an extract from a single event file. The full event file has 60 columns of data (for the 60 sub-panels, and 200 roes of data (for the 200 time steps. The file name contains the date and time that the recording started. For example, the one shown below occurred on October 14, 1982, at 11hours, 37min and 39 seconds (ship time). The header also contains a simple description of the ice conditions and the vessel operations. The integer data represents pressure in psi. The part of the record shown is all essentially zero, and occurred prior to the higher pressures later in the event. For this event, the highest pressure was 1617 psi (11.15 Mpa), and occurred approximately one second after the start of the record. The negative values are a result of small errors in the data reduction algorithm.

Event: R821014_113739 BACKING & RAMMING INTO M.Y. @ 3-4 KT

WL 3-FR 1	WL 3-FR 2	WL 3-FR 3	WL 3-FR 4	WL 3-FR 5	WL 3-FR 6	WL 3-FR 7	WL 3-FR 8	WL 3-FR 9	WL 3-FR 10	WL 4-FR 1	WL 4-FR 2	WL 4-FR 3	WL 4-FR 4	WL 4-FR 5	WL 4-FR 6	WL 4-FR 7	WL 4-FR 8	WL 4-FR 9	WL 4-FR 10	WL 5-FR 1	WL 5-FR 2	WL 5-FR 3	WL 5-FR 4	WL 5-FR 5	WL 5-FR 6	WL 5-FR 7	WL 5-FR 8	WL 5-FR 9	WL 5-FR 10	WL 6-FR 1	WL 6-FR 2	WL 6-FR 3	WL 6-FR 4	WL 6-FR 5	WL 6-FR 6
-4	4	7	-2	6	2	-8	5	0	2	0	3	-1	3	4	1	1	3	-2	1	1	2	-2	4	-4	2	5	1	1	3	-5	-1	-2	1	2	-1
-4	4	7	-2	6	2	-8	5	0	2	0	3	-1	3	4	1	3	2	-2	1	1	2	-2	4	-4	2	5	1	0	3	-5	-1	-2	1	4	-2
-4	4	7	-2	6	2	-8	5	0	2	0	3	-1	3	4	1	1	3	-2	1	1	2	-2	4	-5	2	4	1	-2	4	-5	-1	-2	1	2	-1
-4	4	7	-2	6	2	-8	5	0	2	0	3	-1	3	4	1	1	3	-2	1	1	2	-2	4	-5	2	5	1	-2	4	-5	-1	-3	1	1	-2
-4	4	7	-2	6	2	-8	5	0	2	0	3	-1	3	4	-1	1	3	-2	1	1	2	-2	4	-5	2	5	1	-2	4	-5	-1	-3	1	1	-3
-4	4	7	-2	6	2	-8	5	0	2	0	3	-1	3	4	-1	1	3	-2	1	1	2	-2	4	-4	2	5	1	1	4	-5	-1	-2	0	1	-2
-4	4	5	-2	6	2	-8	5	1	2	0	3	-1	3	4	1	1	3	-2	1	1	2	-2	4	-4	2	5	1	1	4	-5	-1	-2	0	1	-2
-4	4	7	-2	6	2	-8	5	0	2	0	3	-1	3	4	1	1	3	-2	1	1	2	-2	4	-5	2	4	1	-1	4	-5	-1	-2	0	-1	-2
-4	4	7	-2	6	2	-8	5	0	2	0	3	-1	3	4	1	1	3	-2	1	1	2	-2	4	-4	2	5	1	1	4	-5	-1	-2	1	3	-3
-4	4	7	-2	6	2	-8	5	3	1	0	3	-1	3	3	1	0	5	-2	1	1	2	-2	4	-5	2	5	1	2	1	-5	-1	-3	2	0	0
-4	4	7	-2	6	2	-8	5	0	2	0	3	-1	3	4	1	2	5	-2	1	1	2	0	3	-4	2	7	0	2	-1	-5	-1	-3	2	-1	0
-4	4	7	-2	6	2	-8	5	0	2	0	3	-1	3	4	1	0	5	1	0	1	2	-2	4	-5	2	7	0	2	-3	-5	-1	-2	1	4	-1
-4	4	7	-2	6	2	-8	5	0	2	0	3	-1	3	4	1	0	5	1	0	1	2	-2	4	-5	2	7	0	3	-5	-5	-1	-3	1	2	-2
-4	4	7	-2	6	2	-8	2	1	2	0	3	-1	4	4	1	1	3	-2	1	1	2	-2	5	-5	2	5	0	4	-5	-5	-1	-3	2	-2	-1
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-4	4	7	-2	6	2	-8	2	1	2	0	3	-1	4	4	1	1	3	-2	1	1	2	-2	5	-5	2	4	0	2	-7	-5	-2	-2	0	-2	-1
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-4	4	7	-2	6	2	-8	2	1	2	0	3	-1	3	2	-1	1	3	-2	1	1	2	-2	5	-5	2	4	0	2	-7	-5	-2	-2	-1	1	-5
-4	4	7	-2	6	2	-8	2	1	2	0	3	-1	3	4	-1	1	3	-2	1	1	2	-2	5	-5	2	4	1	1	-5	-5	-1	-2	-1	3	-3
-4	4	7	-2	6	2	-8	5	0	2	0	3	-1	3	4	1	1	3	1	0	1	2	-2	4	-5	2	5	0	2	-1	-5	-1	-2	1	2	-3

Annex D – Example of Polar Sea Spatial & Process Pressure-Area Event Files

The table below shows part of the Spatial and Process Pressure-Area Event Files that have been created in this project. For each time step the pressure data was ranked and averaged to produce spatial pressure-area values. A lower threshold pressure of 15 psi (0.1 MPa) was used to eliminate noise. For each time step, both the average pressure (PAV) and the effective total contact area (AT) could then be determined. The PAV/AT values constitute the process pressure-area values.

Event: R821014_113739						3 BACKING & RAMMING INTO M.Y. @ 3-4 KT					
	p_lc	F_max [MN	P_max [Mpa]								
	15	4.952546	11.14922								
				cells	1	2	3	4	5	6	7
				As [m2] ->	0.1516	0.3032	0.4548	0.6064	0.758	0.9096	1.0612
steps	seconds	FT [MN]	AT [m2]	PAV [Mpa]	PMAX [Mpa]						
1	0.03125	0.07526	0.4548	0.16548	0.20685	0.182718	0.16548				
2	0.0625	0.062717	0.3032	0.20685	0.22064	0.20685					
3	0.09375	0.072124	0.3032	0.237878	0.255115	0.237878					
4	0.125	0.098257	0.4548	0.216043	0.310275	0.258563	0.216043				
5	0.15625	0.134841	0.4548	0.296485	0.351645	0.341303	0.296485				
6	0.1875	0.1662	0.4548	0.365435	0.544705	0.444728	0.365435				
7	0.21875	0.18606	0.4548	0.409103	0.71708	0.499888	0.409103				
8	0.25	0.173517	0.4548	0.381523	0.737765	0.479203	0.381523				
9	0.28125	0.122298	0.3032	0.403358	0.586075	0.403358					
10	0.3125	0.086758	0.3032	0.286143	0.35854	0.286143					
11	0.34375	0.062717	0.3032	0.20685	0.213745	0.20685					
12	0.375	0.056445	0.3032	0.186165	0.213745	0.186165					
13	0.40625	0.061672	0.3032	0.203403	0.213745	0.203403					
14	0.4375	0.09303	0.4548	0.204552	0.227535	0.22064	0.204552				
15	0.46875	0.116026	0.4548	0.255115	0.310275	0.2758	0.255115				
16	0.5	0.141113	0.4548	0.310275	0.49644	0.35854	0.310275				
17	0.53125	0.169336	0.4548	0.37233	0.6895	0.451623	0.37233				
18	0.5625	0.165155	0.4548	0.363137	0.70329	0.458518	0.363137				
19	0.59375	0.104528	0.3032	0.34475	0.475755	0.34475					

Annex E – Listing of CONTACT_6

I C E   E D G E   C O N T A C T   M O D E L
Modeling Flake Fracture and Extrusion

Author : Claude Daley

Date: Oct. 31, 1994
Revision: Feb ,2004
File: CONTACT_6.ms
Language : MAPLE V, Rel. 9

Ice Parameters : definition of case
units : MPa, m, MN,

Set All Initial Problem Parameters

```
   >  restart;     # Set all initial parameters
        `Ice Parameters`:
```

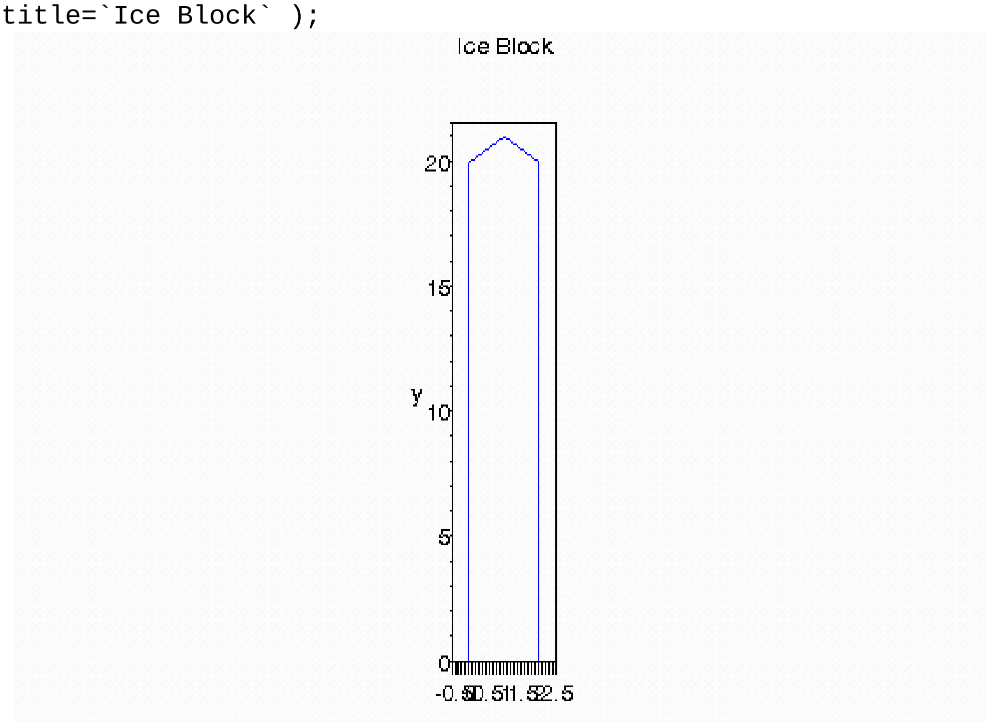
Ice Parameters

```
        orig_profyle := [[0,0],[0,7],[1,8],[2,7],[2,0]]:
        profyle:=orig_profyle:
   >
   > Pc := 20 :                 # Pc is the direct contact pressure
   > c:= .8:                    # ice shear strength (Coulomb)
   > phi[1]:=evalf(0*deg):      #  ice friction angle (Coulomb - solid ice)
   > mu[1]:=tan(phi[1]):        # ice friction factor (on ice failure surface)
   >
   > deg:= evalf(Pi/180):       # conversion of radians to degrees
   > phi[2]:= 10*deg:         #10 solid ice - structure contact friction angle
   > mu[2]:=evalf(tan(phi[2])): # friction factor in direct contact
   > phi[3]:= 0*deg:            #6 granular ice - structure friction angle
   > mu[3]:=evalf(tan(phi[3])): # friction fact in granular ice-structure
contact
   > phi[4]:= 0*deg:            # granular ice - ice contact friction angle
   > mu[4]:=evalf(tan(phi[4])): # friction factor in granular ice-ice contact
   > alpha_lim:= phi[3]+phi[4]: # if alpha < alpha_lim then exponential
   extrusion takes place
   > sl_sail := .8:             # slope of the extruded material above ice
   > sl_keel :=  .8:            # slope of the extruded material below ice
   > phi[5]:=arctan(sl_sail):   # internal friction angle of granular
   material in the sail (deg)
   > phi[6]:=arctan(sl_keel):   # internal friction angle of granular
   material in the keel (deg)
   >
   > g:= 9.8:                                 #  gravity, m/s^2
   > rho:= 560/1000000:                # mass density   Mkg/m^3
   > gb:=9.8*.12;                         # buoyancy m/s^2
```

Determine Ymax

```
Ymax:=0: Ymin:= 0 : Xmax := 0 : Xmin := 0:
for i from 1 to nops(profyle) do
Y := op(2,op(i,profyle)):  # Y val of point
X := op(1,op(i,profyle)):  # X val of point
if Y > Ymax then Ymax := Y: fi;
if Y < Ymin then Ymin := Y: fi;
if X > Xmax then Xmax := X: fi;
if X < Xmin then Xmin := X: fi;
od:
profyle; Xmin; Xmax; Ymin; Ymax;


plot(profyle,x=Xmin-
.5..Xmax+.5,y=Ymin..Ymax+.5,axes=box,scaling=constrained,color=blue,
title=`Ice Block` );
```




```
>
```

define_contact(contact)

define_contact(contact) : a subroutine to determine the location of the direct contact
contact is the location of the indentor
output: Nl, Nc, Nh

```
      define_contact :=
      proc(contact)
>     local i,inl, A,B,C,D,E;              # local variables
>     global Nl,Nh,Nc;                     # global variables
>        Nl := [0,0];                      # low point of contact
>        Nh := [0,0];                      # high point of contact
```

```
>        Nc := [0,0];                       # centre point of contact
      inl := 0;
>
      for i from 2 to nops(profyle) do
>
>             A := op(1,op(i-1,profyle));    # X val of 1st point
>             B := op(2,op(i-1,profyle));    # Y val of 1st point
>             C := op(1,op(i,profyle));       # X val of 2nd point
>             D := op(2,op(i,profyle));       # Y val of 2nd point
      E := contact-B;
>
>        if B < contact  and   D > contact  then
      Nl := [A+(C-A)/(D-B)*E,contact]: inl:= i : break:
      elif D = contact then
      Nl := [C,D]: inl:= i :break:
      elif B = contact then
      Nl := [A,B]: inl:= i-1 :break:
      fi:
      od:
>
      for i from inl+1 to nops(profyle) do
      if Nl = [0,0] then break : fi:
>             A := op(1,op(i-1,profyle));    # X val of 1st point
>             B := op(2,op(i-1,profyle));    # Y val of 1st point
      C := op(1,op(i,profyle));  # X val of 2nd point
      D := op(2,op(i,profyle));  # Y val of 2nd point
      E := contact-B;
>
>        if B > contact  and   D < contact  then
      Nh := [A+(C-A)/(D-B)*E,contact]: break:
      elif D = contact then
      Nh := [C,D]: break:
      elif B = contact then
      Nh := [A,B]: break:
      fi:
      od:
>
      Nc:=[op(1,Nl)+1/2*(op(1,Nh)-op(1,Nl)),op(2,Nl)+1/2*(op(2,Nh)-
      op(2,Nl))]:
>
> end:      #  END of define_contact
>
```

area_profyle()

area_profyle() : a subroutine to determine the area of the profyle
output : profyle_area

```
      area_profyle :=
      proc(profyle)
> local i, A,B,C,D,E,F;           # local variables
> global profyle_area:              # global variables
      profyle_area:= 0;
      for i from 2 to nops(profyle)-1 do
>
>             A := op(1,op(1,profyle));    # X val of apex point
>             B := op(2,op(1,profyle));    # Y val of apex point
```

```
>             C := op(1,op(i,profyle));     # X val of 2nd point
>             D := op(2,op(i,profyle));     # Y val of 2nd point
      E := op(1,op(i+1,profyle));  # X val of 3rd point
      F := op(2,op(i+1,profyle));  # Y val of 3rd point
>
      profyle_area := profyle_area + 1/2 *abs (A*D+B*E+F*C-D*E-B*C-A*F);
      od;
>
      end:  # END of area_profyle
>
```

adjust_profyle(contact)

adjust_profyle(contact) : a subroutine to redefine the profyle to include the direct contact
output: profyle

```
      adjust_profyle :=
      proc(contact)
>    local i, temp_profyle, Temp:       # local variables
>    global profyle,Nl,Nh:        # global variables
      Temp := 0;
      temp_profyle:= NULL:
      for i from 1 to nops(profyle) do
>       if op(2,op(i,profyle)) > contact then   Temp :=1:
      elif op(2,op(i,profyle)) < contact and Temp =1 then
      temp_profyle:=temp_profyle,Nl,Nh,op(i,profyle): Temp :=0:
      else temp_profyle:=temp_profyle,op(i,profyle):
      fi;
      od:
      ### WARNING: `profyle` might conflict with Maple's meaning of that
      name
      profyle:= [temp_profyle]:
>
> end:      # END of adjust_profyle
>
```

extr_profyle(AreaH,AreaL)

extr_profyle(AreaH,AreaL) : a subroutine to determine the profyle of the extruded material
output : profyle

```
      extr_profyle :=
      proc(Ahi,Alo)
      local i,ih,il,nph,npl,contact,A,B,C,D,E,F, Bt,Dt,
>       profyle_Ex,he,hp,lp,le,Arhi,Arlo,Arb,Area_btw;     #local
variables
> global profyle, profyle_EH, profyle_EL:        # global variables
> profyle_EH:=0; profyle_EL:=0 ;           # extrusion profyles, high and
low
      Arhi :=0; Arlo:=0; ih:=nops(profyle)-1;il:=2; #temp variables to sum
      the area
      contact := op(2,Nh);
>   for i from 2 to nops(profyle)-1 do      # find counters for edges of
contact
>
```

```
      if op(i,profyle)=Nh then ih:=i; fi;
      if op(i,profyle)=Nl then il:=i; fi;
      od;
>
      profyle_EH := Nh;
      profyle_EL := Nl;
> #                          lprint(`ih = `,ih);
> for i from ih to nops(profyle)-1 do         # calc profyle for high
extrusion
>       A := op(1,op(i,profyle));              # X val of 1st point
>       B := op(2,op(i,profyle));              #Y val of 1st point
>       C := op(1,op(i+1,profyle));            #X val of 2nd point
>       D := op(2,op(i+1,profyle));            #Y val of 2nd point
      Bt := contact-B;
      Dt := contact-D;
>
      Area_btw :=(Dt^2-Bt^2)*sl_sail/2+((contact-B)+(contact-D))*(C-A)/2;
>
      if Area_btw +Arhi < Ahi then
>       Arhi := Arhi+Area_btw;             #### end point is not in this
segment
      profyle_EH := profyle_EH,[C,D];
>
> else                                  #### end point is in this segment
      Arb:= Ahi-Arhi;
>
      if A=C then
      E:=A;
      F:= 1/2*B+1/2*contact-1/2*(((B^2-2*contact*B+contact^2)
      *sl_sail+4*Arb) /sl_sail)^.5;
      else
      he:= B-D;
      le:= C-A;
      hp:=1/(-sl_sail-le/he)*((Bt*sl_sail+Bt*le/he)-((-(Bt*sl_sail*he)^2
      +2*sl_sail*he^2*Arb+Bt^2*le^2+2*Arb*he*le)^(1/2))/he);
>
      lp:=hp*le/he;
      E:= A+lp;
      F := (E-C)*(B-D)/(A-C)+D;
      fi;
>
>
      profyle_EH := profyle_EH,[E,F], [E+(contact-F)*sl_sail,contact];
      nph:= nops([profyle_EH]);profyle_Ex:= NULL;
>       for i from 2 to nph-1 do               # add face points to
profyle_EH
      profyle_Ex := profyle_Ex,[op(1,op(nph-i,[profyle_EH])),contact];
      od;
      profyle_EH := [profyle_EH,profyle_Ex];
>
>
      break;
      fi;
      od;
>
>
```

```
> for i from 1 to (il-1) do                   # calc profyle for low
extrusion
>      A := op(1,op(il-i+1,profyle));         # X val of 1st point
>      B := op(2,op(il-i+1,profyle));         # Y val of 1st point
>      C := op(1,op(il-i,profyle));           # X val of 2nd point
>      D := op(2,op(il-i,profyle));           # Y val of 2nd point
      Bt := contact-B;
      Dt := contact-D;
>
      Area_btw :=((contact-D)^2-(contact-B)^2)*sl_keel/2+((contact-
      B)+(contact-D))*(A-C)/2;
>
      if Area_btw +Arlo < Alo then
      Arlo := Arlo+Area_btw;
      profyle_EL := profyle_EL,[C,D];
>
      else
      Arb:= Alo-Arlo;
>
      if A=C then
      E:=A;
      F:= 1/2*B+1/2*contact-1/2*(((B^2-
      2*contact*B+contact^2)*sl_keel+4*Arb) /sl_keel)^.5;
      else
>
      he:= B-D;
      le:= A-C;
      hp:=1/(-sl_keel-le/he)*((Bt*sl_keel+Bt*le/he)-((-(Bt*sl_keel*he)^2
      +2*sl_keel*he^2*Arb +Bt^2*le^2+2*Arb*he*le)^(1/2))/he);
      lp:=hp*le/he;
      E:= A-lp;
>
      F := (E-C)*(B-D)/(A-C)+D;
      fi;
      profyle_EL := profyle_EL,[E,F],[E-(contact-F)*sl_keel,contact];
>
>
>  #    profyle_EH := profyle_EH,[E,F], [E+(contact-F)*sl_sail,contact];
      npl:= nops([profyle_EL]);profyle_Ex:= NULL;
>      for i from 2 to npl-1 do                 # add face points to
profyle_EH
      profyle_Ex := profyle_Ex,[op(1,op(npl-i,[profyle_EL])),contact];
      od;
      profyle_EL := [profyle_EL,profyle_Ex];
>
>
      break;
      fi;
      od;
>  #   lprint(`Alo = `,Alo);
> end:   # END of extr_profyle
      # extr_profyle(1,1);
>
```

extr_forces()

extr_forces() : a subroutine to determine the pressures and forces in the extruded material
output : ice_forc_Hv, ice_forc_Hh, ice_forc_Lv, ice_forc_Lh

H : air side, L : water side, v : vertical direction, h : horizontal direction

```
      extr_forces :=
      proc()
      local i,ih,il,nph,npl,contact,
      A,B,C,D,As,Bs, Xts,Yts,
      gama, gamab,ls,ms,li,mi,
      pr,W0,W1, k, H,
      Fsh,Fsv,Fih,Fiv,
      Force_to_x20, Force_to_H, Force_total,
      slope, Po,P1,x_20,
>                                    Ka, alpha   ;            #local
variables
>
>
      global profyle, profyle_EH, profyle_EL,kp,
>   ice_forc_Hv,ice_forc_Hh,ice_forc_Lv,ice_forc_Lh;        # global
variables
>
      # extrusion pressure list, high and low
      # lprint(`a`,Nh);
      contact := op(2,Nh);
>  gama:= rho*g;    # weight density above water
      gamab:=rho*gb;  # buoyant density below water
>    k:= (a,p) ->  (tan(45*deg+p/2*(1-a/(90*deg))))^2;   # k factor
(lateral soil pressure coefficient)
>
      ice_forc_Hv:= 0; ice_forc_Hh:= 0;
      ice_forc_Lv:= 0;  ice_forc_Lh:= 0;
>
      ms:= cos(phi[3]); # direction cosines at structure (horiz)
>             ls:= sin(phi[3]);    #                 "
(vert)
>
> ih:=nops(profyle_EH)/2;                        # high side values
>
>      Xts := op(1,op(ih+1,profyle_EH));         # X val of top of slope
>      Yts := op(2,op(ih+1,profyle_EH));         # Y val of top of slope
>
      pr:= 0;
>
> for i from 1 to (ih-1) do                   # calc profyle for low
extrusion
>
>      A := op(1,op(ih-i+1,profyle_EH));         # X val of 1st point
>      B := op(2,op(ih-i+1,profyle_EH));         # Y val of 1st point
>      C := op(1,op(ih-i,profyle_EH));            # X val of 2nd point
>      D := op(2,op(ih-i,profyle_EH));            # Y val of 2nd point
>      As := op(1,op(ih+i,profyle_EH));    # X val of point on structure
paired with 1st point
>      Bs:= op(2,op(ih+i,profyle_EH));     # Y val of point on structure
paired with 1st point
>
      if A-C=0 then slope:=1E10 else slope:=(D-B)/(A-C); fi;
>       alpha:= arctan(slope);             # angle of edge segment
>
      # lprint(` i=`, i);
```

```
>  # lprint(`[A,B] [C,D],As`,[A,B],[C,D],As);lprint(`   slope=   `,slope,
`alpha =`, alpha, ` alpha_lim =`, alpha_lim);
>
>
      if i=1 then
      W0:=0 ;  W1:= contact - B;
      else
      W0:= contact-B ;
      W1:= contact - D;
      fi;
>
>
      mi:= cos(phi[4])*cos(alpha)+sin(phi[4])*sin(alpha);  #dirn cosines
      at ice (hor)
>   li:= -cos(phi[4])*sin(alpha)+sin(phi[4])*cos(alpha); #      "
(vert)
>
>            H:=As-C;    # m
      Po:=op(i,[pr]);  #MPa
>
      if alpha < alpha_lim then
      # lprint(` doing the exponential  extrusion part`);
      Ka:= k(alpha,phi[5])*(mu[4]+(mu[3]*cos(alpha)-sin(alpha))
      /(cos(alpha)+mu[3]*sin(alpha)));
>
      x_20:= (1-(20/Po)^(-alpha/Ka))*W0/alpha;
>   #     lprint(` k = `,k(alpha,phi[5]),`  Ka =`,Ka, `x_20 = `, x_20,
`H=`,H, `W0=`,W0);     ============
>
>
      if x_20 > H then
>       # lprint(` no re-consol`);                   # no re-
consolidation
      P1:= Po*(1-alpha*H/W0)^(-alpha/Ka);
      # lprint(` Po = `, Po,` P1 = `,P1 );
      pr:= pr,P1;
      Force_to_H:= (W0*Po*(1+((1-alpha*H/W0)^(-Ka/alpha+1)))/(alpha-Ka));
      Fsh:= Force_to_H*k(alpha,phi[5]);
      Fsv:= Fsh*ls/ms;
> #      lprint(`ms == `,ms, ` ls == `,ls); lprint(`    Fsh == `,Fsh, `
Fsv == `,Fsv);
      Fih:= Fsh;
      Fiv:= Fih*li/mi;  #(neg = outward)
      #  lprint(`33  mi == `,mi, ` li == `,li); lprint(`Fih == `,Fih, `
      Fiv == `,Fiv);
      ice_forc_Hv:= ice_forc_Hv + Fiv;
      ice_forc_Hh:= ice_forc_Hh + Fih;
>
      else
>          #  lprint(` re-consol`);          # reconsolidation
>
      P1:= 20;
> #          lprint(` P1 = `,P1);
      pr:= pr,P1;
>
>
```

```
      Force_to_x20:= (W0*Po*(1+(max((1-alpha*x_20/W0),0)^(-
      Ka/alpha+1)))/(alpha-Ka));
>
> #  lprint(`    Force_to_x20 `,Force_to_x20, ` (1-alpha*x_20/W0) = `,(1-
alpha*x_20/W0));
>
      Force_total:= Force_to_x20+(H-x_20)*20 ;
>
      Fsh:= Force_total*k(alpha,phi[5]);
      Fsv:= Fsh*ls/ms;
      # lprint(`  44a ms == `,ms, ` ls == `,ls); lprint(` Fsh =c= `,Fsh, `
      Fsv =c= `,Fsv);
      Fih:= Fsh;
      Fiv:= Fih*li/mi;  #(neg = outward)
>   #   lprint(`  44  mi =  `,mi, ` li = `,li); lprint(`Fih = c = `,Fih, `
Fiv = c = `,Fiv);
      ice_forc_Hv:= ice_forc_Hv + Fiv;
>
      ice_forc_Hh:= ice_forc_Hh + Fih;
      # lprint (` finished re-con`);
      fi;

>
      else
>
>
      P1:=
      (Po*(W0+H/2*k(alpha,phi[5])*(ls/ms+li/mi))+gama*(W0+W1)/2*H)/(W1-
      H/2*k(alpha,phi[5])*(ls/ms+li/mi));
      pr:= pr,P1;
>
>
      Fsh:= (Po+P1)*H*k(alpha,phi[5])/2;
      Fsv:= Fsh*ls/ms;
      Fih:= Fsh;
      Fiv:= Fih*li/mi;  #(neg = outward)
>        #        lprint(`Fih = `,Fih, ` Fiv = `,Fiv, `
k=`,k(alpha,phi[5]));
>  #            lprint(`Fsh = `,Fsh, ` Fsv = `,Fsv);
> #    lprint(`Po = `,Po, ` P1 = `,P1);lprint(`W0 = `,W0, ` W1 =
`,W1);lprint(`ls,ms = `,ls,ms, ` li,mi = `,li,mi);
>
>
      ice_forc_Hv:= ice_forc_Hv + Fiv;
      ice_forc_Hh:= ice_forc_Hh + Fih;
> #             lprint( `******************************`,i);
>
>
      fi;  # end of simple wedge part
      # lprint(`i = `,i);
      od;
>
      #lprint(`pressures = `,[pr]);
>
      # lprint(`2  ice_forc_Hv = `,ice_forc_Hv, `  ice_forc_Hh =
      `,ice_forc_Hh);
>
```

```
>
>
> #                      ...............   LOW SIDE values
.....................
>
      il:=nops(profyle_EL)/2;
>
>      Xts := op(1,op(il+1,profyle_EL));         # X val of top of slope
>      Yts := op(2,op(il+1,profyle_EL));         # Y val of top of slope
>
      pr:= 0;
      # lprint(` il =`, il);
> for i from 1 to (il-1) do                      # calc profyle for low
extrusion
>
>      A := op(1,op(il-i+1,profyle_EL));     # X val of 1st point
>      B := op(2,op(il-i+1,profyle_EL));     # Y val of 1st point
>      C := op(1,op(il-i,profyle_EL));       # X val of 2nd point
>      D := op(2,op(il-i,profyle_EL));       # Y val of 2nd point
      As := op(1,op(il+i,profyle_EL));  # X val of point on structure
      paired with 1st point
>      Bs:= op(2,op(il+i,profyle_EL));   # Y val of point on structure
paired with 1st point
>
      if C-A=0 then slope:=1E10 else slope:=(D-B)/(C-A); fi;
>       alpha:= arctan(slope);            # angle of edge segment
>
      # lprint(` i=`, i);
> # lprint(`[A,B] [C,D],As`,[A,B],[C,D],As);lprint(`   slope=   `,slope,
`alpha =`, alpha, ` alpha_lim =`, alpha_lim);
>
>
      if i=1 then
      W0:=0 ;  W1:= contact - B;
      else
      W0:= contact-B ;
      W1:= contact - D;
      fi;
>
>
      mi:= cos(phi[4])*cos(alpha)+sin(phi[4])*sin(alpha);  #  direction
      cosines at ice (horiz)
>    li:= -cos(phi[4])*sin(alpha)+sin(phi[4])*cos(alpha);   #  "
(vert)
>
>             H:=C-As;    # m
      Po:=op(i,[pr]);  #MPa
>
      if alpha < alpha_lim then
> #       lprint(` doing the exponential  extrusion part`);
      Ka:= k(alpha,phi[5])*(mu[4]+(mu[3]*cos(alpha)-sin(alpha))
/(cos(alpha)+mu[3]*sin(alpha)));
>
      x_20:= (1-(20/Po)^(-alpha/Ka))*W0/alpha;
> #   lprint(` k = `,k(alpha,phi[5]),`  Ka =`,Ka, `x_20 = `, x_20,
`H=`,H);
>
```

```
>
      if x_20 > H then
      # no re-consolidation
      P1:= Po*(1-alpha*H/W0)^(-alpha/Ka);
> #     lprint(` Po = `, Po,` P1 = `,P1 );
      pr:= pr,P1;
      Force_to_H:= (W0*Po*(1+((1-alpha*H/W0)^(-Ka/alpha+1)))/(alpha-Ka));
      Fsh:= Force_to_H*k(alpha,phi[5]);
      Fsv:= Fsh*ls/ms;
> #  lprint(`ms == `,ms, ` ls == `,ls); lprint(`    Fsh == `,Fsh, ` Fsv ==
`,Fsv);
      Fih:= Fsh;
      Fiv:= Fih*li/mi;  #(neg = outward)
> #     lprint(`mi == `,mi, ` li == `,li); lprint(`Fih == `,Fih, ` Fiv ==
`,Fiv);
      ice_forc_Lv:= ice_forc_Lv + Fiv;
      ice_forc_Lh:= ice_forc_Lh + Fih;
>
      else
      # reconsolidation
>
      P1:= 20;
> #           lprint(` P1 = `,P1);
      pr:= pr,P1;
      Force_to_x20:= (W0*Po*(1+((1-alpha*x_20/W0)^(-Ka/alpha+1)))/(alpha-
      Ka));
      Force_total:= Force_to_x20+(H-x_20)*20 ;
>
      Fsh:= Force_total*k(alpha,phi[5]);
      Fsv:= Fsh*ls/ms;
> #      lprint(`ms == `,ms, ` ls == `,ls); lprint(`    Fsh =c= `,Fsh, `
Fsv =c= `,Fsv);
      Fih:= Fsh;
      Fiv:= Fih*li/mi;  #(neg = outward)
>  #    lprint(`mi == `,mi, ` li == `,li); lprint(`Fih =c= `,Fih, ` Fiv
=c= `,Fiv);
>
      ice_forc_Lv:= ice_forc_Lv + Fiv;
      ice_forc_Lh:= ice_forc_Lh + Fih;
>
      fi;
>
      else
>
>
      P1:=
      (Po*(W0+H/2*k(alpha,phi[5])*(ls/ms+li/mi))+gamab*(W0+W1)/2*H)/(W1-
      H/2*k(alpha,phi[5])*(ls/ms+li/mi));
      pr:= pr,P1;
>
>
      Fsh:= (Po+P1)*H*k(alpha,phi[5])/2;
      Fsv:= Fsh*ls/ms;
      Fih:= Fsh;
      Fiv:= Fih*li/mi;  #(neg = outward)
> #               lprint(`Fih = `,Fih, ` Fiv = `,Fiv, `
k=`,k(alpha,phi[5]));
```

```
> #              lprint(`Fsh = `,Fsh, ` Fsv = `,Fsv);
      #lprint(`Po = `,Po, ` P1 = `,P1);lprint(`W0 = `,W0, ` W1 =
      `,W1);lprint(`ls,ms = `,ls,ms, ` li,mi =`,li,mi);
>
>
      ice_forc_Lv:= ice_forc_Lv + Fiv;
      ice_forc_Lh:= ice_forc_Lh + Fih;
> #            lprint( `******************************`);
>
>
      fi;  # end of simple wedge part
>
      od;
>
      #lprint(`pressures = `,[pr]);
      #lprint(`ice_forc_Lv = `,evalf(ice_forc_Lv));
      #lprint(`ice_forc_Lh = `,ice_forc_Lh);
>
> end:   # END of extr_forces
      # extr_forces();profyle_EL;
      # Ncr;
```

define_crack()

define_crack() : a subroutine to determine the critical point between each pair of points on [[profyle]] where a potential crack may occur.

Output : Ncr, Force

```
      define_crack :=
      proc()
      local i,j, A,B,C,D,E,F,G,H,L,Stressmax,
      Fnor,Fshear,Sig,Tau,Stress,
      theta,alpha,slope1,slope2,end_point,
>           temp, AngL, AngH,muL,muH, muc;          #local variables
>
      global Pc,c,ForcHv,ForcHh,ForcLv,ForcLh,Nl,Nh,Nc,Ncr,profyle,pro_cr,
      ice_forc_Hv,ice_forc_Hh,ice_forc_Lv,ice_forc_Lh;  # global variables
>
>             G := op(1,Nc);    # X val of Nc
>             H := op(2,Nc);    # Y val of Nc
      Stressmax:= 0: Ncr:= [0,0]:
>          ForcHh := (op(1,Nh)-op(1,Nl))* Pc/2+ice_forc_Hh;    # contact
force
>          ForcHv := ForcHh*mu[2]+ice_forc_Hv;    # contact force
      # lprint(` ice_forc_Hh =`,ice_forc_Hh );
>          ForcLh := (op(1,Nh)-op(1,Nl))* Pc/2+ice_forc_Lh;    # contact
force
>          ForcLv := ForcLh*mu[2]+ice_forc_Lv;    # contact force
>
>          muH := ForcHv/ForcHh;    # contact force
>          muL := ForcLv/ForcLh;    # contact force
>          AngH := arctan(muH);    # contact force
>          AngL := arctan(muL);    # contact force
>
      for j from  1  to nops(profyle)-1 do
>
>             A := op(1,op(j,profyle));    # X val of 1st point
>             B := op(2,op(j,profyle));    # Y val of 1st point
```

```
      C := op(1,op(j+1,profyle));  # X val of 2nd point
      D := op(2,op(j+1,profyle));  # Y val of 2nd point
>
      if [A,B]=Nl and [C,D]=Nh then next; fi;  # cracks do not run back to
      contact (L would be 0)
>
      ### find slope of (A,B) -> (C,D)
>
      if A=C then
      if D>B then slope1 := infinity;
      else slope1:= -infinity;
      fi;
> else slope1 := (D-B)/(C-A);    # slope of the segment
      fi;
>    theta := arctan(slope1);    # angle of the segment
>
      if evalf(theta) <0 then
>                    alpha := (theta-phi[1]+AngH)/2;      # Air
CHANGED
      muc :=muH;
      else
>                   alpha := (theta+phi[1]-AngL)/2 ;     # Water
CHANGED
      muc :=muL;
      fi;
> slope2 := evalf(tan(alpha)):   ## slope of the candidate crack
      ### find (E,F)
>
      if slope1 = infinity or slope1=-infinity then
      E:= A:
      else
      E:= ((-slope2*G+H-B)/slope1+A)/(1-slope2/slope1):
      fi;
      F:= slope2*(E-G)+H;
>
      ###limit (E,F) to segment of profyle
>
      if B<D then
      if F>D then F:=D; E:=C; fi;
      if F<B then F:=B; E:=A; fi;
      else
      if F<D then F:=D; E:=C; fi;
      if F>B then F:=B; E:=A; fi;
      fi;
> # lprint(` pt 2 in define crack`);   ======================
>        L:= sqrt((G-E)^2+(H-F)^2);    # print(` L crack`,L);   length of
crack
      Fnor:= ForcHh*(abs(G-E)/L-muc*(H-F)/L); # normal force on potential
      crack
      Fshear:=ForcHh*((H-F)/L+muc*abs(G-E)/L); # shear force on potential
      crack
>              Sig:= Fnor/L;  #      nornal stress
>              Tau:= Fshear/L;     # shear stress
>         Stress := Tau-Sig*tan(phi[1]);           # Coulomb stress
      if Stress > Stressmax then
      Stressmax := Stress:  # lprint(`tau =`,Tau,` sigf
      =`,Sig*tan(phi[1]),` Stress = `,Stress); #=====
```

```
>                  Ncr:= [E,F]: pro_cr:=j:    # select crack with highest
stress
      fi:
>
>
>  od;                           # end loop over each segment of profyle
>
> if Stressmax < c then      # if stress is less than failure stress then
no crack
      Ncr := [no,crack];
      fi:
      #lprint(` Ncr= `,Ncr);
> end:      #  END of define_crack
      # define_crack();
>
```

adjust_crack()

adjust_crack() : a subroutine to redefine the profyle to exclude the flake
output: profyle

```
      adjust_crack :=
      proc()
>    local i, temp_profyle, Temp:                    # local variables
>    global profyle,Nl,Nh,Nc,Ncr,pro_cr,side:        # global variables
      Temp := 0;
      temp_profyle:= NULL:
      if Ncr <> [no,crack] then
      #lprint(` Ncr= `,Ncr);
> if op(1,op(pro_cr,profyle)) > op(1,Nc) then    #  crack to high side
>
      for i from 1 to nops(profyle) do
>
>     if op(1,op(i,profyle)) > op(1,Nc) and   op(2,op(i,profyle)) >
op(2,Ncr) and           Temp = 0 then
      temp_profyle:=temp_profyle,Nc,Ncr: Temp := 1:
>
>      elif op(1,op(i,profyle)) > op(1,Nc) and  op(2,op(i,profyle)) >
op(2,Ncr) and     Temp = 1 then
>
      else temp_profyle:=temp_profyle,op(i,profyle):
      fi;
      od:
> Nh:=Nc:    side:= `high`:  # crack on high side
      Nc:=[op(1,Nl)+1/2*(op(1,Nh)-op(1,Nl)),op(2,Nl)+1/2*(op(2,Nh)-
      op(2,Nl))]:
>
> else    # crack on low side
>
      for i from 1 to nops(profyle) do
>
>              if op(1,op(i,profyle)) < op(1,Nc) and    op(2,op(i,profyle))
> op(2,Ncr) and        Temp = 0 then
temp_profyle:=temp_profyle,Ncr,Nc: Temp := 1:
>
```

```
>            elif op(1,op(i,profyle)) < op(1,Nc) and   op(2,op(i,profyle))
> op(1,Ncr) and     Temp = 1 then
>
      else temp_profyle:=temp_profyle,op(i,profyle):
      fi;
      od:
>
>     Nl:= Nc:   side:= `low`:  # crack on low side
      Nc:=[op(1,Nl)+1/2*(op(1,Nh)-op(1,Nl)),op(2,Nl)+1/2*(op(2,Nh)-
      op(2,Nl))]:
      fi:
      ### WARNING: `profyle` might conflict with Maple's meaning of that
      name
      profyle:= [temp_profyle]:
      else:
>
      fi:
> end:      # END of adjust_crack
>
```

Main_Loop()

Main_Loop(): iterates as the indentor moves

```
      Main_Loop :=
      proc()
> local i, contact,lin1,lin2,Forcei,Forceip,ForcePh,
Area1,Area2, Area3,AreaC,AreaH,AreaL;                    # local variables
>    global profyle,surface_prof, Nl, Nh, Nc,llist,llist2,
ForceT,ForceF,ForceEx,Ymax,AreaCr,AreaHi,AreaLo;       # global variables
      llist:= NULL: llist2:= orig_profyle:
      ### WARNING: `profile` might conflict with Maple's meaning of that
      name so `profyle` is used
profyle:=orig_profyle;
>    ForceT:=[0,0]; ForcePh:=[0,0]; ForceEx:=[0,0];AreaHi:=[0,0];
AreaLo:=[0,0]; AreaC:=0;  AreaH:=.010;   AreaL:=.010;
>    define_contact(Ymax,profyle);     extr_profyle(0,0);
      for i from 1 to 53 do
      contact := Ymax - i/200*1:
      area_profyle(profyle); Area1:= profyle_area;
      define_contact(contact,profyle);
      adjust_profyle(contact);
      area_profyle(profyle); Area2:= profyle_area;
      AreaC:= Area1-Area2;
>
>
      extr_forces();
>         define_crack():      # define [Ncr]
>
>
>
      if Ncr <> [no,crack] then
      ForcePh:= ForcePh, [Ymax-contact,(ForcLh+ForcHh)]:
      ForceEx:= ForceEx, [Ymax-contact,(ice_forc_Lh+ice_forc_Hh)]:
>
```

```
        ForceT:= ForceT, [Ymax-contact,(ForcLh+ForcHh)],[Ymax-
        contact,(ForcLh+ForcHh)/2]:
        lin1:=[Ncr,Nc]; llist:=llist,lin1:
        else
        #lprint(Ncr);
        fi:
>            lin2:=[Nl,Nh];    llist:=llist,lin2:
        adjust_crack():
        area_profyle(profyle); Area3:= profyle_area;
>
        if side = `high` then
        AreaH := AreaH + Area2-Area3;
        else
        AreaL := AreaL + Area2-Area3;
        fi;
        AreaH := AreaH + AreaC/2 :  AreaL := AreaL + AreaC/2:
>            AreaHi:= AreaHi,[i,AreaH]:   AreaLo := AreaLo,[i,AreaL]:
>   #  lprint(`i = `,i, `   AreaL = `,AreaL);
>
        extr_profyle(AreaH,AreaL);
        extr_forces();
        od:
        # extr_profyle(AreaH,AreaL);
>
        Forcei := op(2,op(1, [ForcePh])); Forceip := op(2,op(2, [ForcePh]));
        ForceF:=[Forcei,Forceip];
        for i from 2 to nops([ForcePh])-1 do
        Forcei:= op(2,op(i, [ForcePh]));  Forceip := op(2,op(i+1,
        [ForcePh]));
        ForceF:= ForceF,[Forcei,Forcei], [Forceip,Forcei];
        od;
> end:      #  END of Main_Loop
>
> Main_Loop():         # runs program
```

Determine Ymin, Xmax,Xmin for plots

```
        prof:=[op(profyle_EL),op(profyle_EH)]:
        Ymin:= Ymax : Xmax := 1 : Xmin := 1:
        for i from 1 to nops(prof) do
        Y := op(2,op(i,prof)):  # Y val of point
        X := op(1,op(i,prof)):  # X val of point
        if Y < Ymin then Ymin := Y: fi;
        if X > Xmax then Xmax := X: fi;
        if X < Xmin then Xmin := X: fi;
        od:
```

Determine Yend

```
        Yend:= 0:
        for i from 1 to nops(prof) do
        Y := op(2,op(i,prof)):  # Y val of point
        if Y > Yend then Yend := Y: fi;
        od:
```

************* PLOTS **************

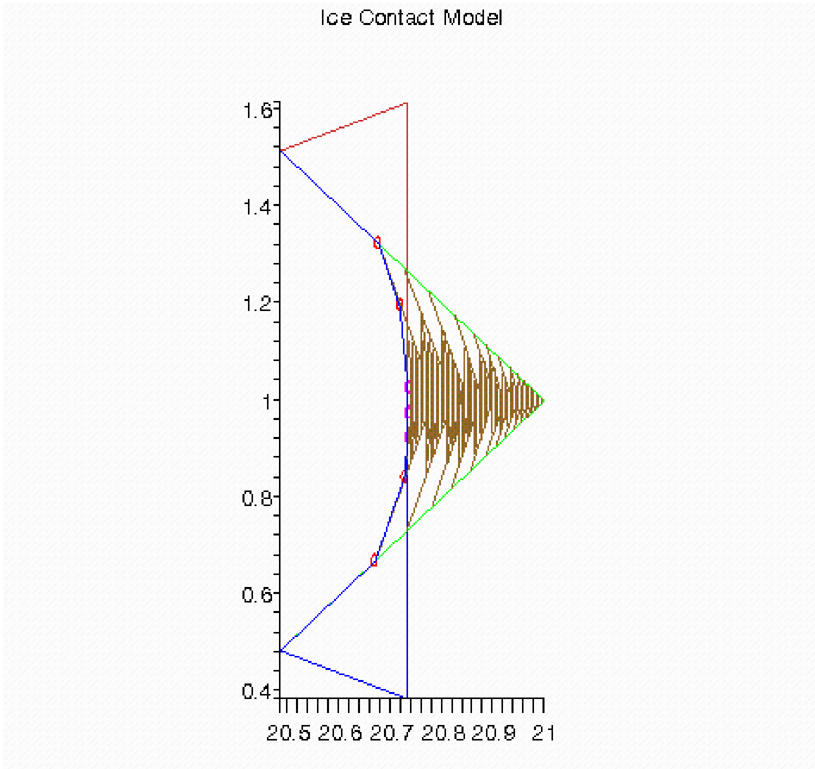
```
>
>
```

ReverseXY(profyle)

```
      reverseXY :=
      proc(list_s)
> local i,j,X,Y,rev_profyle,rev_list,a_profyle;              # local
variables
      rev_list := NULL; rev_profyle:=NULL;
      if (op(0,op(1,op(1,list_s))) = `list`) then
>
>
      for j from 1 to nops(list_s) do
>     a_profyle:=op(j,list_s);   rev_profyle:=NULL;
      for i from 1 to nops(a_profyle) do
>
      Y := op(2,op(i,a_profyle)):  # Y val of point
      X := op(1,op(i,a_profyle)):  # X val of point
      rev_profyle := rev_profyle,[Y,X] :
      od:
      rev_profyle:=[rev_profyle];
      rev_list:= rev_list,rev_profyle;
>
      od:
      rev_profyle:=rev_list;
      else
>
      for i from 1 to nops(list_s) do
      Y := op(2,op(i,list_s)):  # Y val of point
      X := op(1,op(i,list_s)):  # X val of point
      rev_profyle := rev_profyle,[Y,X] :
      od:
>
      rev_profyle:=[rev_profyle];
      fi:
      RETURN(rev_profyle);
      end:
>
>
```

plots

```
      p1:=
      plot(reverseXY(profyle),y=Ymin..Ymax,x=Xmin..Xmax,axes=none,color=bl
      ue,scaling=constrained, axes = frame):
      p2:= plot(reverseXY(profyle),style=point,symbol=circle,color=red):
      p3:= plot(reverseXY(llist2), color=green):
      p4:= plot(reverseXY([Nl,Nc,Nh]),style=line,color=magenta):
      p5:=
      plot(reverseXY([Nl,Nc,Nh]),style=point,symbol=circle,color=magenta):
      p6:= plot({reverseXY([llist])}, color=sienna):
      p7:= plot(reverseXY(profyle_EH),color=orange):
      p8:= plot(reverseXY(profyle_EL),color=blue):
>
      plset:={p1,p2,p3,p4,p5,p6,p7,p8}:
      plots[display](plset,title=`Ice Contact Model`);
```



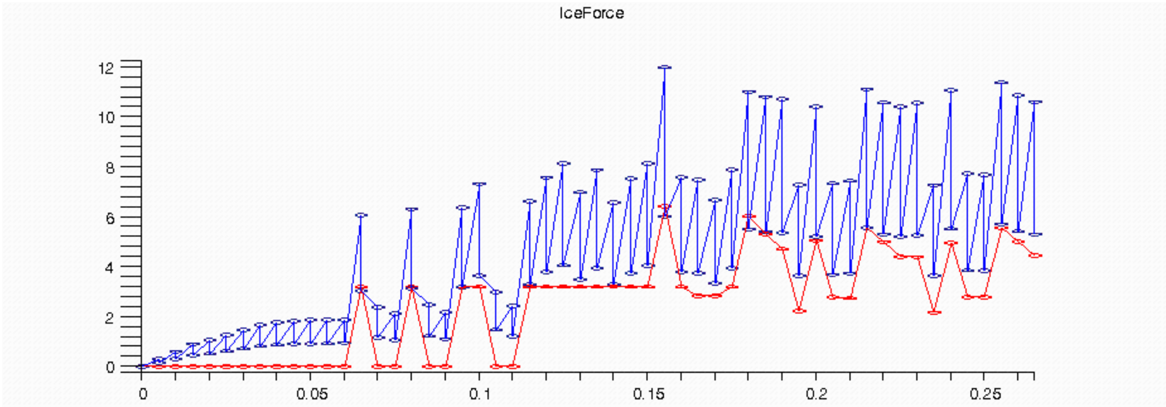
>

Determine Fmax for plot

```
Fmax:=0:
for i from 1 to nops([ForceT]) do
F := op(2,op(i,[ForceT])):  # Force at i
#lprint(F, Fmax);
if F > Fmax then Fmax := F: fi;
od:
```

>

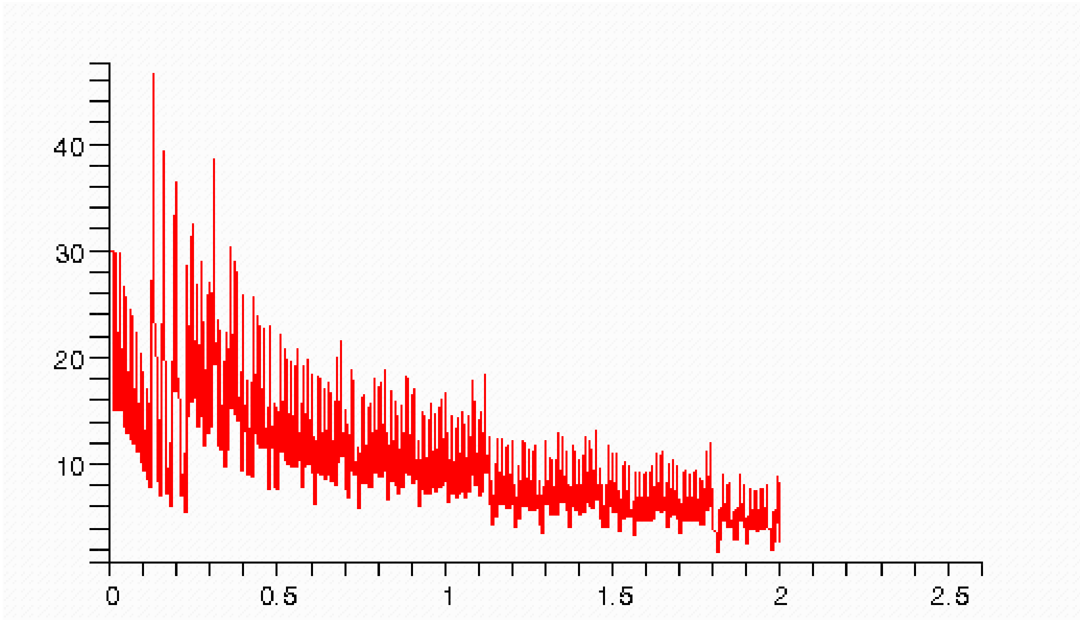
```
pf:= plot([ForceT],x=0..(Ymax-
Yend),color=blue,scaling=unconstrained,axes=frame, style=line):
pfp:= plot([ForceT],x=0..(Ymax-
Yend),color=navy,scaling=unconstrained,axes=frame, style=point):
pe:= plot([ForceEx],x=0..(Ymax-
Yend),color=red,scaling=unconstrained,axes=frame,  style=line):
pep:= plot([ForceEx],x=0..(Ymax-
Yend),color=red,scaling=unconstrained,axes=frame, style=point):
plots[display]({pf,pe,pfp,pep},title=` IceForce`);
```




```
      `Fmax=`,Fmax;
      `Pc = `, Pc  , ` c = `, c;
> `phi[1]`,phi[1]/deg, `  deg   ice friction angle (Coulomb - solid
ice)`;
> `phi[2]`,phi[2]/deg, `  deg  solid ice - structure contact friction
angle`;
> `phi[3]`,phi[3]/deg, `  deg  granular ice - structure friction angle`;
> `phi[4]`,phi[4]/deg, `  deg  granular ice - ice contact friction
angle`;

      PA:=[evalf(log10(.01)),evalf(log10(50))];nops([ForceT]);nops([ForceE
      x]);
      for i from 2 to nops([ForceT]) do
>
>
      Fi := op(2,op(i,[ForceT]));  # Crush Force at i
      Fr := op(2,op(i,[ForceEx])):  # Extr Force at i
>
      F:=Fi+Fr:
      peni := op(1,op(i,[ForceT]));  # penetration at i
      penr := op(1,op(i,[ForceEx]));  # penetration at i
      lprint(i,peni,Fi,penr,Fr):
      A:=min(2,pen*2);  # Nominal Area
>
      P:=F/A;
      #lPA := lPA,[log(A),log(P)]:
      PA := PA,[A,P]:
      #lprint(P,A):
      od:

      PA:
      Amax:= op(1,op(nops([lPA]),[lPA])):  # Crush Force at i
      #PA:=[PA]:
      plot([PA],x=0..2.6,color=red,scaling=unconstrained,axes=frame);
```


40
30
20
10
0
0.5
1
1.5
2
2.5

>