IACS Unified Requirements for Polar Ships

Background Notes to

Shell Plating Thickness

Prepared for:

IACS Ad-Hoc Group on Polar Class Ships

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SUMMARY

The shell plating is one of the most important elements of ice strengthening structures; the correct regulation of its thickness is of high importance: if the requirements level is not insufficient, frequent repair is required (to remove the impermissible shell plating corrugation), if the requirements level is excessive, ship deadweight is decreased due to ice strengthening weight increase. It was shown during the Harmonization Process that there is certain reconciliation in the requirements to the shell plating thickness imposed by Canadian and Russian Rules: in both standard systems the ultimate strength criterion is used and the local character of the ice load is taken into account. This fact has made it possible to develop a validated analytic relationship for shell plating thickness definition basing on the consideration of the ultimate state of the locally loaded plate with account of the ice load peakness effect.

REGULATION OF THE ICE STRENGTHENING SHELL PLATING PERMISSIBLE NET THICKNESS ON THE BASIS OF THE ANALYTICAL MODEL FOR PLATE TRANSITION INTO THE ULTIMATE STATE UNDER THE LOCAL ICE LOAD

1. INTRODUCTION

Below the method for regulation of the ice strengthening shell plating net thickness (i.e. the thickness less the abrasion - corrosion additions) is considered,. The separate Background covers the matter of the abrasion - corrosion additions. In order to define the permissible net thickness, the ultimate strength criterion is utilized. To describe the plate ultimate state, the kinematic method of the ultimate balance theory (UBT) [1, 2, 3] is applied. A plate in the ultimate state is simulated as a set of rigid parts connected by rectilinear plastic hinges formed by two-side corners of the plate surface kink [4]. The plate load is idealized with account of the peakness effect.

2. THE ANALYTIC MODEL OF PLATE TRANSITION INTO THE ULTIMATE STATE UNDER THE LOCAL LOAD

Let us consider a rectangular plate with sides s and l (s is a short side) fixed on the contour under a local load of intensity p, applied in compliance with the scheme in Fig. 2.1, a. It was mentioned above that in this case can be used the approximate version of the kinematic method of the ultimate balance theory based on introducing the concepts of rectilinear yield hinges formed by two-side corners of surface plate kink. The possibility of such approach application in problems of plates ultimate balance does not require additional justifications.

The main difficulties of kinematic method application in problems with local loads are caused by definition of the plates failure true mechanism. Strict enough solution can be based on plate surface idealization in the form of the grid of possible yield hinges and further searching for the failed surface form and the ultimate load by application of the numerical procedure of linear programming. However, in the examined case the required accuracy can be reached on the basis of approximate specification of the failed surface form.

It is familiar that at load distribution on all the plate field, the plate is failed by so called envelope (Fig. 2.1, b). The level of ice load localization on the plate's field can be defined by the following relationship:

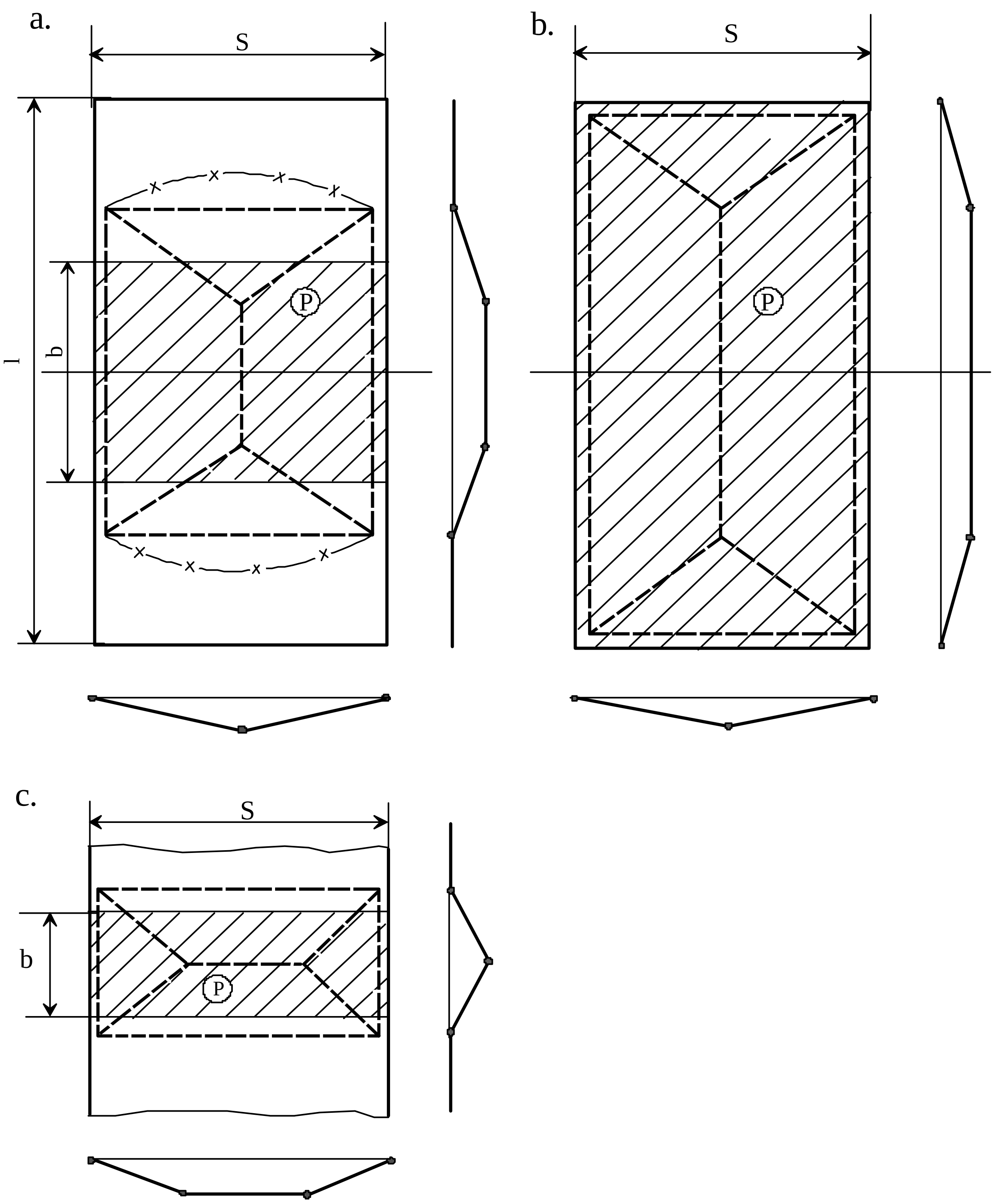

Fig.2.1 The scheme of ice load application to the ice belt shell plating plate and possible forms of plate deformed surface at its transition into the ultimate state

a, c - local load; b - uniformly distributed load

– – – –, • lines of yield hinges

——x—— edge hinges form defined from the condition of ultimate load minimum

$$b \geq S,$$

where b is the ice load distribution height.

If ice load is local, is possible the failure in the envelope form distributing on the part of the plate surface (Fig. 2.1, a) with forming edge yield hinges (not coinciding with the supporting contour). At the strict approach, the form of the edge hinges is defined on the basis of the extreme principles of the ultimate balance theory from the condition of the plate ultimate load minimum. However, special assessments show that for the practical use rectilinear edge hinges are permissible to be specified (Fig. 2.1.a), as far as in this case an error in ultimate load definition does not exceed several percents, and the final solution is simplified appreciably. Therefore, further the plate deformed surface (plastic hinges) is examined in the envelope form with rectilinear yield hinges; the envelope length m satisfies the following condition (Fig. 2.2):

$$b < m \leq l. \qquad (2.1)$$

At the specified plate size and load form, the geometry of the deformed surface is defined completely by specification of two parameters: length of the failure area m and height of the triangular sector n (see Fig. 2.2). The values of these parameters, as well as the ultimate load (pressure) value p_u are defined within the procedure of the kinematic method reducing to analysis of external and internal forces work for the plastic mechanism varied kinematically (Fig. 2.2) on displacement (deflection) f.

Presence of problem symmetry permits to be limited by examination of sector ABCD displacement.

Internal forces work in the plastic hinge is calculated by the following familiar relationship:

$$T_i = -\delta \frac{FYt^2}{4} \theta_i d_i \qquad (2.2)$$

where $\delta = \frac{2}{\sqrt{3}}$,

i is a plastic hinge number;

θ_i is an angle of the kink in the i-th plastic hinge;

d_i is the length of the i-th plastic hinge.

Let us accept the following hinges numeration:

AB-i=1; BC-i=2; DO-i=3; OB-i=4

then $\theta_1 = \frac{f}{n}; \quad d_1 = 0.5S;$

$$\theta_2 = \frac{f}{0.5S}; \quad d_2 = 0.5m; \qquad (2.3)$$

$$\theta_3 = \frac{f}{0.5S}; \quad d_3 = 0.5m - n$$

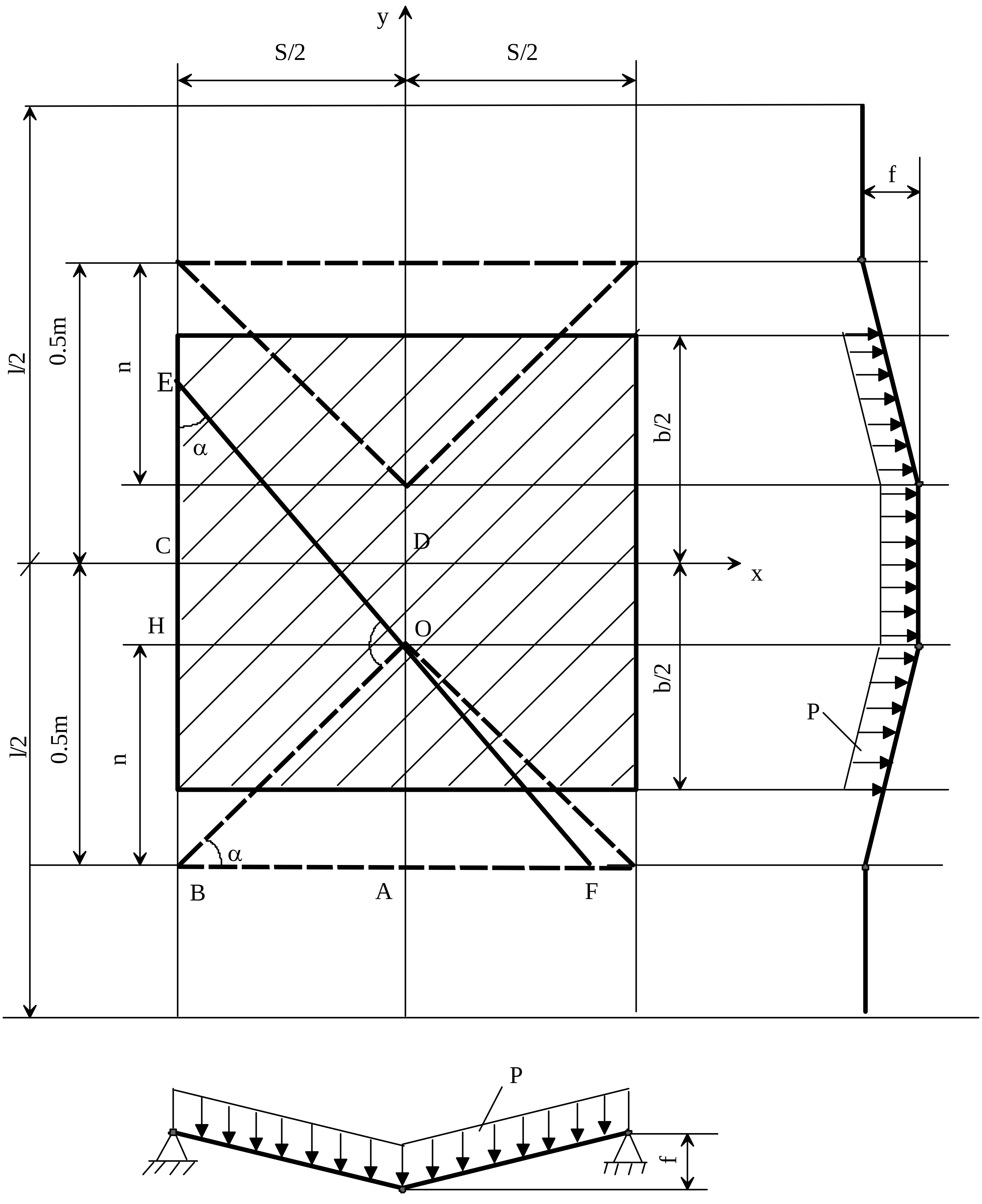

Fig.2.2 The form of plate deformed surface with rectilinear edge hinges
• , ▬ ▬ ▬ ▬ ines of yield hinges lines of yield hinges

To calculate θ_4, a straight line $EF \perp BO$ was drew through point O. From geometrical consideration we have the following (Fig. 2.2)

$$BO = \sqrt{0.25S^2 + n^2};$$

$$OF = BO \cdot tg\alpha = \frac{\sqrt{0.25S^2 + n^2} \cdot n}{0.5S};$$

$$OE = BO \cdot ctg\alpha = \frac{\sqrt{0.25S^2 + n^2} \cdot 0.5S}{n}$$

Then

$$\theta_4 = \frac{f}{OE} + \frac{f}{OF} = \frac{f}{\sqrt{0.25S^2 + n^2}} \cdot \left(\frac{0.5S}{n} + \frac{n}{0.5S} \right) \qquad (2.4)$$

The full work of the plastic forces in the hinge is as follows:

$$T = -\sum_i T_i \qquad (2.5)$$

Substituting relationships (2.2) - (2.4) into (2.5), the following is obtained:

$$T = -\delta \cdot FY \cdot t^2 \cdot \frac{f}{2} \left(\frac{0.5S}{n} + \frac{m}{S} \right) \qquad (2.6)$$

External forces work on deflection f of the kinamatic mechanism is as follows:

$$V = \int_F p_u W(x, y) dF; \qquad (2.7)$$

where

p_u - is the ultimate pressure;

$W(x, y) = f \cdot \varphi(x, y)$ is plate deflection in a point with coordinates (x, y);

$\varphi(x, y) \le 1$ is the piece-linear function;

F is the area of the plate's deformed part.

Let (Fig. 2.3)

$$F = F_{ABO} + F_{BOH} + F_{HODC} \qquad (2.8)$$

Integrals on areas $F_{ABO}, F_{BOH}, F_{HODC}$ are calculated from the following obvious relationships (Fig. 2.3):

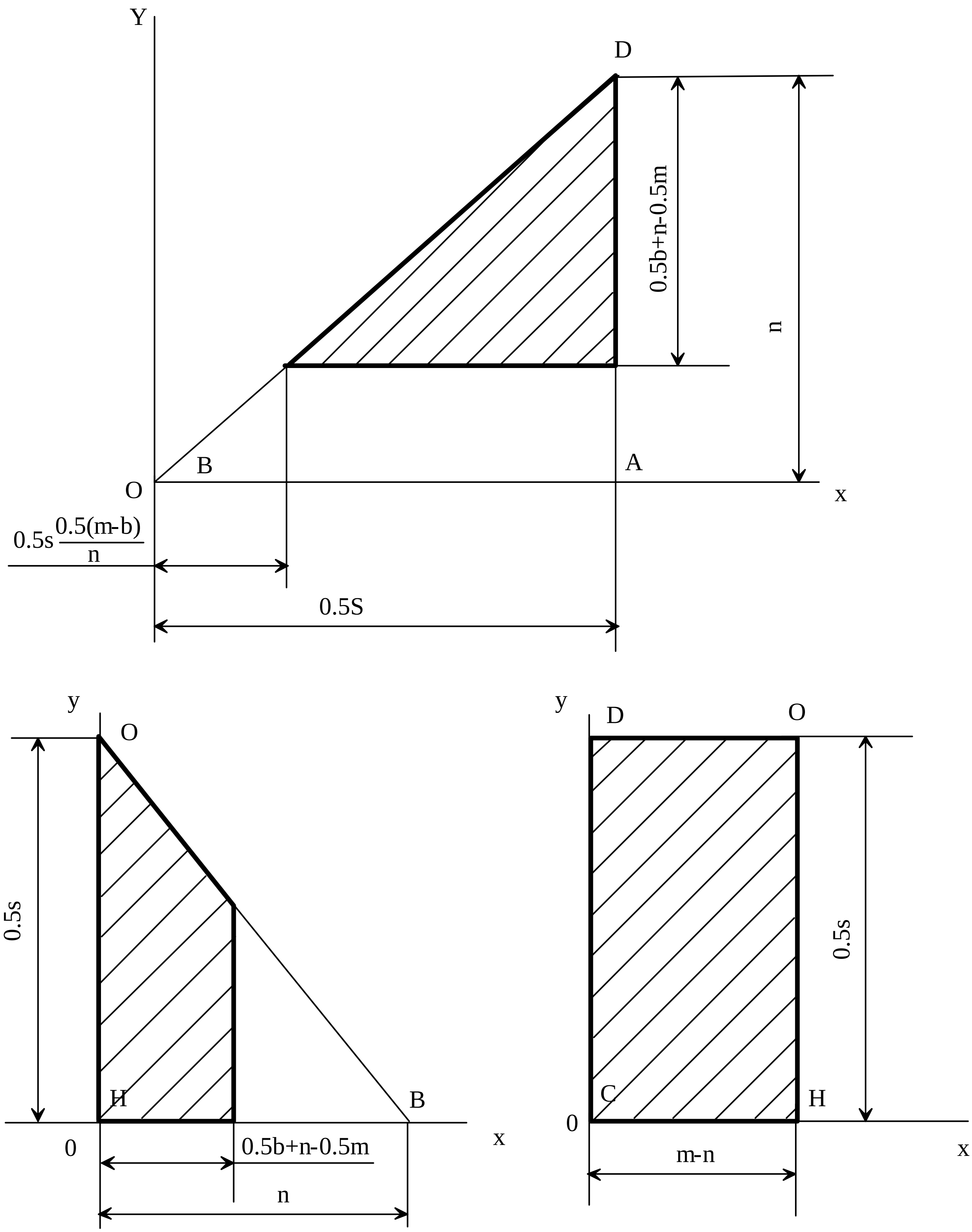

Fig.2.3 Computation of external forces work

$$V_{ABO} = \int\limits_{0.5n\frac{0.5m-0.5b}{n}}^{n} \int\limits_{0.5(m-b)}^{\frac{n}{0.5S}x} p \cdot f \cdot \frac{y}{n} dxdy = \frac{1}{12} pfS^2 \left(\frac{n}{S} - 3\frac{(0.5m-0.5b)^2}{n \cdot S} + \frac{2(0.5m-0.5b)^3}{n^2 S} \right) \quad (2.9)$$

$$V_{BOH} = \int\limits_{0}^{0.5b-0.5m+n} \int\limits_{0}^{0.5S\left(1-\frac{x}{n}\right)} p_u \cdot f \cdot \frac{y}{0.5S} dxdy = \frac{1}{12} pfS^2 \left(\frac{n}{S} - \frac{(0.5m-0.5b)^3}{n^2 \cdot S} \right) \quad (2.10)$$

$$V_{HODC} = \int\limits_{0}^{0.5m-n} \int\limits_{0}^{0.5S} p \quad \cdot f \cdot \frac{y}{0.5S} dxdy = \frac{1}{4} pfS^2 \frac{0.5m-n}{S} \quad (2.11)$$

External forces full work is as follows:

$$V = V_{ABO} + V_{BOH} + V_{HODC} = \frac{1}{8} pfS^2 \left[\frac{m}{s} - \frac{1}{3} \cdot \frac{n}{0.5S} + \frac{1}{3} \cdot \frac{(0.5m-0.5b)^3}{n^2 \cdot 0.5S} - \frac{(0.5m-0.5b)^2}{n \cdot 0.5S} \right] \quad (2.12)$$

By the kinematic method methodology, the equations system relatively the unknown parameters m, n and p_u is as follows:

$$\frac{\partial(T+V)}{\partial f} = 0; \quad \frac{\partial(T+V)}{\partial n} = 0; \quad \frac{\partial(T+V)}{\partial m} = 0. \quad (2.13)$$

Substituting here expressions (2.6), (2.12) and differentiating, we obtain the following expressions after elementary manipulation:

$$\delta\left(\overline{m} + \frac{1}{\overline{n}} \right) - 0.25\overline{p}\left[\overline{m} - \frac{\overline{n}}{3} + \frac{(\overline{m}-\overline{b})^3}{3\overline{n}^2} - \frac{(\overline{m}-\overline{b})^2}{\overline{n}} \right] = 0$$

$$\delta - 0.25\overline{p}\left(1 - \frac{\overline{m}-\overline{b}}{\overline{n}} \right)^2 = 0 \quad (2.14)$$

$$\frac{\delta}{\overline{n}^2} + 0.25\overline{p}\left[\frac{1}{3} + \frac{2}{3}\left(\frac{\overline{m}-\overline{b}}{\overline{n}} \right)^3 - \left(\frac{\overline{m}-\overline{b}}{\overline{n}} \right)^2 \right] = 0$$

where

$$\overline{m} = \frac{m}{S}; \quad \overline{b} = \frac{b}{S}; \quad \overline{n} = \frac{n}{0.5S}; \quad \overline{p} = \frac{p_u S^2}{FYt^2}; \quad \delta = \frac{2}{\sqrt{3}}$$

The results of numerical solution of nonlinear system (2.14) are cited in Fig. 2.4.

It follows from the data of Fig. 2.4 that the failure area length m only slightly exceeds the load application area height b, and at

$$b \to \infty; \qquad m \to b$$

Let us obtain the approximate problem solution basing on the following assumption

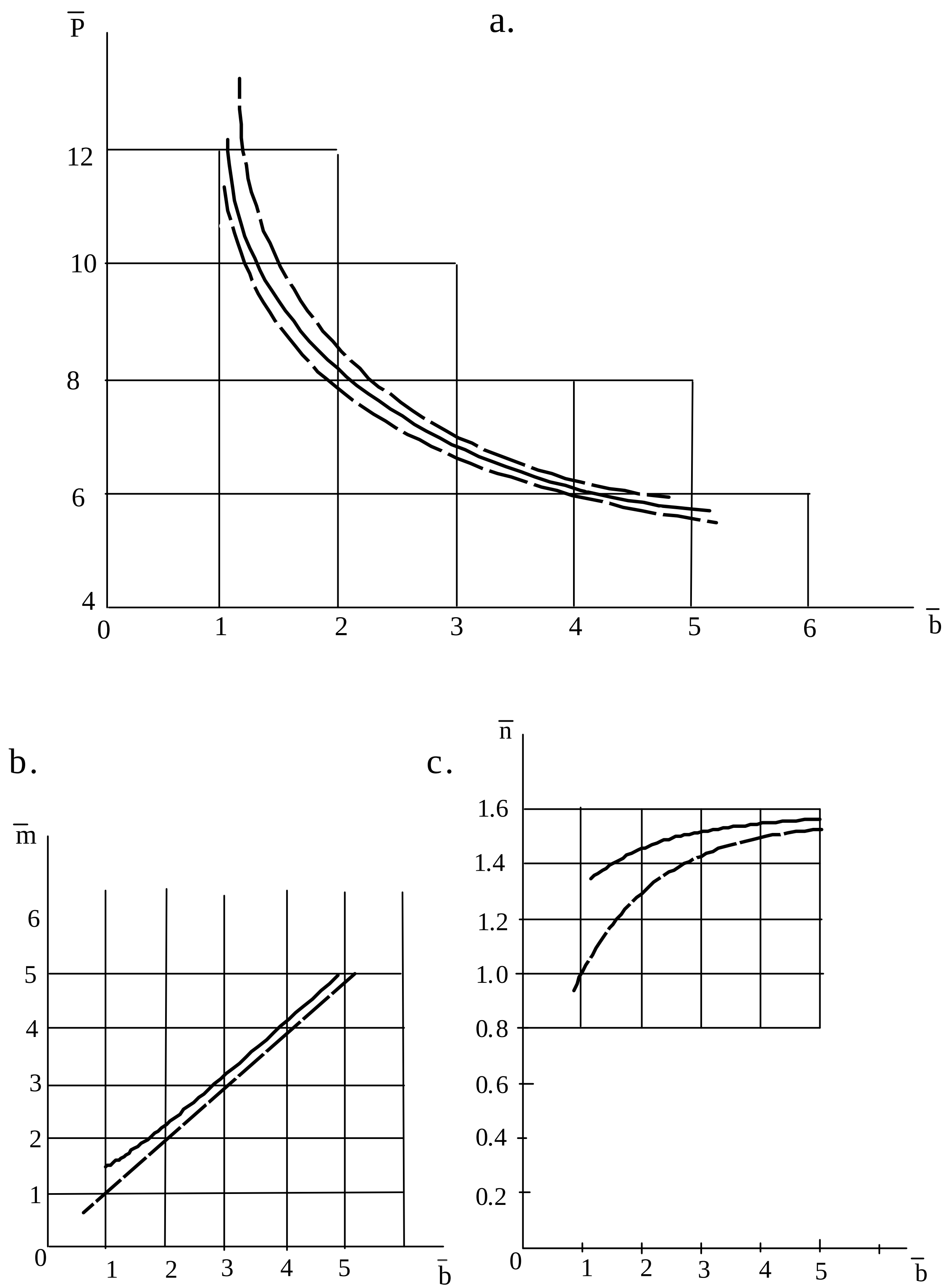

Fig.2.4 Function of the ultimate state parameters versus the relative height of the load application area

——— numerical solution of system (2.15)

– – – – approximate solution of (2.18) at $\overline{m} = \overline{b}$

—·—· approximation formula (2.20)

$$m = b \tag{2.15}$$

It gives

$$\left.\begin{aligned} \frac{\partial(T+V)}{\partial f} &= \delta\left(\bar{b} + \frac{1}{n}\right) - 0.25\bar{p}\left(\bar{b} - \frac{\bar{n}}{3}\right) = 0 \\ \frac{\partial(T+V)}{\partial n} &= -\frac{\delta}{\bar{n}^2} + \frac{0.25\bar{p}}{3} = 0 \end{aligned}\right\} \tag{2.16}$$

The (2.16) solution is as follows

$$\bar{p} = 4\delta\left(\frac{1}{\sqrt{3\bar{b}}} + \sqrt{\frac{1}{3\bar{b}^2} + 1}\right)^2, \tag{2.17}$$

$$\bar{n} = \sqrt{\frac{12\delta}{\bar{p}}} \tag{2.18}$$

Taking into account the fact that normally for ice going ships

$$\frac{1}{3\bar{b}^2} << 1,$$

the following can be written instead of (2.17)

$$\bar{p} = 4\delta\left(1 + \frac{1}{\sqrt{3\bar{b}}}\right)^2, \tag{2.19}$$

Formula (2.19) gives the results coinciding with the numerical solution (2.14) with accuracy of 1.2% (see Fig. 2.4, a).

At the same time, the data of Fig. (2.4, a) characterize clearly the factor of ultimate pressure critically increase and decrease of the load distribution height.

It is familiar that the kinematic method gives the increased assessment for the ultimate load. Hence, solution of (2.14) based on the hypothesis of rectilinear plastic hinges results in an error always directed in the dangerous side. Therefore, preparing the RR regulation requirement to the shell plating thickness basing on (2.19), it was decided to use more caution assessment in order to weaken parameter $\bar{b}$ influence upon the ultimate load.

Equally with this it was taken into consideration that in the old Russian Rules coefficient δ was ignored at shell plating thickness regulation. Therefore, preparing the standing Russian Rules and IACS UR Draft, in order to provide for succession in the relationship of the requirement to the shell plating and framing, it was decided to ignore this factor as well. In the result, the formula for the ultimate load (2.19) takes the following form:

$$\bar{p}_1 = 4\left(1 + \frac{0.5}{\bar{b}}\right)^2, \tag{2.20}$$

In the typical area of parameter $\bar{b} = 1 \div 3$ variation the following relationship between magnitudes $\bar{p}$ and $\bar{p}_1$ take place:

$$\bar{p}_1 = (0.8 \div 0.85)\bar{p} \tag{2.21}$$

Therefore, in the IACS UR requirements is put down the more cautious assessment of shell plating ultimate strength, than it follows directly from the described theoretic solution.

Such a decision seems to be justified enough as far as the actual plate loads are characterized by the highest changeability.

It should be also noted that the obtained solution is based on consideration of the form of the plate deformed surface cited in Fig. 2.1, a. At small distribution heights b another form of the deformed surface is possible when the yield hinge in the plate center is oriented perpendicularly to the plate long edges (Fig. 2.2, c). With account of this, formula (2.20) takes the following form:

$$\bar{p}_1 = 4c^2, \tag{2.22}$$

where $c = \begin{cases} 1 + 0.5/\bar{b} & \text{at } \bar{b} \geq 1 \\ 0.5 + 1/\bar{b} & \text{at } \bar{b} < 1 \end{cases}$

It should be noted that in the area

$$b < 0.5s$$

relationship (2.22) results in an appreciable error. However, so small load distribution heights are not met practically and there is no necessity in (2.22) further refinement. In the UR the requirement to the shell plating thickness is generated on the relationship (2.21), as far as condition $b > s$ is practically always valid. The resultant function for the shell plating thickness is as follows:

$$t_{ice} = 0.5s\sqrt{\frac{p}{FY}} \cdot \frac{1}{1 + 0.5\frac{s}{b}} \tag{2.23}$$

In order to consider the ice load pattern peakness, the peakness factor PPF is introduced. To take the frames (transverse framing, longitudinal framing, inclined frames) orientation into account, the orientation factor OF is introduced into the regulation relationship. In the result the formula for the net thickness takes the following form:

$$t_{net} = 0.5s \cdot OF\sqrt{\frac{PPF \cdot p}{FY}} \cdot \frac{1}{1 + 0.5\frac{s}{l}} \tag{2.24}$$

Relationship (2.24) differs from the regulation formula of the IACS UR Draft by notation and dimensional representation.

3. CONCLUSION

Applicability of relationship (2.24) for regulation of the ice strengthening shell plating thickness within the IACS UR (with account of the abrasion - corrosion additions) was verified by numerous comparisons with the requirements contained in the Russian, Canadian and Finnish-Sweden Rules.

The obtained regulation relationship provides the high level of service reliability at minimal metal consumption and deadweight loss.

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