# Oblique Ice Collision Loads On Ships Based On Energy Methods 

C. G. Daley ${ }^{1}$<br>${ }^{1}$ Faculty of Engineering and Applied Science, Memorial University, St. John's, Canada, AlB $3 X 5$<br>email: cdaley@engr.mun.ca


#### Abstract

A new IACS (International Association of Classification Societies) standard for Polar Ship design, in the form of a Unified Requirement is being developed by an international working group with representatives from many classification societies and with the active participation of many polar nations. The new standard bases the level of structural strength on the design ice collision loads for the particular class. This paper describes the underlying methods for determining the ice collision loads. The methods are founded on energy principles, which are robust and result in analytical expressions. This approach is well suited for developing design standards.


| NOMENCLATURE |  |
| :---: | :---: |
| $a_{n}$ | normal acceleration of collision point |
| $b$ | height of design load patch |
| ex | exponent in pressure/area equation |
| fa | hull shape factor |
| $i, j, k$ | Cartesian unit vectors |
| ka | angle factor |
| $l, m, n$ | direction cosines |
| $p$ | design patch pressure |
| $r x, r y, r z$ | radii of gyration for ship |
| w | length of design load patch |
| $x, y, z$ | Cartesian coordinates |
| $x_{p}, y_{p}, z_{p}$ | coordinates of collision point |
| A | area |
| AMpit, | added mass factor for pitch |
| AMrol, | added mass factor for roll |
| AMyaw | added mass factor for yaw |
| $A M x$, | added mass factors for surge |
| $A M y$, | added mass factors for sway |
| AMz | added mass factors for heave |
| AR | design patch aspect ratio |
| $B$ | beam of ship |
| Cb | block coefficient |
| $C_{C_{C}}$ | class factor |
| $C F_{D}$ | class factor |
| Cm | midbody section coefficient |


| Co | mass reduction factor |
| :--- | :--- |
| $C w p$ | waterplane coefficient |
| $E_{\text {crush }}$ | energy expended to crush the ice edge |
| $F_{n}$ | normal force, |
| $H$ | load patch height, height of ship |
| $H_{\text {nom }}$ | nominal load patch height |
| $K E_{n}$ | normal kinetic energy of the ship |
| $L$ | ship length |
| $M_{e}$ | effective mass at collision point |
| $M_{\text {ship }}$ | mass of ship |
| $P$ | pressure on an area |
| $P o$ | pressure on $1 \mathrm{~m}^{2}$ |
| $Q$ | design line load |
| $T$ | ship draft |
| $V_{n}$ | normal velocity at collision point |
| $V_{\text {ship }}$ | ship velocity |
| $W$ | load height of triangular patch |
| $W_{\text {non }}$ | nominal load height of rectangular patch |
| $\alpha$ | waterline angle |
| $\beta$ | true frame angle <br> $\beta$ |
| frame angle |  |
| $\delta$ | ice edge penetration |
| $\delta_{m}$ | maximum ice edge penetration |
| $\phi$ | ice edge opening angle |
| $\gamma$ | stem angle |
| $\eta 1$ | collision point moment arm for roll |
| $\lambda 1$ | collision point moment arm for pitch |
| $\mu 1$ | collision point moment arm for yaw |

## 1. INTRODUCTION

Design is the process of specifying capability to satisfy anticipated demands. When designing ships for operation in ice, it is necessary to anticipate the extent of the ice forces on the hull. This paper describes a method to calculate ice loads on the bow and shoulders of a ship as it strikes an ice edge. The ice may be the edge of a track created by an icebreaker, or may be the edge of a large ice floe. This approach was developed as part of the development of the new Unified Requirements (UR) for Polar Ships, developed by the International Association of Classification Societies (IACS) [IACS, 2001]. The ice load concepts for the UR are described in [Daley,2000]. The structural requirements for the UR which make use of the ice load are described in [Daley, Kendrick and Appolonov, 2001]

## 2. ICE COLLISION FORCE

In the following material, the force that results from a ship striking the ice edge is derived. The ice is assumed massive. Flexural failure is treated separately. The mechanics are based on the Popov collision model [Popov et.al. 1969] but are modified to include a wedge shaped ice edge and a pressure/area ice indentation model. Popov assumed a round ice edge, but a wedge shaped edge is more representative of most ice features. A general discussion of energy based collision forces, with a set of solutions for a variety of ice and structure geometry is described in [Daley 1999].

The force is found by equating the normal kinetic energy with the ice crushing energy,

$$
\begin{equation*}
K E_{n}=E_{\text {crush }} \tag{1}
\end{equation*}
$$

The crushing energy is found by integrating the normal force over the penetration depth,

$$
\begin{equation*}
E_{\text {crush }}=\int_{0}^{\delta} F_{n}(\delta) \cdot d \delta \tag{2}
\end{equation*}
$$

The normal kinetic energy combines the normal velocity with the effective mass (see the Annex for calculation of the effective mass) at the collision point,

$$
\begin{equation*}
K E_{n}=\frac{1}{2} M_{e} \cdot V_{n}^{2} \tag{3}
\end{equation*}
$$

combining these two terms gives

$$
\begin{equation*}
\frac{1}{2} M_{e} \cdot V_{n}^{2}=\int_{0}^{\delta m} F_{n}(\delta) \cdot d \delta \tag{4}
\end{equation*}
$$

where

$$
\begin{aligned}
& \delta=\text { normal ice penetration } \\
& \begin{aligned}
F_{n} & =\text { normal force } \\
M_{e} & =\text { effective mass } \\
& =M_{\text {ship }} / C o
\end{aligned} \\
& C o=\text { mass reduction coefficient (see Annex) } \\
& \begin{aligned}
V_{n} & =\text { normal velocity } \\
& =V_{\text {ship }} l
\end{aligned} \\
& l=\text { direction cosine }
\end{aligned}
$$

The ice penetration geometry together with the pressure-area relationship is the basis of finding the force. The nominal area is found for a penetration $\delta$ (see Figure 1).


Figure 1 Nominal contact geometry during oblique collision with an ice edge.

The nominal contact area is

$$
\begin{equation*}
A=W / 2 \times H \tag{5}
\end{equation*}
$$

The width $(W)$ and height $(H)$ of the nominal contact area can be determined by the normal penetration depth $(\delta)$ along with the normal frame angle $\left(\beta^{\prime}\right)$ and the ice edge angle ( $\phi$ ),

$$
\begin{align*}
& W=2 \delta \tan (\phi / 2) / \cos \left(\beta^{\prime}\right)  \tag{6}\\
& H=\delta /\left(\sin \left(\beta^{\prime}\right) \cos \left(\beta^{\prime}\right)\right) \tag{7}
\end{align*}
$$

Hence the area is

$$
\begin{equation*}
A=\delta^{2} \tan (\phi / 2) /\left(\cos ^{2}\left(\beta^{\prime}\right) \sin \left(\beta^{\prime}\right)\right) \tag{8}
\end{equation*}
$$

This simple relationship assumes that the ship side is a flat surface, which is adequate for the limited extent of the contact zone. The average pressure is found from the pressurearea relationship (see [Sanderson 1988] for a general discussion of pressure-area concepts, and [Daley 1994] for a review of ice indentation data and pressure-area effects.);

$$
\begin{equation*}
P=P o A^{e x} \tag{9}
\end{equation*}
$$

The normal force is

$$
\begin{equation*}
F_{n}(\delta)=P A=P o A^{1+e x} \tag{10}
\end{equation*}
$$

Substituting the expression for area (8) gives

$$
\begin{align*}
F_{n}(\delta) & =\operatorname{Po}\left(\delta^{2} \tan (\phi / 2) /\left(\cos ^{2}\left(\beta^{\prime}\right) \sin \left(\beta^{\prime}\right)\right)\right)^{1+e x}  \tag{11}\\
& =\text { Po } k a^{1+e x} \delta^{2+2 e x} \tag{12}
\end{align*}
$$

where we define the angle factor $k a$ as

$$
\begin{equation*}
k a=\tan (\phi / 2) /\left(\cos ^{2}\left(\beta^{\prime}\right) \sin \left(\beta^{\prime}\right)\right) \tag{13}
\end{equation*}
$$

We can now solve the energy balance equation ((12) into (4)) to find the maximum penetration,

$$
\begin{equation*}
\frac{1}{2} M_{e} \cdot V_{n}^{2}=P o \cdot k a^{1+e x} \int_{0}^{\delta m} \delta^{2+2 \cdot e x} \cdot d \delta \tag{14}
\end{equation*}
$$

We can extract the maximum penetration,

$$
\begin{equation*}
\delta_{m}=\left(1 / 2 M V_{n}^{2}(3+2 e x) /\left(\text { Po ka }^{1+e x}\right)\right)^{1 /(3+2 e x)} \tag{15}
\end{equation*}
$$

This is substituted into the expression for force, (12), to give

$$
\begin{equation*}
F_{n}=P o k a^{l+e x}\left(1 / 2 M V_{n}^{2}(3+2 e x) /\left(\text { Po ka }^{l+e x}\right)\right)^{(2+2 e x) /(3+2 e x)} \tag{16}
\end{equation*}
$$

This can be somewhat simplified to give

$$
\begin{equation*}
F_{n}=P o^{1 /(3+2 e x)} k a^{(1+e x) /(3+2 e x)}\left(1 / 2 M V_{n}^{2}(3+2 e x)\right)^{(2+2 e x) /(3+2 e x)} \tag{17}
\end{equation*}
$$

Substituting for $M_{e}$ and $V_{n}$, we get
$F_{n}=P o^{l /(3+2 e x)} k a^{(1+e x) /(3+2 e x)}\left(l^{2} /(2 C o)\right)^{(2+2 e x) /(3+2 e x)}\left(M_{\text {ship }} V_{\text {ship }}{ }^{2}(3+2 e x)\right)^{(2+2 e x) /(3+2 e x)}$

We can collect all shape related terms (comprising $k a$ and the terms with $C o$ and $l$ ) into a single term $f a$,

$$
\begin{equation*}
f a=(3+2 \cdot e x)^{\frac{2+2 \cdot e x}{3+2 \cdot e x}} \cdot\left(\frac{\tan (\phi / 2)}{\sin \left(\beta^{\prime}\right) \cdot \cos ^{2}\left(\beta^{\prime}\right)}\right)^{\frac{1+e x}{3+2 \cdot e x}} \cdot\left(\frac{1}{2 \cdot C o} \cdot l^{2}\right)^{\frac{2+2 \cdot e x}{3+2 \cdot e x}} \tag{19}
\end{equation*}
$$

With $f a$, we can write the force equation as

$$
\begin{equation*}
F_{n}=f a \cdot P o^{\frac{1}{3+2 \cdot e x}} \cdot V_{\text {ship }}{ }^{\frac{4+4 e x}{3+2 \cdot e x}} \cdot M_{\text {ship }} \frac{2+2 \cdot \cdot e x}{3+2 \cdot e x} \tag{20}
\end{equation*}
$$

Which, for $e x=-0.1$ (as used in the UR, see [Daley 2000]) gives;

$$
\begin{equation*}
F_{n}=\text { fa Po }^{0.36} V_{\text {ship }}{ }^{1.28} M_{\text {ship }}{ }^{0.64} \tag{21}
\end{equation*}
$$

Equation (21) represents only the crushing force. The flexural failure force must also be included in the design force, as a limit on the collision force. Refer to [Daley, 2000] and [Daley, Kendrick and Appolonov, 2001] for material on the flexural force.

## 3. ICE LOAD PATCH AND PRESSURES

The ice load patch is found from $F_{n}$. Using (20) and (10), we can solve for the nominal contact area,

$$
\begin{equation*}
A=\left(\frac{F_{n}}{P o}\right)^{\frac{1}{1+e x}} \tag{22}
\end{equation*}
$$

At this point, we introduce a change in load patch shape from triangular to rectangular. This is done to keep the design process manageably simple. We will assume that the load patch is $H_{\text {nom }} \times W_{\text {nom }}$, with an area $A$. The aspect ratio $A R$ (which is $W_{\text {nom }} / H_{\text {nom }}$ ) is

$$
\begin{align*}
A R & =2 \tan (\phi / 2) \sin \left(\beta^{\prime}\right) \\
& =7.46 \sin \left(\beta^{\prime}\right) \quad[\text { assumes } \phi=150 \mathrm{deg}] \tag{23}
\end{align*}
$$

Therefore, we can write;

$$
\begin{equation*}
A=H_{\text {nom }} H_{\text {nom }} A R \tag{24}
\end{equation*}
$$

and using (22) we can describe the dimensions of the nominal load patch in terms of the force;

$$
\begin{gather*}
H_{\text {nom }}=\left(\frac{F_{n}}{P o \cdot A R^{1+e x}}\right)^{\frac{1}{2+2 e x}}  \tag{25}\\
W_{\text {nom }}=\left(\frac{F_{n}}{P o \cdot A R^{1+e x}}\right)^{\frac{1}{2+2 e x}} \cdot A R \tag{26}
\end{gather*}
$$

At this point, we introduce a reduction in the size of the load patch (see Figure 2). This reduction is done to account for the concentration of force that takes place as ice edges spall off, reducing the size of the contact patch. When the patch size is reduced, the force is unchanged, so the design pressure rises correspondingly. The rule (or design) patch length w is;

$$
\begin{equation*}
w=W_{n o m}{ }^{\text {wex }}=F_{n}{ }^{\text {wex }(2+2 e x)} P o^{-w e x /(2+2 e x)} A R^{\text {wex } / 2} \tag{27}
\end{equation*}
$$

where, with wex $=0.7$ and ex $=-0.1$ (values used in the IACS UR, see Daley 2000), we have;

$$
\begin{equation*}
w=F n^{0.389} \text { Po }^{-0.389} A r^{0.35} \tag{28}
\end{equation*}
$$

The design load height is;

$$
\begin{equation*}
b=\frac{w}{A R} \tag{29}
\end{equation*}
$$

or

$$
\begin{equation*}
b=F n^{0.389} P o^{-0.389} A R^{-0.65} \tag{30}
\end{equation*}
$$

The nominal and design load patches have the same aspect ratio. The load quantities used in the scantling calculations include the line load;

$$
\begin{equation*}
Q=F_{n} / w \tag{31}
\end{equation*}
$$

and the pressure,

$$
\begin{equation*}
p=Q / b \tag{32}
\end{equation*}
$$

We can solve for $Q$ and $p$ by using (20) and (22-30). The line load becomes;

$$
\begin{equation*}
Q=\frac{F_{n}^{1-\frac{w e x}{2+2 e x}} \cdot P o^{\frac{w e x}{2+2 e x}}}{A R^{w e x / 2}} \tag{33}
\end{equation*}
$$

The pressure is;

$$
\begin{equation*}
p=\frac{F_{n}^{1-\frac{w e x}{1+e x}} \cdot P o^{\frac{w e x}{1+e x}}}{A R^{w e x-1}} \tag{34}
\end{equation*}
$$

For the rule formula we use $e x=-0.1$, and wex $=0.7$. This gives;

$$
\begin{equation*}
Q=F_{n}^{0.611} P o^{.389} A R^{-0.35} \tag{35}
\end{equation*}
$$

and

$$
\begin{equation*}
p=F_{n}^{0.222} P o^{.778} A R^{0.3} \tag{36}
\end{equation*}
$$

## 4. CLASS FACTORS

All ice class rules specify various levels of ice strengthening according to ice classes. The ice class is intended to reflect the severity of the ice conditions. The class factors include ice thickness and strength parameters, as well as vessel speed. The following class factors are to be found in the IACS UR:

$$
\begin{array}{lc}
\text { Crushing class factor } & C F_{C}=P o^{0.36} V_{\text {ship }}^{1.28} \\
\text { Patch class factor } & C F_{D}=P o^{.389}
\end{array}
$$

With these class factors, we can express the force (eqn 21) as;

$$
\begin{equation*}
F_{n}=f a C F_{C} M_{\text {ship }}^{0.64} \tag{39}
\end{equation*}
$$

The line load (eqn 35) and pressure (eqn 36) become;

$$
\begin{align*}
& \mathrm{Q}=F_{n}^{0.611} C F_{D} A R^{-0.35}  \tag{40}\\
& p=F_{n}^{0.222} C F_{D}{ }^{2} A R^{0.3} \tag{41}
\end{align*}
$$

respectively. In this way the design load patch is expressed as a function of displacement, a hull shape function and a set of class factors. The class factors reflect the operational conditions; ice conditions and velocity.


Figure 2. Nominal and design rectangular load patches.

## 5. CONCLUSION

The derivation of the ice load equations found in the IACS Unified Requirement for Polar Ships is given. The solutions are presented in a general analytical form. The equations are then simplified by the specification of an assumed pressure-area exponent, and further simplified by collecting certain terms into class factors. The UR specifies specific class factors. The paper explains every step of the creation of the class factors and shows the rationale of the UR. Further, by presenting all the steps, revision of the UR to account for the results of experience and new knowledge will made easier.

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## Annex: Mass reduction coefficient.

This annex describes the calculation of the mass reduction coefficient Co. This approach was developed by Popov (1969). A collision taking place at point (x,y,z) (see Figure A1), will result in a normal force $F_{n}$. The point will accelerate, and a component of the acceleration will be along the normal vector, with a magnitude $a_{n}$. The collision can be modeled as if point P were a single mass (1 degree of freedom system) with an equivalent mass $M_{e}$ of;

$$
\begin{equation*}
M e=F n / a_{n} \tag{A1}
\end{equation*}
$$

The effective mass is a function of the inertial properties (mass, radii of gyration, hull angles and moment arms) of the ship. The effective mass is linearly proportional to the mass (displacement) of the vessel, and can be expressed as;

$$
\begin{equation*}
M e=M_{\text {ship }} / C o \tag{A2}
\end{equation*}
$$

where $C o$ is the mass reduction coefficient.


Figure A1. Collision point geometry
The inertial properties of the vessel are as follows,
Hull angles at point:
$\alpha=$ waterline angle
$\beta=$ frame angle
$\beta^{\prime}=$ normal frame angle
$\gamma=$ sheer angle


Figure A2. Hull angle definitions.
The various angles are related as follows,

$$
\begin{align*}
& \tan (\beta)=\tan (\alpha) \tan (\gamma)  \tag{A3}\\
& \tan \left(\beta^{\prime}\right)=\tan (\beta) \tan (\alpha) \tag{A4}
\end{align*}
$$

Based on these angles, the direction cosines, $l, m, n$ are

$$
\begin{align*}
& l=\sin (\alpha) \cos \left(\beta^{\prime}\right)  \tag{A5}\\
& m=\cos (\alpha) \cos \left(\beta^{\prime}\right)  \tag{A6}\\
& n=\sin \left(\beta^{\prime}\right) \tag{A7}
\end{align*}
$$

and the moment arms are

$$
\begin{align*}
& \lambda l=n y_{p}-m z_{p} \quad(\text { roll moment arm })  \tag{A8}\\
& \mu l=l z_{p}-n x_{p} \quad(\text { pitch moment arm })  \tag{A9}\\
& \eta l=m x_{p}-l y_{p} \quad(\text { yaw moment arm }) \tag{A10}
\end{align*}
$$

The added mass terms are as follows;

$$
\begin{align*}
& A M x=\text { added } \text { mass factor in surge }=0  \tag{A11}\\
& A M y=\text { added mass factor in sway }=2 T / B  \tag{A12}\\
& A M z=\text { added } \text { mass factor in heave }=2 / 3\left(B C w p^{2}\right) /(T(C b(1+C w p))  \tag{A13}\\
& A M r o l=\text { added mass factor in roll }=0.25  \tag{A14}\\
& A M p i t=\text { added mass factor in pitch }=B /((T(3-2 C w p)(3-C w p))  \tag{A15}\\
& A M y a w=\text { added mass factor in yaw }=0.3+0.05 L / B \tag{A16}
\end{align*}
$$

The mass radii of gyration (squared) are;

$$
\begin{align*}
& r x^{2}=C w p B^{2} /(11.4 C m)+H^{2} / 12 \quad(\text { roll })  \tag{A17}\\
& \left.r y^{2}=0.07 C w p L^{2} \quad \text { (pitch }\right)  \tag{A18}\\
& r z^{2}=L^{2} / 16 \quad \text { (yaw) } \tag{A19}
\end{align*}
$$

With the above quantities defined, the mass reduction coefficient is

$$
\begin{align*}
C o= & l^{2} /(l+A M x)+m^{2} /(1+A M y)+n^{2} /(l+A M z) \\
& +\lambda l^{2} /\left(r x^{2}(l+\text { AMrol })+\mu l^{2} /\left(r y^{2}(1+\text { AMpit })\right)+\eta l^{2} /\left(r z^{2}(1+\text { AMyaw })\right)\right. \tag{A20}
\end{align*}
$$

