

IACS Unified Requirements for Polar Ships

Background Notes to

Grillages

Prepared for:

IACS Ad-Hoc Group on Polar Class Ships

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April 2000

SUMMARY

The problem of definition of the required scantlings of side grillages that suffer ice loads effects, occupies traditionally one of the central places in the global problem of hull ice strength ensuring for ships and icebreakers. Insufficient side grillage strength can result in serious hull damage dangerous for the whole ship. Presence of excess strength margins results in both unwarranted increase of the ice strengthening structures weight and appearance of considerable technological difficulties rising with class growth.

The mentioned above considerations determine the two main principles that are required to be taken into consideration at developing methodology of side grillage designing within the framework of IACS UR.

* Using criteria and methods of strength assessments taking into account reserves of plastic deforming and actual ice damage forms, provides for guarantee of side grillages reliability. Only such an approach ensures equal strength of different grillage types, as well as of various structural elements being a part of one grillage with reference to loads capable to induce actual damages. Naturally, practical realization of this approach meets some difficulties of methodological character in comparison to more traditional methods of analyzing structures work in the elastic domain.

* Derivation of rational solutions is based on a flexible design procedure taking into account force interaction of member types constituting the grillage as well as the various level of state of loading of main and web framing beams. Realization of such a procedure results in some complication of the total analytic relationships, as well as necessity to regulate some standard set of side grillages schemes.

In this paper is considered a grillage with the transverse framing including both primary frames and load-carrying stringers, as well as web frames. Such a grillage type is main for the ice strengthening structures. The IACS UR requirements to grillages with the longitudinal framing systems are based on the same methodological basis.

DESIGN OF TRANSVERSE FRAMING GRILLAGES WITH WEB FRAMES BY THE ULTIMATE STRENGTH CRITERION

1. INTRODUCTION

The Draft IACS UR for Polar ships has been developed basing on modern methods of strength of ships that take into account actual reserves of material plastic deforming. It has been shown [1, 2, 3] that for relatively rigid ice strengthening structures utilization of the ultimate strength criterion [4, 5, 6] is reasonably. By this criterion structure failure corresponds to realization of the ultimate state described with the help of the linear ultimate balance theory [7, 8].

Consideration of the plastic deforming reserves for frames is performed in compliance with the requirements of both the Russian Register Rules [9] and Canadian Rules [10]. At the same time, for the Canadian Rules wide utilization of the single frame analytic model is typical, while the

Russian Rules use a more progressive methodology basing on which Draft IACS UR has been generated.

2. MAIN ASSUMPTIONS

Below the main assumptions accepted for developing the considered methodology are given. The majority of the accepted assumptions are traditional enough for grillages analysis; the others are natural for examination of ice strengthening structures with the help of the ultimate strength criterion.

2.1 Assumptions of grillage topology

2.1.1 The intersecting frames are mutually perpendicular, the grillage supporting contour has a rectangular shape.

2.1.2 The frames are oriented either horizontally or vertically.

2.1.3 The frames of the same type have same cross sections, placed at the same distance each one from another, secured on the grillage supporting contour in the same way (intermediate frames for which is taken into account the difference in their supporting conditions in comparison with primary frames are exception).

2.2 The frame hierarchical levels and assumptions on ice load resistance by them

2.2.1 Ordinary frames (main and intermediate ones) resisting the ice load directly and bearing against the load-carrying stringers are listed among the first level ($i = 1$).

2.2.2 Load-carrying stringers supporting the primary frames and bearing against the web frames are listed among the second level ($i = 2$).

2.2.3 Web frames supporting the load-carrying stringers and bearing against the grillage supporting contour only are listed among the third level ($i = 3$).

2.3 The examined types of cinematic mechanisms

2.3.1 Frame cinematic mechanisms of the first type describe the ordinary frames transition into the ultimate state as single frames failed between the load-carrying stringers. The load-carrying stringers and web frames remain non deformed. By considering this type of cinematic mechanisms, the requirements to primary frame scantlings are specified as it is cited in a special Background.

2.3.2 The cinematic mechanisms of the second type describe joint transition of primary frames and load-carrying stringers into the ultimate state within one web frame spacing. The web frames remain non deformed. Provided the requirements to ordinary frames are specified, examination of the second type cinematic mechanisms allows to formulate the requirements to the load-carrying stringers that depend on both the load parameters and grillage geometry, as well as the ordinary frame actual parameters.

2.3.3 The cinematic mechanisms of the third type describe grillage transition into the ultimate state in a whole when all the three frame levels including the ordinary frames, load-carrying stringers and web frames, are involved in deforming. In a result of the third type cinematic mechanisms examination, the requirements to web frame scantlings are specified that also take into account the available strength margins for load-carrying stringers and ordinary frames.

2.4 The design loads

2.4.1 The ice load design pattern is triangular.

2.4.2 In order to take into account the difference of the actual ice load pattern from the rectangular one, a localization factor KL_i is introduced (the peakness factor), where i is a number of the frame hierarchical level.

2.4.3 At definition of KL_i value the possibility of installation of load-separating stringers providing adjacent ordinary frames joint deforming, should be taken into account.

2.4.4 For all types of the cinematic mechanisms the ice load position is considered design if the pattern center is on the intersection of the vertical and horizontal axes of the examined plastic mechanism symmetry.

2.5 The supporting sections

2.5.1 All frames, excepting the intermediate frames, are assumed fixed in the supporting sections; the intermediate frames are assumed simply supported if they are terminated in the supporting structure and are not connected with its framing.

2.5.2 If a local plastic mechanism which does not reach the supporting contour, is formed, the non deformed frames of the second and the third types are considered as quasi-supporting structures. The deformed frames are fixed in the quasi-supporting structures.

2.5.3 Supporting sections position on the quasi-supporting structures does not depend on brackets presence or absence and are installed in the place of intersection of the next hierarchical level frame shear area.

2.5.4 The frame supporting section position on the supporting contour at without-bracket connection is specified as follows:

- on the supporting structure plating plane if the frame is connected with the plating only or the framing is installed from the other side of the supporting structure;
- on the supporting structure framing free flange installed from the side of the deformed frame and connected to it.

2.5.5 The frame supporting section position on the supporting contour at bracket connection is specified as follows:

- on the end of a bracket with the straight edge;
- in the middle of a bracket with the rounded edge.

3. Algorithm of side grillage design

The stated considerations permit to describe in the general form the algorithm for side grillage design by the ultimate strength criterion.

Let the examined grillage includes beams of the three levels. The set of geometrical sizes determining unambiguously mutual disposition of the beams in the grillage, as well as the parameters of the design ice load are specified..

For convenience of further expounding, the following system of notation is introduced:

i is the index of a plastic mechanism type and a beam level;

p , b are the design ice load parameters regulated by the HR;

$p_i = KL_i \cdot p$ is the design pressure used at analysis of i type plastic mechanisms ultimate equilibrium;

KL_i are the coefficients of design load localization;

SA_i , PM_i , (SA_{if} , PM_{if}) are the required (actual) web plate area and plastic section modulus for the cross section of i level beam;

$\{g_i\}$ is a vector of grillage and ice load pattern geometric sizes used at consideration of i type plastic mechanisms.

With account of the introduced notation, the criterion of side grillage ultimate strength is written as follows:

$$p_{ui} \geq KL_i \cdot p \quad i = 1, 2, 3 \quad (3.1)$$

where p_{ui} is the ultimate load for i type plastic mechanisms.

According to the TUE extreme principles, p_{ui} is determined as the minimum load among the ultimate loads of i type plastic mechanisms:

$$p_{ui} = \min(p_{ui1}, \dots, p_{uiv}) \quad (3.2)$$

where v is the number of possible plastic mechanisms of i type.

The value of p_{ui} is a function of the vector of grillage and ice load pattern geometric sizes, material yield stress FY , cross section scantlings of the beams forming the i type plastic mechanisms as follows:

$$p_{ui} = p_{ui}(\{g_i\}, FY, SA_1, \dots, SA_i, PM_1, \dots, PM_i), \quad i = 1, 2, 3 \quad (3.3)$$

However, as it was mentioned in division 2.3, p_{ui} definition and later check of criterion (3.1) satisfaction, that is the normal procedure of checking strength calculation of a separate structure, is not the purpose of this investigation. Therefore, there is no necessity to generate functions (3.3). Instead of this, solution of a reverse problem is required, namely, to define the beam cross section characteristics that provide for satisfaction of the ultimate strength criterion (3.1) for all the types of the plastic mechanisms occurring during the grillage transition into the ultimate state.

Solution of the reverse problem is carried out on the basis of the following algorithm.

From consideration of the first type plastic mechanisms, the requirements to the geometrical characteristics of first level beam cross sections are defined:

$$\begin{aligned} SA_1 &= \gamma_1 \cdot SA_{10} \cdot KL_1 \\ PM_1 &= \alpha(\gamma_1) \cdot PM_{10} \cdot KL_1 \end{aligned} \quad (3.4)$$

where $SA_{10} = C1(\{g_i\}, FY) \cdot p$,

$PM_{10} = D1(\{g_i\}, FY) \cdot p$

$C1(\{g_i\}, FY)$, $D1(\{g_i\}, FY)$ are functions defined from solution of the problem on the first type plastic mechanisms ultimate equilibrium;
 $\gamma_1 \geq 1$ is an independent varied parameter.

Section selection basing on (3.4) is carried out by varying parameter γ_1 . After definition of the cross section sizes, coefficients φ_1 and ψ_1 are calculated that characterize actual beam material margins in comparison to the ones required by (3.4), as well as the difference between pressures p and p_1 caused by the factors of load localization as follows:

$$\varphi_1 = \frac{SA1_f}{SA1_0} \geq \gamma_1 \cdot KL_1, \quad \psi_1 = \frac{PM1_f}{PM1_0} \geq \alpha(\gamma_1) \cdot KL_1, \quad (3.5)$$

From consideration of the second type plastic mechanisms, the requirements to the geometrical characteristics of the second level beam cross sections are defined:

$$\begin{aligned} SA2 &= \gamma_2 SA2_0 \cdot KL_2 \\ PM2 &= \alpha(\gamma_2) PM2_0 \cdot KL_2 \end{aligned} \quad (3.6)$$

where $SA2_0 = C2(\{g_2\}, FY, \varphi_1, \psi_1)p$,

$PM2_0 = D2(\{g_2\}, FY, \varphi_1, \psi_1)p$

$C2(\{g_2\}, FY, \varphi_1, \psi_1)$, $D2(\{g_2\}, FY, \varphi_1, \psi_1)$ are functions defined from solution of the problem on the second type plastic mechanisms ultimate equilibrium;
 $\gamma_2 \geq 1$ is the independent varied parameter.

The main difference of (3.6) from (3.4) consists in the fact that the required geometrical characteristics of the second level beams depend on the actual scantlings of the first level beams as well. After definition of the scantlings, the coefficients are calculated as follows:

$$\varphi_2 = \frac{SA2_f}{SA2_0} \geq \gamma_2 \cdot KL_2, \quad \psi_2 = \frac{PM2_f}{PM2_0} \geq \alpha(\gamma_2) \cdot KL_2, \quad (3.7)$$

From consideration of the third type plastic mechanisms, the requirements to the geometrical characteristics of the third level beam cross sections are defined:

$$\begin{aligned} SA3 &= \gamma_3 SA3_0 \cdot KL_3 \\ PM3 &= \alpha(\gamma_3) PM3_0 \cdot KL_3 \end{aligned} \quad (3.8)$$

where $SA3_0 = C3(\{g_3\}, FY, \varphi_1, \psi_1, \varphi_2, \psi_2)p$,

$PM3_0 = D3(\{g_3\}, FY, \varphi_1, \psi_1, \varphi_2, \psi_2)p$

$C3(\{g_3\}, FY, \varphi_1, \psi_1, \varphi_2, \psi_2)$, $D3(\{g_3\}, FY, \varphi_1, \psi_1, \varphi_2, \psi_2)$ are functions defined from

solution of the problem on the third type plastic mechanisms ultimate equilibrium;

$\gamma_3 \geq 1$ is the independent varied parameter.

Relations (3.8) depend on the actual scantlings of the first and second level beams.

In general the stated algorithm (3.4) - (3.8) permits not only to decrease the sizes of the second and third level beams due to presence of excessive material margins in the first and second level beams. Basing on the algorithm, account of the differences in the design load values for the different level beams is carried out directly that is rather important for the HR, as well as purposeful redistribution of the material among the beams of different levels can be carried out for optimization of the grillage size. Inclusion in the design procedure of possibilities to vary in wide range the relationships between the cross section bending and shearing characteristics through parameter γ_i varying, facilitates largely the process of developing the reconciled approach to side grillage strength regulation in the IACS UR.

Difficulties in realization of the described algorithm are generally connected with the necessity to generate the complicated approximation relationships for functions C_i and D_i . However, as it will be further shown, these difficulties can be overcome.

4. GENERATION OF REGULATION RELATIONSHIPS

To generate the Draft IACS UR regulation relationships, by general methodology stated in chapter 3, it is required to evaluate the grillage ultimate balance for all the three types of the plastic mechanisms and to specify the structure of the relationships for C_i and D_i .

4.1 Ordinary frames

Because of the primary frames high importance for the ice strength, some circumstances not following from the ultimate balance theory are also considered at their regulation. The separate Background covers this matter.

4.2. Load-carrying stringers

4.2.1. Formulation of the problem

In compliance with the statements presented in sections 2., 3., the requirements to the geometric characteristics of load-carrying stringers (beams of the second level) cross sections are defined by consideration of second type plastic mechanisms describing transitions into the ultimate state of a grillage part within one web frame spacing.

The web frames restricted the plastic mechanism from two sides remain not deformed. Ice load within one spacing is supposed uniformly distributed on a stripe of height b and length S_w (web frame spacing). Uniform pressure on the stripe $b \times S_w$ is determined in compliance with IACS UR. Horizontal axis of the load pattern coincides with the horizontal symmetry axis of the plastic mechanism.

Relatively small main framing (ordinary frames) spacing is a feature of ice ships side grillages.

Therefore, examining the second type plastic mechanism, is made the following assumption on character of its beams components force interaction:

- ice load is resisted directly by ordinary frames;
- load-carrying stringers are loaded by reactions in stringers intersection with the frames only.

For the considered grillage consisting of similar ordinary frames and similar load-carrying stringers all the intersection reactions should be similar.

Due to the local character of ice load, is unknown in advance the number of load-carrying stringers transiting into the ultimate state together with the frames. Therefore, in accordance with the TUE kinematic theorem, true structure ultimate load is defined from the following condition:

$$p_{u2} = \min (p_{u21}, p_{u22}, \dots, p_{u2j}, \dots p_{u2m}) \quad (4.2.1)$$

where m is a number of the load-carrying stringers in a grillage;

$j = 1, \dots, m$ is an index of the j -th plastic second type mechanism, equal to the number of the load-carrying stringers that have transmitted into the ultimate state;

p_{u2j} is the ultimate load of the j -th second type plastic mechanism;

p_{u2} is the true ultimate load for all the second type plastic mechanisms.

Analytic schemes of the ordinary frame and the load-carrying stringer for the j -th plastic mechanism (at $j = 3$) are cited in Fig. 4.2.1. It follows from Fig. 4.2.1 that the load-carrying analytic scheme is reduced to an isolated beam rigidly attached under load per unit uniformly distributed on the length $S \times n$ (n is a number of the ordinary frames in the web frame spacing)

$$q = Q_j/S \quad (4.2.2)$$

where Q_j is reaction transferring by the ordinary frame to the load-carrying stringer.

Applying the ultimate strength criterion to the load-carrying stringer

$$p_{u2} = KL_2 \cdot p,$$

expressions, similar to the ones cited as per 4.1.6, for the required geometric characteristics of the load-carrying stringer cross section can be obtained easily

$$SA_2 = KL_2 \cdot SA_{20} \quad (4.2.3)$$

$$PM_2 = KL_2 \cdot PM_{20} \cdot \alpha(\gamma_2)$$

where $SA_{20} = \frac{1}{\sqrt{3}} \cdot \frac{p \cdot S \cdot b}{FY} \cdot n \cdot \bar{Q}$

$$PM2_0 = \frac{1}{8} \frac{p \cdot S^2 \cdot b}{FY} n(n+2) : \bar{Q}$$

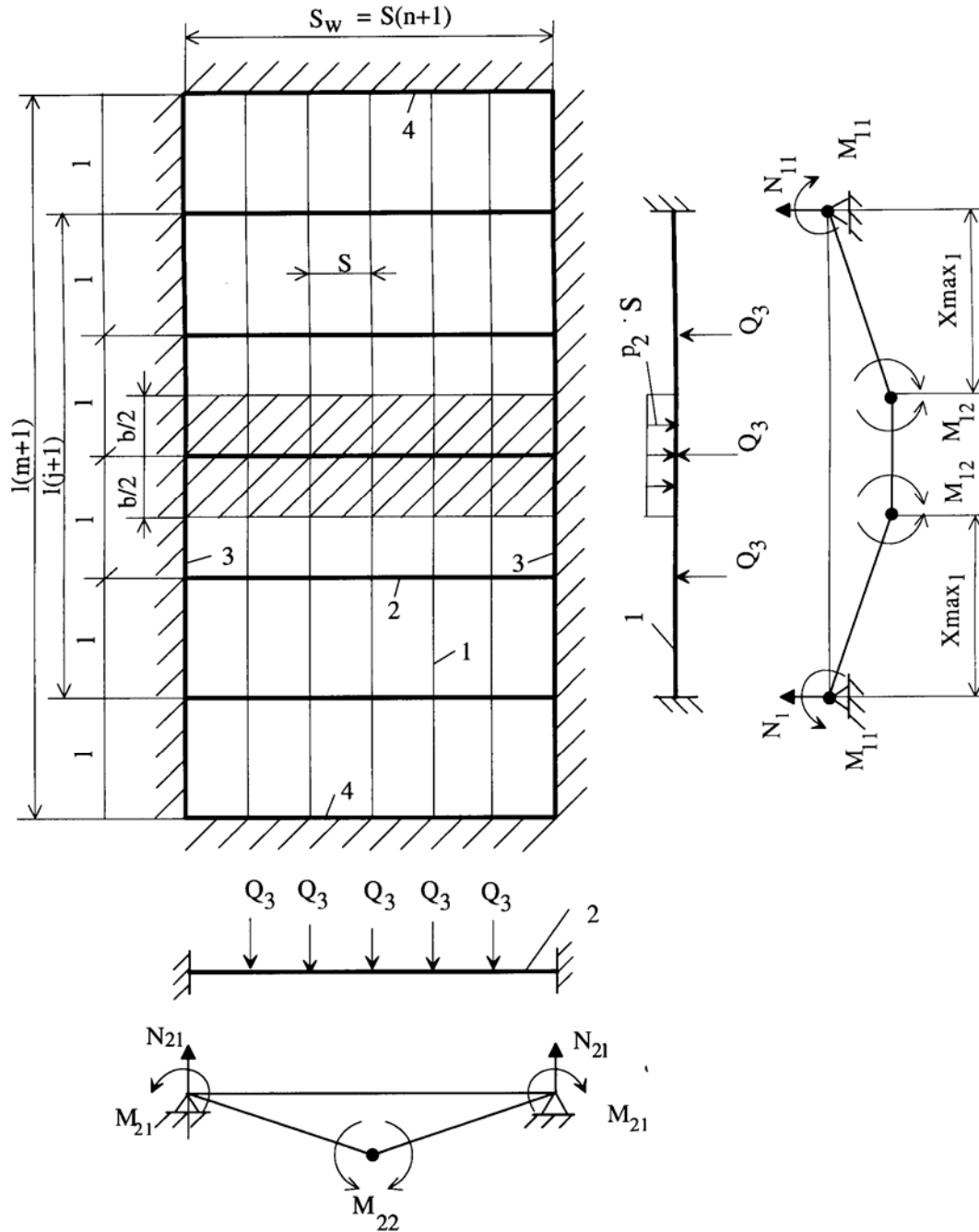


Fig.4.2.1. Scheme of the second type plastic mechanism at $j = 3$
 (three load-carrying stringers transmit into the ultimate state)

- area of ice load application P_2
- - plastic hinges
- 1 - ordinary frames; 2 - load-carrying frames;
- 3 - web frames; 4 - supporting structures.

$$\bar{Q} = \frac{Q}{p \cdot S \cdot b}$$

$$\gamma_2 = \frac{SA2_f}{SA2}$$

$$\alpha(\gamma_2) = \frac{1}{1 + \sqrt{1 - 1/\gamma_2^2}}$$

Q is the value of intersection reactions corresponding to the true plastic mechanism with ultimate load p_{u2} ;

SA2_f is the actual web plate area of the load-carrying stringer cross section.

Because dimensionless reaction \bar{Q} is in inverse proportion to p_{u2} , the following extreme condition is resulted from (4.2.1):

$$\bar{Q} = \max(\bar{Q}_1, \bar{Q}_2, \dots, \bar{Q}_j, \dots, \bar{Q}_m) \quad (4.2.4)$$

Therefore, to define the required geometrical characteristics of the load-carrying stringers, it is required to obtain the expression for dimensionless reaction \bar{Q}_j ($j = 1, \dots, m$) basing on the analysis of the ultimate state of second type plastic mechanisms and to find the maximum value \bar{Q} . As it was mentioned above, to find directly ultimate loads p_{u2j} and true ultimate loads p_{u2} is not required.

4.2.2 Ultimate state of the second type plastic mechanisms

It will be shown below that to define parameter \bar{Q} , it is enough to analyze plastic mechanisms including not more than three load-carrying stringers. At such restriction the system of equations, including the equilibrium equations, equations of ultimate curves, as well as the additional condition for finding bending plastic hinges location in the ordinary frame span, can be written in the general form (notations correspond to the ones accepted Fig. 4.2.1):

$$\begin{aligned} M(x_{\max 1}) &= M_{11} - N_{11}x_{\max} + 0.5p_{u2}S(x_{\max 1} - 0.5(j+1)l + 0.5b)^2 - \\ &\quad - \parallel_{j>1} Q_j(x_{\max 1} - l) = -M_{12} \end{aligned} \quad (4.2.5)$$

$$N_{11} - 0.5 \cdot p_{u2} \cdot b \cdot S + 0.5 \cdot j \cdot Q_j = 0$$

$$\left(\frac{M_{11}}{M_{f1}}\right)^2 + \left(\frac{N_{11}}{N_{f1}}\right)^2 - 1 = 0$$

$$\frac{M_{12}}{M_{f1}} - 1 = 0$$

$$\frac{dM(x_{\max 1})}{dx_{\max 1}} = 0$$

where M_{11} , M_{12} , N_{11} are absolute values of bending moments and share force acting in supporting sections (index «11») and in the span (index «12») (in section $x_{\max 1}$) of the ordinary frame;

$x_{\max 1}$ is a coordinate of a section where the bending plastic hinge occurs;

M_{f1} , N_{f1} are ultimate bending moment and shear force corresponding to actual sizes of the ordinary frame cross section that were defined by the requirements of section 4.1.

Expressions for M_{01} and N_{01} can be presented as follows [2]:

$$M_{01} = \psi_1 \cdot 0.25 \cdot KL_2 \cdot p \cdot S \cdot b \cdot l(1 - 0.5\beta), \quad \psi_1 = \frac{PM_{1f}}{PM_{10} \cdot KL_2} \quad (4.2.6)$$

$$N_{01} = \varphi_1 \cdot 0.5 \cdot KL_2 \cdot p \cdot S \cdot b, \quad \varphi_1 = \frac{PM_{1f}}{SA_{10} \cdot KL_2}$$

where $\beta = b/l$

p is the design pressure.

Using the ultimate strength criterion, basing on (4.2.5), (4.2.6) the following equations for definition of dimensionless reactions \bar{Q}_j can be obtained:

$$0.25(1 - 0.5\beta)\psi_1 \left(1 + \sqrt{1 - \varphi_1^2 (1 - j\bar{Q}_j)^2} \right) - 0.5(1 - j\bar{Q}_j)\bar{x} + \quad (4.2.7)$$

$$0.5(\bar{x} - 0.5(j+1) + 0.5\beta)^2 - \prod_{j>1} \bar{Q}_j (\bar{x} - 1) = 0$$

where $\bar{Q}_j = \frac{Q_j}{KL_2 \cdot p \cdot S \cdot b}$

$$\bar{x} = \frac{x_{\max 1}}{l} = 0.5(j+1) - \bar{Q}_j \beta \left(0.5j - \prod_{j>1} 1 \right)$$

The following approximation of equation (4.2.7) solution is proposed :

$$\bar{Q}_i(\beta, \psi_1, \varphi_1) \approx \bar{Q}_i(\beta, 1, 1) + \Delta \bar{Q}_i + (0.5, \psi_1, \varphi_1) \quad (4.2.8)$$

$$\Delta \bar{Q}_i(0.5, 1, 1) = 0.$$

In compliance with results of the numerical analysis, the first summand of (4.2.8) can be presented as follows:

$$\bar{Q}_j(\beta, 1, 1) = A_j + B_j\beta \quad (4.2.9)$$

where $A_1=0.2$, $A_2=0.269$, $A_3=0.25$

$B_1=0.132$, $B_2=0.11$, $B_3=0.063$

The second summand is specified by the following polynomial

$$\Delta\bar{Q}_j = c_{j3}(\psi_1 - 1) + c_{j4}(\varphi_1 - 1) + c_{j5}(\psi_1\varphi_1 - 1) \quad (4.2.10)$$

where c_{jk} are unknown coefficients ($k=3, 4, 5$).

Then approximation (4.2.8) can be presented as follows:

$$\bar{Q}_j(\beta, \psi_1, \varphi_1) = c_{j1} + c_{j2}\beta + c_{j3}\psi_1 + c_{j4}\varphi_1 + c_{j5}\varphi_1\psi_1 \quad (4.2.11)$$

where $c_{j1}=A_j-c_{j3}-c_{j4}-c_{j5}$, $c_{j2}=B_j$

To generate approximation relationship (4.2.11), it is advantageous to assess the actual ranges of coefficients φ_1 and ψ_1 variation. For this purpose let represent ψ_1 as follows:

$$\psi_1 = \bar{\psi}_1\alpha(\gamma_1) \quad (4.2.12)$$

where $\bar{\psi}_1 = \frac{KL_1}{KL_2} \cdot \frac{PM1_f}{PM1}$ is a coefficient describing increase of the plastic section modulus due to load localization and additional material margins.

Parameters γ_1 or φ_1 and function $\alpha(\gamma_1)$, as it is shown in [1], can vary in wide ranges:

$$1 \leq \varphi_1(\gamma_1) \leq 1.6 \div 1.9, \quad 0.5 \leq \alpha(\gamma_1) \leq 1 \quad (4.2.13)$$

that should be taken into account at generation of relationships (4.2.11). With account of the values of the localization coefficient accepted at this stage, the upper boundary for value $\bar{\psi}_1$ can be presented at the following level:

$$\bar{\psi}_1 \leq 1.5 \quad (4.2.14)$$

In the presented ranges (4.2.13), (4.2.14), relationships (4.2.11) should give reliable results.

Basing on the numerical analysis, the following values of coefficients c_{jk} , entering relationships (4.2.11) were determined (see Table 4.2.1). Reliability of approximation relationships (4.2.11) at variation coefficients φ_1 , $\bar{\psi}_1$, and parameter β in the specified range is confirmed by the results of mass comparison calculations presented in Table 4.2.2. - 4.2.4.

Table 4.2.1 Values of coefficients c_{jk}

j	k				
	1	2	3	4	5
1	0.003	0.132	0.398	0.584	-0.785
2	0.363	0.11	-0.078	0.186	-0.202
3	0.34	0.063	-0.095	0.073	-0.068

Attention should be paid to the fact that at $j = 3$, due to increase of the frame span length, influence of the shear forces in supporting sections is leveled practically. Therefore, for all \bar{Q}_3 instead of (4.11) can be offered a simpler function providing the required level of accuracy:

$$\bar{Q}_3 = 0.5 + 0.125(\psi_1 - 0.5) - 0.25\psi_1 \quad (4.2.15)$$

The value of reaction \bar{Q} , determining the requirements to load-carrying stringer scantlings is found out from the following condition:

$$\bar{Q} = \max(\bar{Q}_1, \dots, \bar{Q}_m) , \quad m \leq 3 \quad (4.2.16)$$

Naturally that at $m = 1$

$$\bar{Q} = \bar{Q}_1 \quad (4.2.17)$$

At $m = 2$ to open relationship (4.2.16), analysis of the values of the following function is carried out:

$$\delta_{Q_2} = \frac{\bar{Q}_2}{\bar{Q}_1} \quad (4.2.18)$$

at parameters ψ_1 and φ_1 variation. The results of this analysis is illustrated in Fig. 4.2.2. by a system of the following plots:

$$\delta_{Q_2} = \delta_{Q_2}(\varphi_1, \psi_1) \quad (4.2.19)$$

where $\psi_1 = \bar{\psi}_1 \cdot \alpha(\varphi_1)$

$$\bar{\psi}_1 = 1.0; 1.1; 1.2; 1.3$$

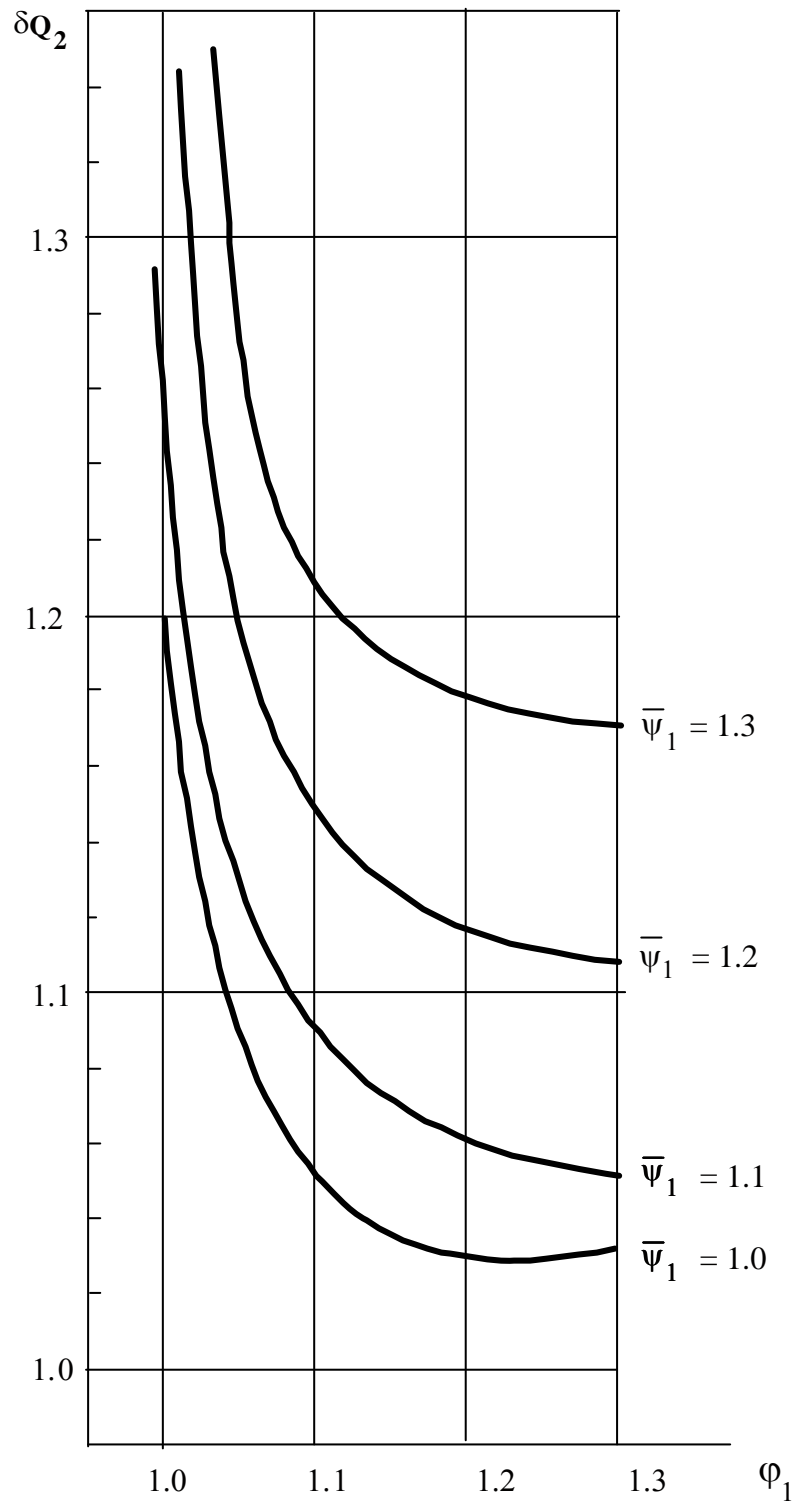


Fig.4.2.2 Plot of function

$$\delta Q_2(\phi_1, \psi_1) \Big|_{\psi_1 = \bar{\psi}_1 \cdot \alpha(\phi_1)}$$

Table 4.2.2 Assessment of an error of function \bar{Q}_1 approximation with the help of relationship (4.2.11)

b	1/j ₁	$\bar{\psi}_1$									
		1.1		1.2		1.3		1.4		1.5	
		accurate value	error (%)	accurate value	error (%)	accurate value	error (%)	accurate value	error (%)	accurate value	error (%)
0.2	0.90	0.335	5.9	0.289	2.4	0.246	1.1	0.206	4.7	0.169	8.1
0.2	0.80	0.379	5.5	0.333	3.2	0.288	0.8	0.245	1.5	0.205	3.9
0.2	0.70	0.407	1.8	0.360	0.7	0.314	0.3	0.270	1.0	0.288	1.4
0.2	0.60	0.426	4.9	0.379	4.5	0.333	3.8	0.288	2.5	0.244	0.2
0.5	0.90	0.368	3.7	0.326	1.3	0.286	1.0	0.248	3.1	0.213	4.7
0.5	0.80	0.411	3.1	0.368	1.7	0.327	0.5	0.288	0.4	0.250	0.9
0.5	0.70	0.437	0.5	0.394	0.7	0.353	0.6	0.312	0.0	0.274	1.2
0.5	0.60	0.455	6.8	0.412	5.6	0.371	3.9	0.330	1.4	0.290	2.2
0.8	0.90	0.409	3.6	0.370	2.4	0.333	1.5	0.298	1.1	0.266	1.3
0.8	0.80	0.449	2.5	0.410	2.2	0.373	2.3	0.338	2.8	0.304	3.9
0.8	0.70	0.473	1.1	0.435	0.4	0.398	0.9	0.362	2.7	0.327	5.3
0.8	0.60	0.490	7.3	0.452	5.1	0.415	2.3	0.379	1.3	0.344	5.9

Table 4.2.3 Assessment of an error of function \bar{Q}_1 approximation with the help of relationship (4.2.11)

b	1/j ₁	$\bar{\Psi}_1$									
		1.1		1.2		1.3		1.4		1.5	
		accurate value	error (%)	accurate value	error (%)	accurate value	error (%)	accurate value	error (%)	accurate value	error (%)
0.2	0.90	0.384	6.4	0.355	4.8	0.328	3.1	0.301	1.5	0.275	0.1
0.2	0.80	0.417	6.5	0.390	5.4	0.364	4.2	0.338	3.0	0.313	1.8
0.2	0.70	0.437	5.0	0.411	4.2	0.386	3.5	0.361	2.8	0.336	2.0
0.2	0.60	0.450	2.0	0.426	1.7	0.401	1.5	0.377	1.2	0.353	1.0
0.5	0.90	0.402	2.6	0.378	1.9	0.354	1.2	0.331	0.6	0.308	0.1
0.5	0.80	0.430	1.9	0.408	1.5	0.385	1.2	0.363	0.9	0.342	0.6
0.5	0.70	0.447	0.1	0.426	0.1	0.404	0.1	0.383	0.0	0.362	0.1
0.5	0.60	0.458	3.3	0.438	2.9	0.417	2.5	0.397	1.9	0.377	1.3
0.8	0.90	0.421	0.7	0.401	0.4	0.382	0.1	0.362	0.3	0.343	0.9
0.8	0.80	0.444	2.3	0.426	1.9	0.408	1.4	0.390	0.8	0.372	0.0
0.8	0.70	0.458	4.8	0.440	4.1	0.423	3.3	0.406	2.3	0.389	1.3
0.8	0.60	0.467	8.4	0.450	7.3	0.434	6.0	0.417	4.7	0.401	3.2

Table 4.2.4 Assessment of an error of function \bar{Q}_1 approximation with the help of relationship (4.2.11)

b	1/j ₁	$\bar{\Psi}_1$									
		1.1		1.2		1.3		1.4		1.5	
		accurate value	error (%)	accurate value	error (%)	accurate value	error (%)	accurate value	error (%)	accurate value	error (%)
0.2	0.90	0.316	4.3	0.301	3.3	0.286	2.2	0.271	1.2	0.256	0.1
0.2	0.80	0.334	4.2	0.320	3.5	0.306	2.8	0.292	2.0	0.278	1.2
0.2	0.70	0.345	3.2	0.331	2.7	0.318	2.2	0.305	1.6	0.292	1.1
0.2	0.60	0.352	1.3	0.339	1.0	0.326	0.7	0.314	0.4	0.301	0.1
0.5	0.90	0.328	2.1	0.315	1.7	0.302	1.4	0.289	1.1	0.276	0.8
0.5	0.80	0.344	1.4	0.332	1.2	0.320	1.0	0.308	0.9	0.296	0.7
0.5	0.70	0.352	0.0	0.341	0.0	0.330	0.0	0.319	0.0	0.308	0.0
0.5	0.60	0.358	2.1	0.348	1.9	0.337	1.7	0.326	1.5	0.316	1.3
0.8	0.90	0.341	0.1	0.330	0.4	0.319	0.8	0.309	1.2	0.298	1.7
0.8	0.80	0.353	1.3	0.343	0.9	0.334	0.4	0.324	0.0	0.314	0.6
0.8	0.70	0.360	3.0	0.351	2.5	0.342	1.9	0.333	1.4	0.324	0.7
0.8	0.60	0.365	5.4	0.356	4.7	0.348	4.0	0.339	3.2	0.330	2.4

The plots of Fig. 4.2.2 permit to identify unambiguously that

$$\text{at } m=2 \quad \bar{Q} = \bar{Q}_2 \quad (4.2.20)$$

Execution of the similar procedure at $m = 3$ (see Fig. 4.2.3) gives

$$\bar{Q} = \bar{Q}_2$$

Therefore, the expression for dimensionless reaction \bar{Q} is written as follows:

$$\bar{Q} = \bar{Q}_j \quad (4.2.21)$$

where j is an index defined in dependence of the stringers number

$$j = m \quad \text{at } m \leq 2$$

$$j = 2 \quad \text{at } m > 2$$

\bar{Q}_j is a function defined with formulae (4.2.11) and Table 4.2.1.

Therefore, it is ascertained that not more than two load carrying stringers are included in the second type plastic mechanisms.

The obtained relationships for reactions \bar{Q}_j permit to assess quantitatively the supporting effect role created by the ordinary frames for the load carrying stringers. At consideration of the load carrying stringer in accordance with the CR approach (see section 3.2) as an isolated beam resisting all the ice load within the web frame spacing, in relationships (4.2.3) $\bar{Q} \approx 1$ should be written for SA2 and PM2. The obtained solution taking into account force interaction of load carrying stringers and ordinary frames gives $\bar{Q} \approx 0.3 \div 0.4$. Correspondingly, the requirements to the load carrying stringers geometric characteristics will be one-second — one-third as much as by the Canadian Rules [2] approach.

The identified important feature of the second type plastic mechanisms ultimate state that permits to justify moderate requirements to the load carrying stringers, will be illustrated below by some quantitative assessments.

4.2.3. Additional restrictions on load carrying stringer scantlings

In the solution obtained as per 4.2.2 for reaction \bar{Q}_j it is supposed that in the second type plastic mechanisms ordinary frames are supported by load carrying stringers. However, in separate cases, when coefficients φ_1 and ψ_1 are considerably greater than one, ultimate strength of the ordinary frames is so high that the frames are capable to resist ice load p_2 without additional support and load carrying stringers. Formally in this cases $\bar{Q}_j \leq 0$ will be follow from the solution of the system of equations (4.2.5) that does not permit to impose the analytic requirements to the load

carrying

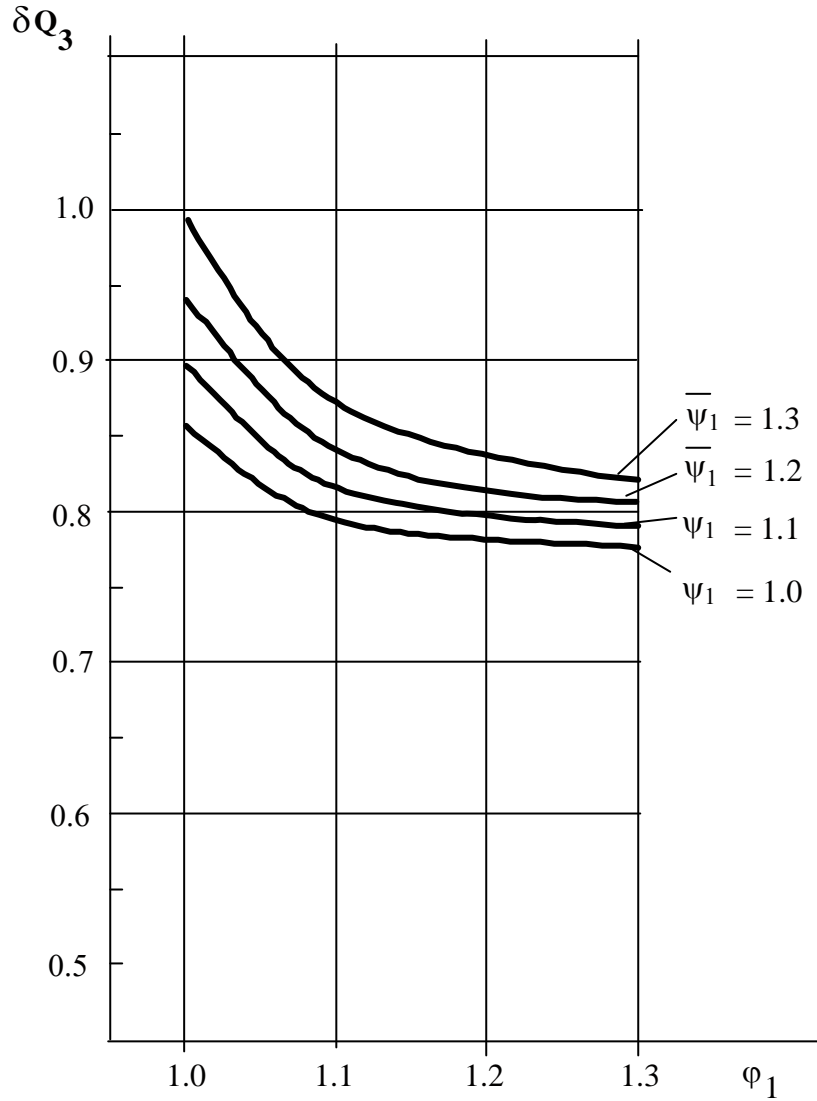


Fig.4.2.3. Plot of function

$$\delta Q_3(\phi_1, \psi_1) \Big|_{\psi_1 = \bar{\psi}_1 \cdot \alpha(\phi_1)}$$

stringer scantlings.

The correct refinement of the solution cited as per 4.2.2 for the mentioned cases consists in examination of the ultimate state of the second type plastic mechanism in which only a part (n_1) of the mostly loaded ordinary frames is involved under varied ice load, and other ($n-n_1$) ordinary frames remain undeformed (Fig. 4.2.4). By this mechanism considering, a minimal values of Q_{\min} can be identified for reactions Q_j , defined by (4.2.5) and entering into analytic functions (4.2.3) for the geometrical characteristics SA2 and PM2. At the same time, however, this results in considerable sophistication of the total analytical relationships. Therefore, in order to simplify the relationships the following can be accepted:

$$n_1=2, \quad p_{v2}=p_1=KL_1p \quad (4.2.22)$$

With account of (4.2.2.2) coefficients φ_1 and ψ_1 , defined before by (4.2.6), the value of reaction Q_{\min} . transferred by the ordinary frames to the load carrying stringer will not depend on coefficients KL_1 , KL_2 . Correspondingly, the range of reaction Q_{\min} variation will decrease in comparison with Q_j . Therefore, for Q_{\min} the following simplified relationship can be accepted with some reserve:

$$Q_{\min}=c \cdot KL_1pSb \quad (4.2.23)$$

where $c \approx 0.5$ is a numerical coefficient.

Basing on (4.2.22) and (4.2.23), relationships (4.2.3) should be added by the corresponding restrictions and to present as follows:

$$SA2=KL_2 \cdot SA2_0 \quad (4.2.24)$$

$$PM2=\alpha(\gamma_2) \cdot KL_2 \cdot PM2_0$$

where $\alpha(\gamma_2) = \frac{1}{1 + \sqrt{1 - 1/\gamma_2^2}}$

$$SA2_0 = \frac{1}{\sqrt{3}} \frac{pSb}{FY} n \cdot \bar{Q}_m$$

$$PM2_0 = \frac{1}{8} \frac{pS^2b}{FY} \cdot n(n+2) \bar{Q}_m$$

$$\bar{Q}_m = \max(\bar{Q}, \bar{Q}_{\min})$$

$$\bar{Q}_{\min} = c \frac{n_1}{n} \cdot \frac{KL_1}{KL_2} = \frac{KL_1}{n \cdot KL_2}$$

\bar{Q} are dimensionless reaction defined with formulae (4.2.2.1).

Alongside with (4.2.23), it is expedient to consider an issue of imposing the HR structural

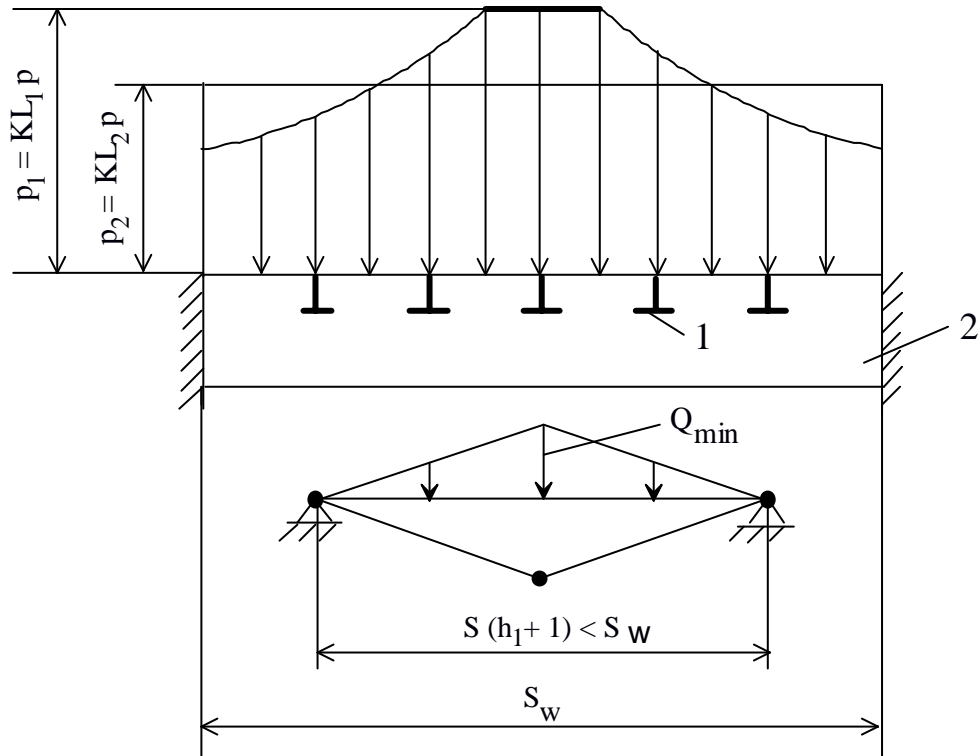


Fig. 4.2.4. Scheme of the second type plastic mechanism when
 only a part of ordinary frames is involved in deforming
 1 - ordinary frames
 2 - load-carrying stringer

requirements to the ordinary frame intersection of a load carrying stringer. In the RR to compensate partial strength losing of the load carrying stringer web plate due to a cutout for running of an ordinary frame, the following requirement is included traditionally: the load carrying cross section height should not be less than the double height of the ordinary frame. Normally, this requirements results in some increase of the actual load carrying stringer geometrical characteristics in comparison to the required ones by (4.2.24). Nevertheless, Russian experience shows that many-years application of this requirement ensures the required level of reliability of ordinary frames and load carrying stringers intersection. Additionally, as it will be shown below, increase of the load carrying stringer actual scantlings grants a possibility to decrease the required web frame scantlings.

4.3. Web frames

4.3.1. Problem formulation

In compliance with the statements presented in section 3.5, the requirements to the geometric characteristics of web frame are defined by consideration of the third type plastic mechanisms describing transitions into the ultimate state of a structure within two web frame spacings under ice load intensity p_3 , distributed uniformly on a strip $2S_w \times b$.

The formulated as per 4.2.1. assumptions on character of load resisting and force interaction of different levels beams as applied to the third type plastic mechanisms, are unchanged for ordinary frames and take the following form for load carrying stringers and web frames:

- each load stringer on the span equal to two web frame spacings is loaded by $2n$ reactions Q transferred from the ordinary frames side and one supporting reaction G applied in a intersection with the web frame;
- a web frame is loaded with ice load p_3S per unit length, as well as by reactions G in the points of intersection with load carrying stringers transmitted into the ultimate state.

As in the case of the second plastic mechanisms, a number of loading stringers (j) involved in plastic mechanism displacements is unknown in advance and is defined from the condition of true ultimate load minimum:

$$p_{u3} = \min(p_{u31}, p_{u32}, \dots, p_{u3j}, \dots, p_{u3m}) \quad (4.3.1)$$

where p_{u3j} is the ultimate load of a plastic with j load carrying stringers.

In accordance with the result obtained as per 4.2.2, in the plastic mechanisms of the second type not more than two load carrying stringers transmit into the ultimate state. Preliminary assessments show that for the plastic mechanisms of the third type the plastic mechanism is the one including all m load carrying stringers. Therefore, condition (4.3.1) takes the following form:

$$p_{u3} = p_{u3m}, \quad m \leq 6 \quad (4.3.2)$$

Correctness of (4.3.2) in the mentioned range of a load carrying stringers number in the grillage ($m \leq 6$) will be demonstrated below on the basis of the total results analysis. In principle deviation

from (4.3.2) is not exceeded in the domain $m > 6$; however, side grillages with such a big number of the load carrying stringers are not met in practice.

4.3.2. Analysis of plastic mechanisms of the third type ultimate state

The analytic model of the third type plastic mechanism (at $m = 3$) is cited in Fig. 4.3.1. As applied to this model, the system of equilibrium equations and ultimate curves will take the following form after substitution of the ultimate strength criterion (3.5.1) into them:

$$p_{u3} = KL_3 \cdot p$$

$$\left. \begin{aligned} M(x_{\max 1}) &= M_{11} - N_{11}x_{\max 1} + 0.5KL_3pS(x_{\max 1} - 0.5(m+1)l + 0.5b)^2 - \\ &\quad - Q_m(y_m x_{\max 1} - z_m \cdot l) = -M_{12} \\ N_{11} - 0.5KL_3pSb + 0.5m \cdot Q_m &= 0 \\ \left(\frac{M_{11}}{M_{f1}}\right)^2 + \left(\frac{N_{11}}{N_{f1}}\right)^2 - 1 &= 0 \\ M_{12} - M_{f1} &= 0 \\ \frac{dM(x_{\max 1})}{dx_{\max 1}} &= 0 \end{aligned} \right\} \quad (4.3.3,a)$$

$$\left. \begin{aligned} M(x_{\max 2}) &= M_{21} - N_{21}x_{\max 2} + \frac{Q'_m}{2S}(x_{\max 2} - 0.5S)^2 = -M_{22} \\ N_{21} + 0.5Gm - Q'_m n &= 0 \\ \left(\frac{M_{21}}{M_{f2}}\right)^2 + \left(\frac{N_{21}}{N_{f2}}\right)^2 - 1 &= 0 \\ M_{22} - M_{f2} &= 0 \\ \frac{dM(x_{\max 2})}{dx_{\max 2}} &= 0 \end{aligned} \right\} \quad (4.3.3,b)$$

$$\left. \begin{aligned} M_{31} = M_{32} - \frac{m+1}{2}lN_{31} + t_m lG_m + \frac{KL_3pSb^2}{8} &= 0 \\ N_{31} - \frac{KL_3pSb}{2} - \frac{m}{2}G_m &= 0 \\ \left(\frac{M_{31}}{M_{f3}}\right)^2 + \left(\frac{N_{31}}{N_{f3}}\right)^2 - 1 &= 0 \\ M_{32} - M_{f3} &= 0 \end{aligned} \right\} \quad (4.3.3,c)$$

where Q'_m are reactions in the points of ordinary frames and load carrying stringers intersection

that

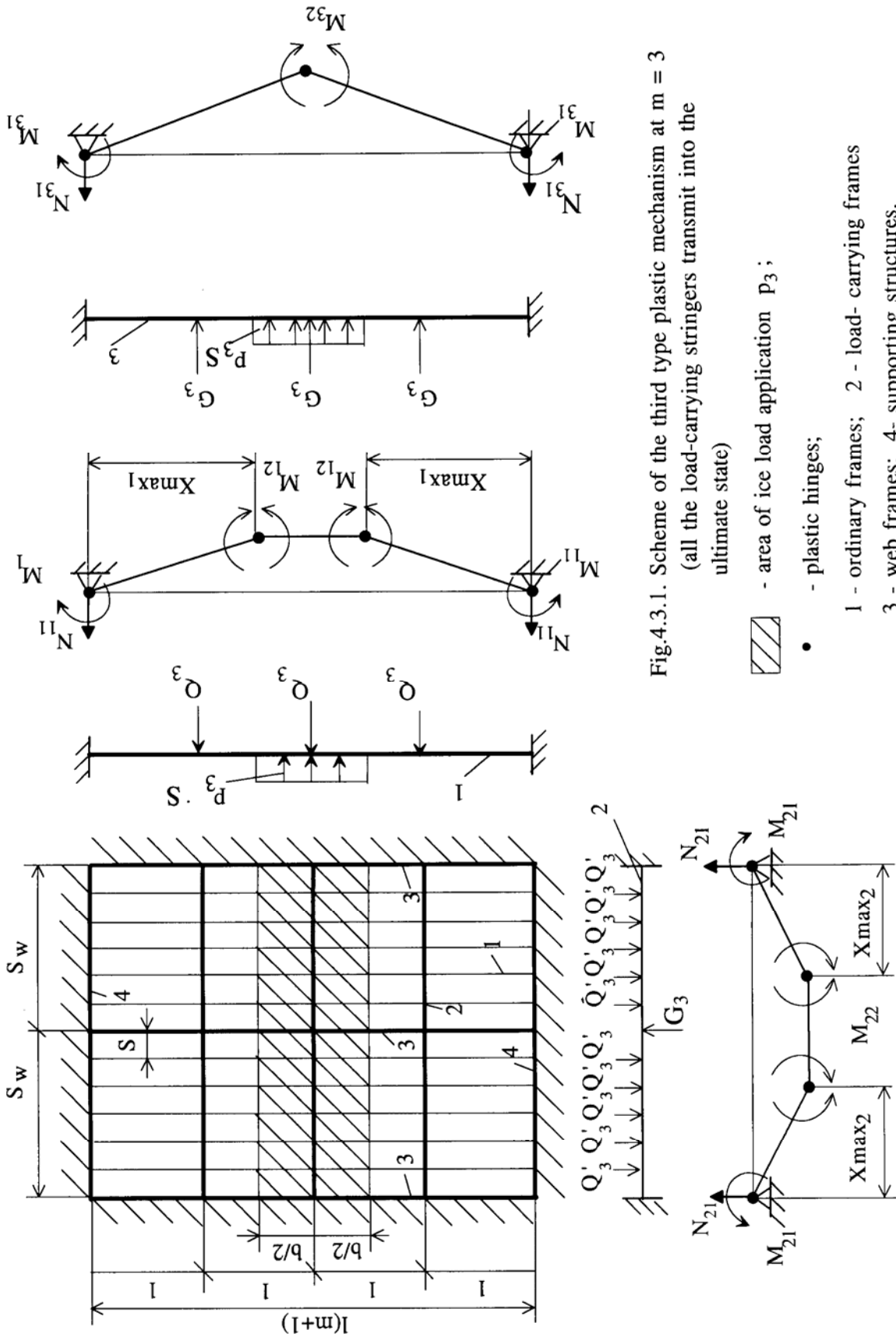


Fig.4.3.1. Scheme of the third type plastic mechanism at $m = 3$
(all the load-carrying stringers transmit into the ultimate state)

- \square - area of ice load application P_3 ;
- \bullet - plastic hinges;
- 1 - ordinary frames; 2 - load-carrying frames
- 3 - web frames; 4 - supporting structures.

occur in the third type plastic mechanisms and, in general case, do not differ from reactions Q_j defined as per 4.2.2;

M_{31} , M_{32} , N_{31} are absolute values of the bending moments and shear forces acting in the supporting section (index "1") and section x_{max2} (index "2");

G_m are reactions of interaction between the load carrying stringers and web frames;

M_{f3} , N_{f3} are ultimate bending moment and shear force in the web frame cross section;

y_m , z_m , t_m are coefficients determined by Table 4.3.1 for grillages with $m \leq 6$.

Table 4.3.1 Values of coefficients y_m , z_m , t_m

m	y_m	z_m	t_m
1	0	0	0
2	1	1	0.5
3	1	1	1
4	2	3	2
5	2	3	3
6	3	6	4.5

The solution of system (4.3.3) is carried out in three sequential stages.

Initially subsystem (4.3. 3, a), is examined. Values M_{f1} and N_{f1} entering in the system are familiar as far as the ordinary frame has been already designed. The difference of the subsystem from system (4.2.5) presented as per 4.2.2 consists in the following:

- plastic mechanisms with a greater number of the load carrying stringers ($m \leq 6$ instead of $j \leq 3$) are considered;
- load $p_3 = KL_3p$ is used instead of $p_2 = KL_2p$.

The derivation of the function for dimensionless reactions \bar{Q}_m in the range $m = 4, 5, 6$ is out of difficulties. Basing on the considerations given as per 4.2.2 at generation, of relationship (4.2.15), the relationship for $m = 4, 5, 6$ can be obtained by the similar way:

$$\bar{Q}_m = c_{m1} + c_{m2}(0.5 \cdot \beta(\psi_1 - 0.5) - \psi_1), \quad m=3, 4, 5, 6 \quad (4.3.4)$$

where c_{mk} are coefficients cited in Table 4.3.2.

Table 4.3.2 Values of coefficients c_{m1} , c_{m2}

m	c_{m1}	c_{m2}
3	0.5	0.25
4	0.417	0.167
5	0.333	0.111
6	0.292	0.083

Taking into consideration the differences in the loads, solution (4.3.3,a) should be presented in the following form:

$$Q'_m = KL_3 p S b \bar{Q}'_m \quad (4.3.5)$$

where $\bar{Q}'_m = c_{m1} + c_{m2}\beta + c_{m3}\psi'_1 + c_{m4}\phi'_1 + c_{m5}\psi'_1\phi'_1$, $m=1, 2$

$\bar{Q}'_m = c_{m1} + c_{m2}(0.5\beta(\psi_1 - 0.5) - \psi_1)$, $m=3, 4, 5, 6$

$$\psi'_1 = \psi_1 \frac{KL_2}{KL_3}$$

$$\phi'_1 = \phi_1 \frac{KL_2}{KL_3}$$

c_{mk} are coefficients defined form Table 4.2.1 (at $m = j = 1, 2$) or form Table 4.3.2 (at $m = 3, 4, 5, 6$)

However, taking into account that the some difference in coefficients KL_2 and KL_3 occur in unconventional cases only (grillages with the increased web frame spacing). In practical calculations it is permissible to assume:

$$\bar{Q}'_m = \bar{Q}_j \quad \text{at } m=j=1, 2 \quad (4.3.6)$$

At the second stage is examined subsystem (4.3,b), where values Q'_m as well as M_{f2} and N_{f2} are considered familiar as far as the load carrying stringer has been already designed. For future analysis, the expressions for as M_{f2} and N_{f2} are convenient to transform to the following form with the help of relationships (4.2.3), (4.3.5), (4.3.6):

$$M_{f2} = \psi_2 \frac{\bar{Q}'_m n(n+2)}{8} KL_3 p S^2 b, \quad \psi_2 = \eta_m \frac{PM2_f}{PM2_0 KL_3} \quad (4.3.7)$$

$$N_{f2} = \phi_2 \frac{\bar{Q}'_m n}{2} KL_3 p S b, \quad \phi_2 = \eta_m \frac{SA2_f}{SA2_0 KL_3}$$

$$\text{where } \eta_m = \frac{\bar{Q}_m}{Q_m} = \begin{cases} \frac{\bar{Q}_m}{Q_m} \approx 1 & \text{at } m = 1,2 \\ \frac{\bar{Q}_2}{Q_m} \approx \frac{\bar{Q}_2}{Q_m} & \text{at } m = 3,4,5,6 \end{cases}$$

PM2_f, SM2_f are actual plastic section modulus and web plate area of the load carrying stringer cross section.

By substituting relationships (4.3.5), (4.3.7) into subsystem (4.3.3,b), the following solution relatively the unknown reaction of interaction in the points of load carrying stringers intersection with the web frame can be obtained:

$$G_m = KL_3 p S b \cdot \bar{G}_m \quad (4.3.8)$$

where $\bar{G}_m = 2n\bar{Q}_m(1 - \bar{R})$

$\bar{R} = \frac{N_{21}}{nQ_m}$ is a dimensionless parameter defined form solution of the following equation:

$$\frac{n+2}{4n} \psi_2 \left(1 + \sqrt{1 - \left(\frac{2\bar{R}}{\varphi_2} \right)^2} \right) - \bar{R} \left(\bar{R} + \frac{1}{n} \right) = 0 \quad (4.3.9)$$

Basing on the numerical analysis the following approximate solution of equation (4.3.9) can be derived:

$$\bar{R} = 0.5 \begin{cases} \sqrt{2\psi_2 - (\psi_2 / \varphi_2)^2} & \text{at } \psi_2 < \varphi_2^2 \\ \varphi_2 & \text{at } \psi_2 \geq \varphi_2^2 \end{cases} \quad (4.3.10)$$

Reliability of (4.3.10) in the actual domain of parameter n, φ_2 variation as well as parameter $\bar{\psi}_2$ introducing by the analogy with (4.2.12)

$$\bar{\psi}_2 = \frac{KL_2}{KL_3} \frac{PM2_f}{PM2} \quad (4.3.11)$$

is confirmed by the results of the comparing computations cited in Table 4.3.3.

The solution obtained for reaction G_m is used at the last stage of global system (4.3.3) analysis that is definition of the required values of the web frame cross section geometric characteristics by solving subsystem (4.3.3,c). Substitution of expressions (4.3.8), (4.3.10) into (4.3.3,c) gives the following after some manipulations:

Table 4.3.3 Assessment of error of function R approximation \bar{R}

n	1/j ₂	$\bar{\psi}_2$									
		1.0		1.2		1.4		1.6		1.8	
		accurate value	error (%)	accurate value	error (%)	accurate value	error (%)	accurate value	error (%)	accurate value	error (%)
3	1.00	0.5000	0.00	0.5000	0.00	0.5000	0.00	0.5000	0.00	0.5000	0.00
3	0.90	0.5000	0.00	0.5285	0.52	0.5462	0.56	0.5545	0.29	0.5546	0.16
3	0.80	0.5000	0.00	0.5392	1.00	0.5697	1.49	0.5927	1.62	0.6091	1.49
3	0.70	0.5000	0.00	0.5458	1.34	0.5843	2.18	0.6164	2.67	0.6431	2.90
3	0.60	0.5000	0.00	0.5504	1.59	0.5943	2.69	0.6328	3.44	0.6667	3.95
5	1.00	0.5000	0.00	0.5000	0.00	0.5000	0.00	0.5000	0.00	0.5000	0.00
5	0.90	0.5000	0.00	0.5277	0.36	0.5453	0.40	0.5541	0.22	0.5550	0.10
5	0.80	0.5000	0.00	0.5375	0.68	0.5671	1.03	0.5898	1.14	0.6064	1.06
5	0.70	0.5000	0.00	0.5435	0.91	0.5802	1.49	0.6112	1.83	0.6373	2.01
5	0.60	0.5000	0.00	0.5475	1.08	0.5891	1.82	0.6257	2.34	0.6581	2.70

$$SA3 = \frac{\sqrt{3}}{2} \frac{pSb}{FY} KL_3 (1 + m\bar{G}_m)$$

$$PM3 = 0.25 \frac{pSbl_w}{FY} \left(1 - \frac{b}{l_w} + k_m \cdot \bar{G}_m \right) \cdot \alpha(\gamma_3) \quad (4.3.12)$$

where $k_m = \frac{m^2 + m - 4t_m}{m + 1}$

$$\alpha(\gamma_3) = \frac{1}{1 + \sqrt{1 - 1/\gamma_3^2}}$$

$$\gamma_3 = \frac{SA3_f}{SA3}$$

l_w is the web frame span equal to the distance between its supporting sections on the supporting structures;

$SA3_f$ is the actual web plate area of the web frame cross section.

Basing on relationships (4.3.12), it is easy to prove the reliability of the assumption accepted as per 4.3.1 consisting in substitution of the global condition of the ultimate load minimum (4.3.1) to the simplified relationship (4.3.2).

By (4.3.8) dimensionless reactions \bar{Q}_m entering relationships (4.3.12) are connected with \bar{Q}_m linearly. Considering \bar{Q}_m , defined by (3.4.5), as functions of m , it can be concluded that the maximum of relationship $\bar{Q}_m(m)$ takes place at $m = 2$, and the function slow decrease takes place at $m > 2$ (see Fig. 4.3.2,a).

Therefore, the web frame cross section geometric characteristics in the range $m \leq 6$ are increasing functions of m (Fig. 4.3.2,b); i. e. the following conditions are satisfied:

$$\max(\overline{SA3}_j, j = 1, \dots, m) = \overline{SA3}_m$$

$$\max(\overline{PM3}_j, j = 1, \dots, m) = \overline{PM3}_m \quad (4.3.13)$$

where $\overline{SA3}_j = 1 + j\bar{G}_j$

$$\overline{PM3}_j = 1 + j - 0.5 \frac{b}{l_w} + (j^2 + j - 4t_j)\bar{G}_j$$

As far as the problem of geometric characteristics definition by the ultimate strength criterion is reverse to the problem of ultimate load determination, it is easy to show that satisfaction of

(4.3.13)

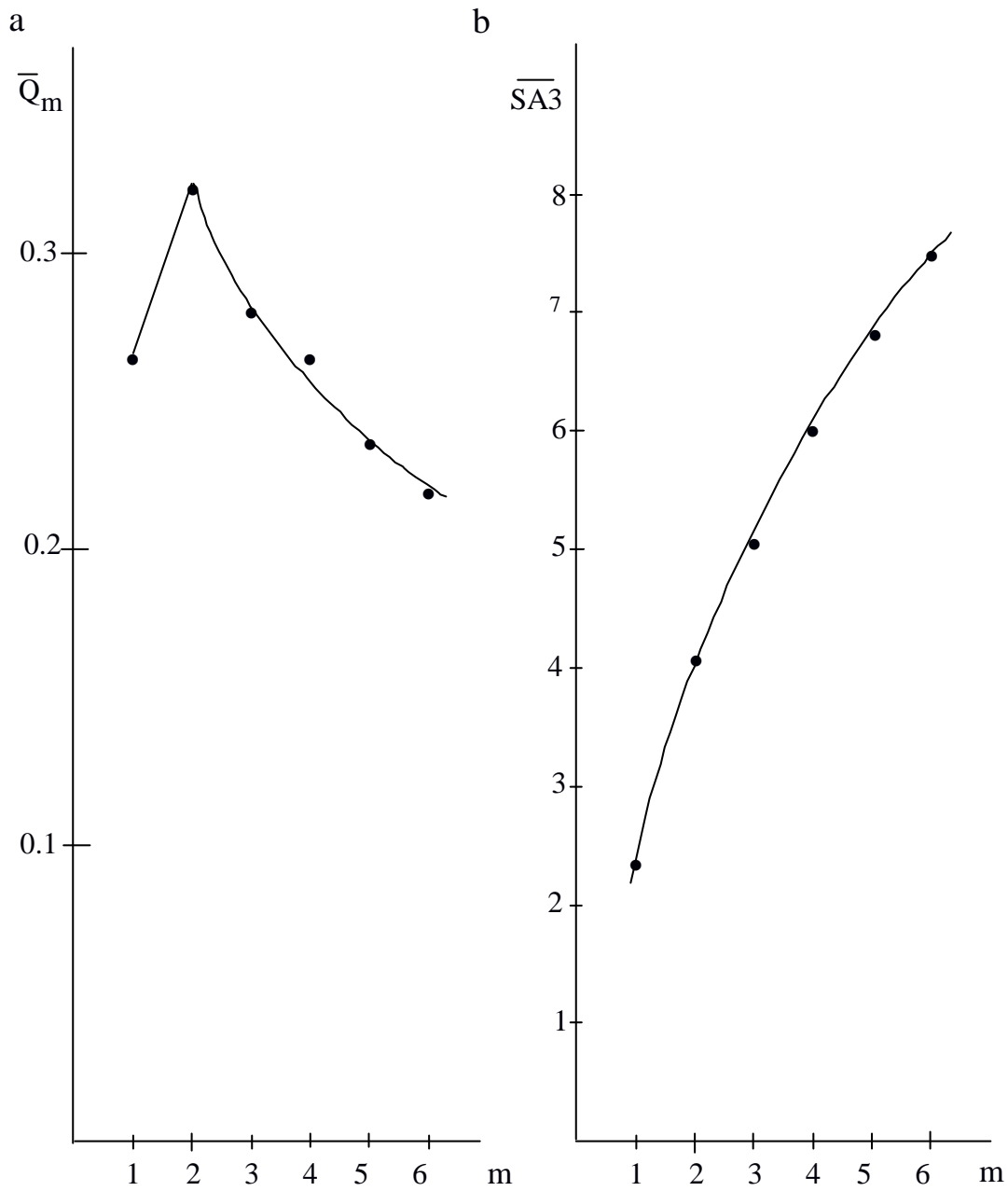


Fig.4.3.2 Plots of functions

$$a - \bar{Q}_m = \bar{Q}_m(m)$$

$$b - \overline{SA3}(m) = 1 + m \bar{G}_m$$

proves validity of substitution of condition (4.3.1) at defining the true (minimum) ultimate load to the simplified condition (4.3.2).

Concluding the executed analysis, it is expedient to note that the obtained relationships demonstrate rather appreciable difference of the ultimate state of the third type plastic mechanisms from the model of an isolated beam accepted in the Canadian Rules [10]. In particular, if in the web frame isolated beam model decrease of web frame scantlings occurs with increasing the load carrying stringers number and with corresponding decreasing the load carrying stringers spacing, the reverse picture is observed in the obtained relationships.

5. CORROBORATION OF THE OBTAINED REGULATION RELATIONSHIPS RELIABILITY

To corroborate the reliability of the results that can be obtained with the help of the regulation relationships derived in chapter 4, a special analysis based on grillages direct calculations by FEM in the area of the material plastic deforming has been carried out. With this purpose were selected three grillages the structural scheme of which corresponded to those utilized in Canada (Fig. 5.1, 5.2) and in Russia (Fig. 5.3); therefore, the performed evaluation involved all the range of actually possible structural schemes. The grillage frame scantlings were defined through the calculations by the formulae of section 4; therefore it was assumed that the design load for which the grillage was designed, was its ultimate load (p_{ul}).

The essence of the verification consisted in the following: the ultimate load p_{ul} value should fall on the kink of the curve «load-deflection» that corresponded to the main idea of the ultimate strength criterion: the ultimate load is a boundary exceeding of which results in quick growth of permanent sets. The FEA was performed with the help of software ANSYS by Canadian [11] and Russian [12] experts. The results of the nonlinear analysis [11], [12] presented in Fig. 5.1 - 5.3 confirm completely the reliability of the developed methodology for side grillages design. Design (ultimate) load action induces occurrence of moderate permanent plastic strains in the grillages (the maximum permanent set is about 20 - 30 mm) that are not required immediate removal and do not decrease the ship serviceability. At the same time, the grillages keep the ability for further resisting the growing load.

Further the similar data were obtained by experts of Lloyd's Register [13] utilizing ABAQUS finite element program for the nonlinear analysis.

Therefore, the FEA results corroborate the possibility of utilization of the formulae obtained in section 4 for regulating the web frame scantlings for Polar ship ice strengthening structures.

6. CONCLUSION

The methodology based on the ultimate strength criterion has been developed for regulating the web frame scantlings for Polar ship ice strengthening grillages. The main merits of the accepted regulation system in comparison with the simpler schemes are as follows:

- the possibility to consider the actual strength reserve capacity of the lower hierarchical level frames at designing;

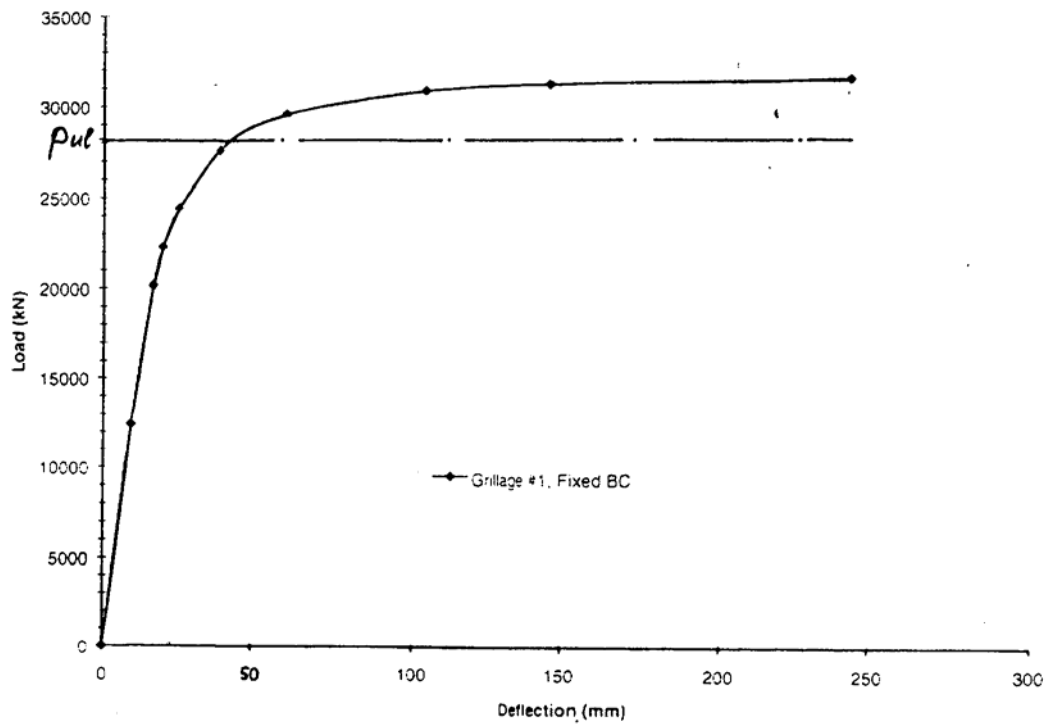
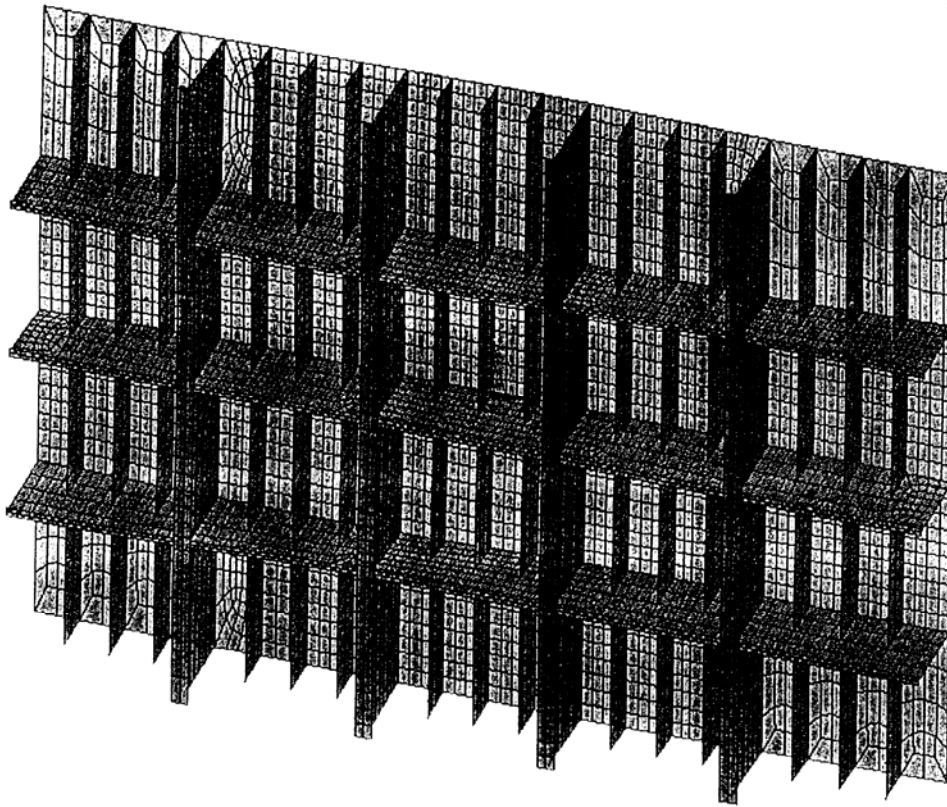


Fig.5.1 Grillage #1, Reserve Capacity: Load v/s Deflection Results

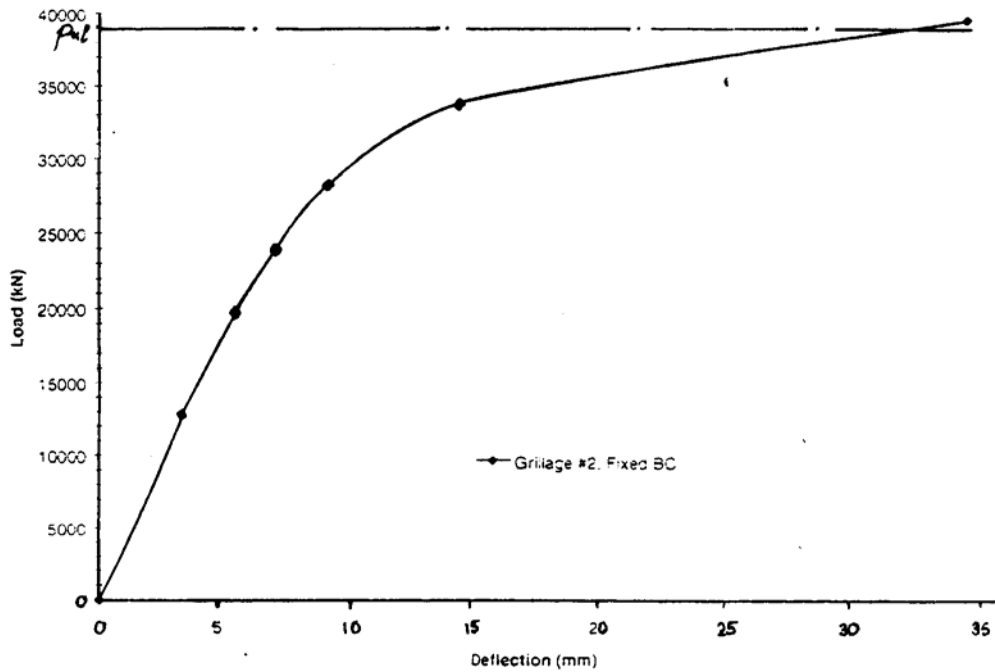
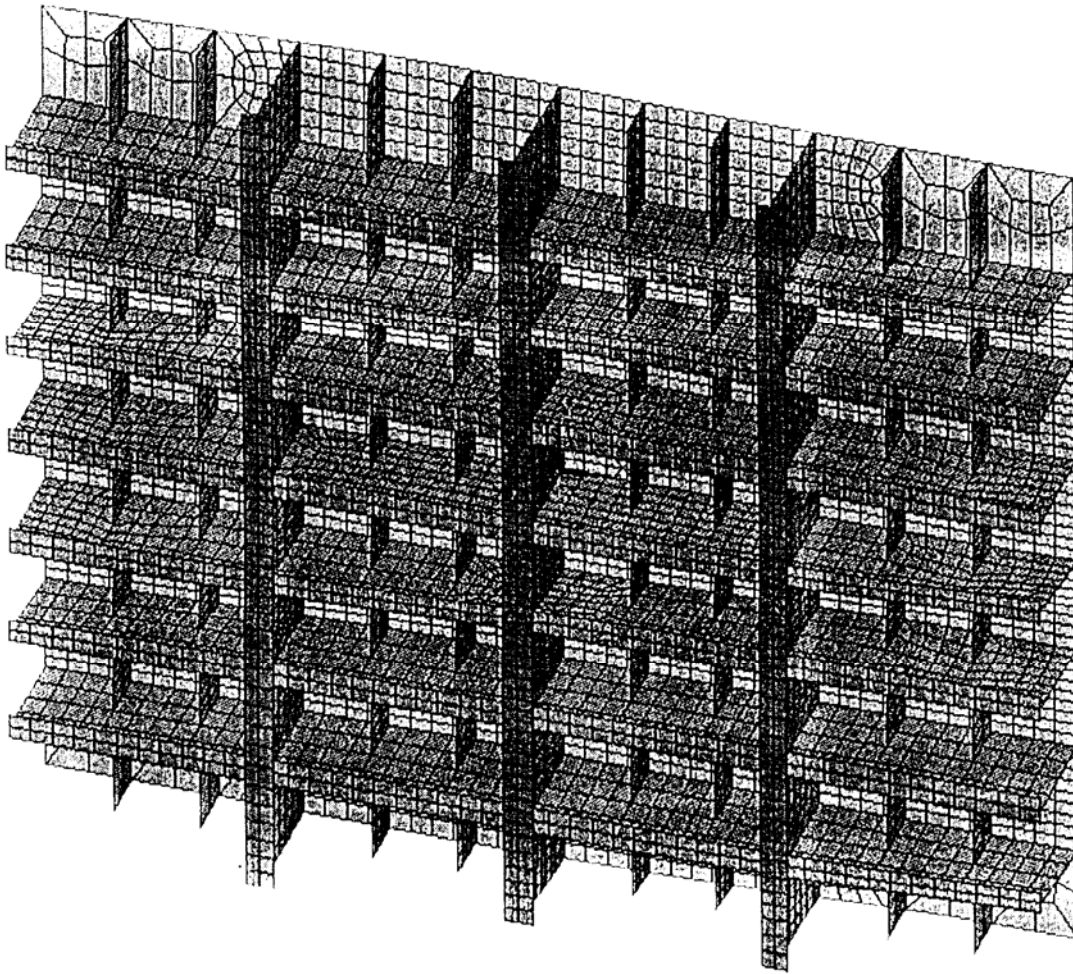
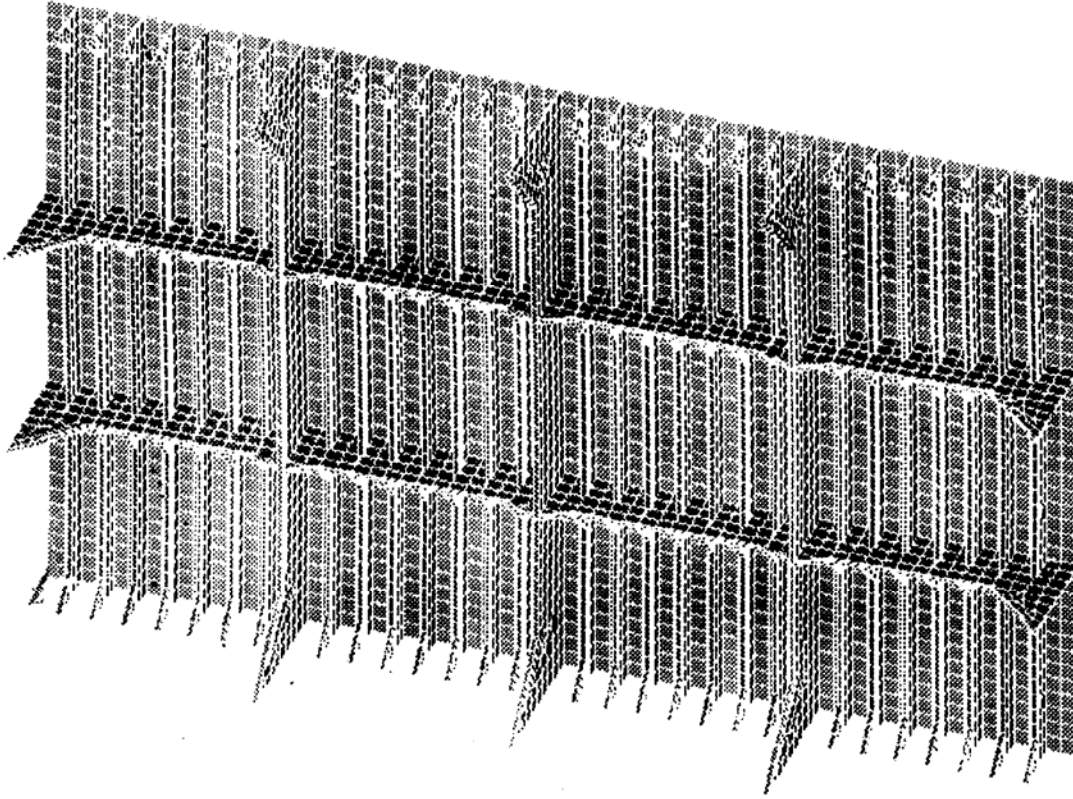


Fig.5.2 Grillage #2, Reserve Capacity: Load v/s Deflection Results



General view of the finite element model of the grillage.

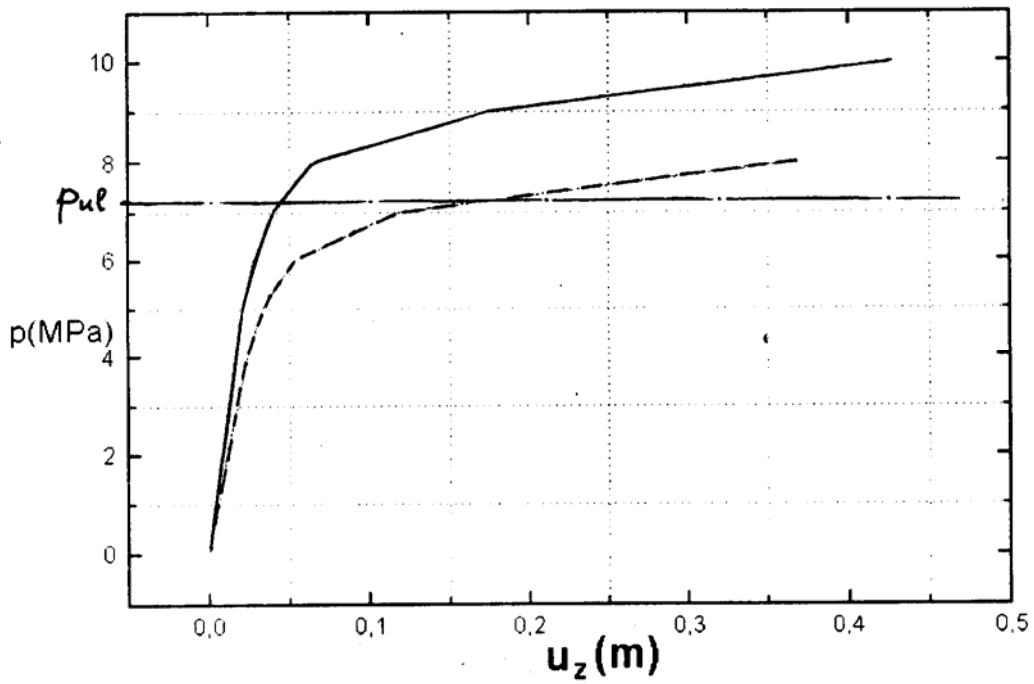


Fig.5.3 Diagram pressure-maximal deflection for the grillage with free vertical deflection on the upper boundary. Black line is the basic case.

the adaptable design procedure permitting to increase the grillage ultimate strength due to the frames scantling, increase of which is of the most structural expediency;

- correct taking into account the sophisticated character of grillage frames interaction and the ice load pattern peakness.

The performed comparison with the direct FEA has corroborated the possibility to apply the developed methodology in IACS UR for Polar ships.

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