

**UNIFIED REQUIREMENTS LOAD MODEL -
'SYNTHESIZED APPROACH'**

Prepared for:

**IACS Polar Rules Harmonization
Semi-Permanent Working Group**

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on behalf of the:

**Canada/Russia Bilateral Project
Steering Committee:
Institute for Marine Dynamics,
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Russian Maritime Register**

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1. INTRODUCTION

Since the initiation of the Harmonization efforts, it has been apparent that there are several significantly different philosophies regarding the mechanisms by which ship/ice interactions generate structural loads.

The exchange of technical reports, experimental data, and service experience over the course of the OWG and SWG meetings has served to highlight areas of agreement and disagreement on the underlying physics. In turn, this has allowed an understanding of the reasons underlying the resulting differences in selecting critical parameters and identifying how these influence the resulting loads.

Meanwhile, however, joint discussions have also resulted in general agreement on how the actual structural requirements should vary across the range of Polar Classes envisaged under the new system. This reflects the service experience of the (limited) set of polar ships already in existence, and a realistic approach to extending this to vessels of different size, shape, and mission in future years. Suitable analytical tools for verifying the acceptability of local structural components have also been selected, and have in turn led to agreement on the nature of the load idealizations which will be used with them.

Thus, although it is probable that considerable additional research will be needed to reach a consensus on load modelling, it has been considered to be feasible to construct a 'synthesized approach' which has the following characteristics:

- a) it can provide outputs which lead to the desired scantlings;
- b) its workings are broadly compatible with several proposed methodologies;
- c) its form makes it simple to introduce future refinements, as our knowledge of ice interaction improves.

At the last SWG meeting in Oslo, February 1998, it was agreed that the Canadian team supporting Transport Canada would aim to produce such a synthesis. This paper describes the resulting approach.

It should be noted that an implicit assumption of the paper is that the glancing impact scenario, which it addresses, represents the design loading case both for the forebody, and also (through empirical hull area factors) for the rest of the ship hull. This is not necessarily valid for all ships, and the Unified Requirements need to consider the incorporation of other scenarios for some or all hull areas, now or in future editions.

2. OVERVIEW

2.1 General

It should be appreciated that the ship/ice interactions which any Polar Class vessel will experience through life will be a complex mix. In principle, the approach to setting structural design requirements would be probabilistic, establishing a target damage probability which could be matched to the joint distributions of interaction parameters. In practice, the necessary tools and data for the construction of a probabilistic approach are lacking. The glancing impact (and ramming) scenarios are thus treated deterministically in the load models proposed for the first edition of the Unified Requirements. The parameter values used for each class are selected to be reasonable representations of combinations which might be expected to lead to severe loading conditions (see Section 5).

The generation of structural requirements through the glancing impact scenario can be represented using Figure 1 below:

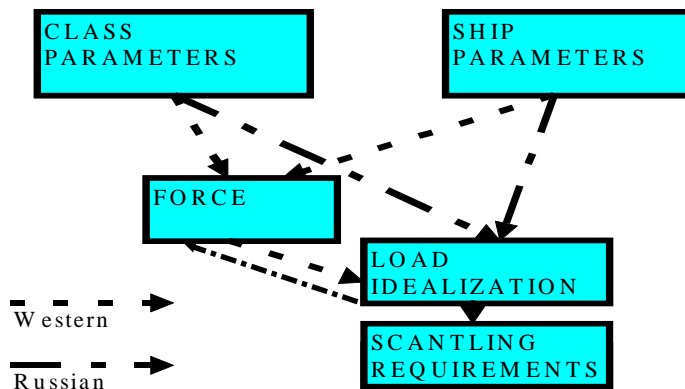


Figure 1 - Load Models

Under both the 'Western' and 'Russian' visualizations of the scenario, similar sets of class and ship parameters define the scenario, namely:

- | | | |
|------------------|---|--|
| Class parameters | - | ice strengths ('crushing', flexure)
ice thickness
ice floe size, shape
impact speed |
| Ship parameters | - | displacement
hull form |

It should be noted that, in the actual URs, it is anticipated that the class parameters will be collapsed into a single 'class factor'.

The (simplified) Western view, expressed in ASPPR [1] and elsewhere, is that these parameters define the impact force, which in turn allows an idealized set of design loads on structural components to be generated. Under the Russian approach [2], [3], it is not necessary to work through the force, and the loads are derived directly from the class and ship parameters. Force can, if desired, be generated from the local loads and their distribution.

2.2 Total Force

As explained above, in order to accommodate both the Russian and Western approaches, the synthesized approach needs to incorporate an overall force model. The proposed Rule model is described in detail at Section 3. It is based on the collision models traditionally used by Russian scientists and engineers in deriving ship/ice interaction results. It is considered to be a simplification of adequate accuracy for current purposes. Hull form effects have been incorporated through further simplifications, which may have a restricted range of validity to 'radical' hull shapes. The necessary sophistication of representation of hull shape warrants further discussion. Similarly, ice floe shape has been handled by assuming a floe of effectively infinite size (lateral extent), whose thickness varies with class, and whose edge geometry is represented as a wedge, as opposed to the Russian assumption of a rounded edge. All such assumptions are somewhat arbitrary, but the wedge is considered at least equally valid, and more tractable by analytical methods.

The force model incorporates an ice strength representation in the form typically used in the West, in both ship and offshore structure design, where average instantaneous pressure is an empirical function of total contact area, i.e.

$$p_{av} = CA^e$$

The selection of values for the constant and exponent has aimed to ensure that the resulting local loads are also reasonably compatible with Russian expectations for parametric dependencies. As noted earlier, the values will not be directly visible to most users of the URs, as they will be incorporated in the class factors and in the system of equations. This report, and its subsequent extension into the overall Background Document for the URs, will provide the understanding which will be needed to allow for future updates of the formulae and values as consensus on principles improves.

2.3 Contact Area

The contact model allows calculation of the geometric overlap between ship and ice, and thus the apparent contact area and its shape. It is known that not all of this apparent area

will actually carry a load at any moment in an interaction. The synthesized model has therefore applied a correction factor to the apparent area, and further simplified it to provide a rectangular patch. The nature of the correction, and its qualitative justification, are described at Section 4. The idealization to rectangular shape is considered reasonable, and is in line with all current Rule systems.

2.4 Local Loads

Using the Western philosophy, the average pressure at any point during the interaction can be derived from the force and from the contact area. This is concentrated first by the reduction from 'apparent' to 'true' contact area described at 2.3 above, and second by the nature of the distribution within the true area, which is known to be non-uniform. The non-uniformity is manifested in somewhat different ways in existing rule systems, but is an explicit or implicit element in all resulting scantling calculations.

The form of the concentration factor which has been proposed here is based on the formulations used in the ASPPR, modified for better correlation with the current and proposed Russian Rules. It is described in more detail at Section 4. It should be noted that this factor does not attempt to model any of the theories of ice failure exactly, but yields results which can be consistent with several of these, to the levels of accuracy required by the structural formulae.

2.5 Scantling Formulae

The approach outlined above has been used to generate scantling requirements using the formulae presented to the SWG at the New York meetings [4]. These are plastic collapse limit state formulae, which will be supplemented by structural stability criteria [5], [1]. Stability is not checked against in the analyses presented at Section 6.

It should also be noted that no additional strength reserves are built into the framing design criteria; i.e. framing reaches its limit state at the identical load to plating, assuming that both will see identical loads. This does not account for the possibility of 'bridging', as assumed in the Baltic Rules [6], or of the potential desirability of higher safety factors for framing design. Both these issues may warrant further discussion.

3. FORCE MODEL

3.1 Impact Scenario

The SWG has agreed that three loading scenarios should be incorporated in the load assumptions of the Unified Requirements:

1. head-on (ramming) impacts;
2. glancing impacts;
3. loadings under pressured ice conditions.

The first of these will be used as for global strength checks, which may only influence scantlings on some (higher polar class) vessels. The treatment of the third has not been discussed in detail, but it is anticipated that it could provide a lower bound requirement for midbody areas of certain ship classes and sizes. The glancing impact scenario, analyzed in detail for the bow and applied to other part of the hull through hull area scaling factors will be the basis for most scantling requirements. This scenario is the focus of the approach described here.

Under the basic scenario, the ship is assumed to have only ahead velocity. Other glancing impacts are possible, for example as the ship turns or bounces off one piece of ice onto another. These may represent potentially more severe cases, and for the hull aft of the point of maximum beam some sway or yaw motion is obviously required to generate an impact in the first place. Neglecting the reflected impact case is assumed to be reasonably valid for cargo-carrying vessels, but it may be necessary to take some account of this for icebreakers and ice management vessels, where it has a higher probability of occurrence. For other hull areas, it is envisaged that a more generally valid impact modelling approach may be developed in the future. This would allow direct assessment of the loads rather than the area factor approach.

3.2 Contact Idealization

Hull form effects are generally acknowledged to influence ice loads, though they are not treated explicitly in all current ice rules, or consistently in the rules where they do apply. It has been accepted that the URs should include form effects to acknowledge their importance.

The hull angles of interest are those of the locus of the interaction, but in a simple model this locus must be treated as a point. In current Russian rules [2] and previous harmonization proposals, the variation in shape over the bow is handled by analyzing a set of hull sections, and choosing the one with the 'worst' properties. In the approach described here, it is assumed that the overall bow shape is adequately defined by the set of angles at the stem; i.e. that the stem angles will be strongly related to the shape

elsewhere in the forebody and can thus substitute for these. This leads to the angle dependencies which are included in the formulae. In the version of the presented in New York, and in previous Russian approaches, they have been further simplified by linearizing their trigonometrical forms. These two simplifications have been questioned by a number of participants in the SWG, and work is continuing to assess their validity and/or range of applicability. Thus, in the version presented here the hull angle function has been omitted altogether, and a single-valued coefficient (representative of an icebreaking form) is used to provide sample calculations.

The current version of the angle formulations is based on the following definitions and derivations:

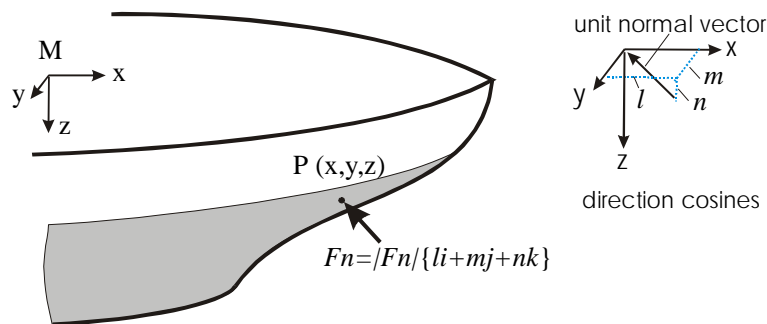


Figure 3.1. Collision Geometry

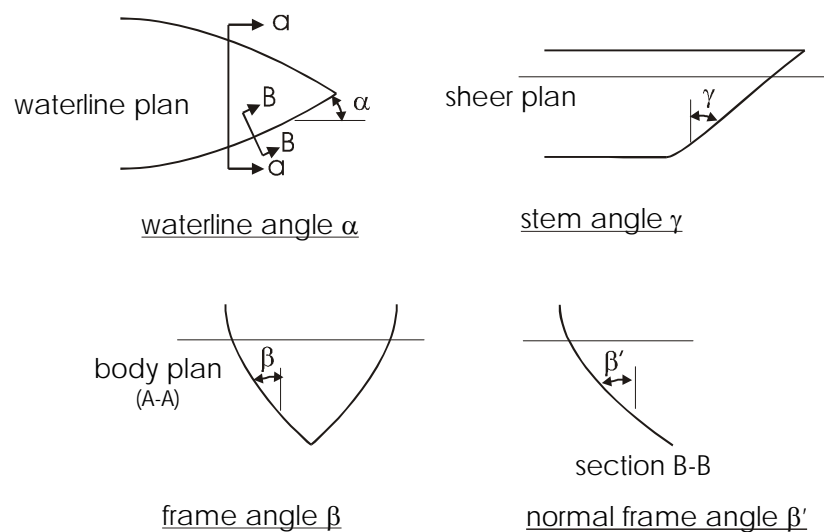


Figure 3.2. Definition of Hull Angles

3.3 Collision Modeling

A rigid body has 6 degrees of freedom in 3 dimensions. Modeling elastic deformations requires more degrees of freedom. Ship/ice impact requires at least six-degrees of freedom problem for both of the bodies involved - the ship and the ice floe. Collision modeling also requires an understanding of the contact process between the bodies, and the behavior of the bodies in their environment. General collision modeling techniques are available, but require relatively complex numerical models. Difficulties and uncertainties arise from the contact process as well as the environmental influences (i.e. fluid added mass). The modeling approaches used in Canada and Russia have employed a variety of different numerical approaches. The most advanced Canadian and Russian models are based on different (and not always compatible) assumptions

For the Unified Requirements, it has therefore been agreed that the load model should be based on a relatively simple approach which can meet the criteria listed in Section 1. A familiar model to many researchers is that of Popov [7]. This equates the normal kinetic energy lost in the collision to the energy absorbed in destroying the ice. Normal kinetic energy uses the effective mass of the ship, and its velocity in the direction of the ice indentation; i.e. the local normal to the hull. This is described in more detail in Appendix A. Effective mass is calculated using the Popov approach, which is outlined in Appendix B. The normal velocity is a function of the hull angles at the point (centre) of the contact, which is idealized as described above.

'Popov' models can use a variety of representations of ice 'crushing' energy, his own early work applying a constant value. More recent Russian work incorporates terms which reflect the 'hydrodynamic' model. In the current model an alternative representation has been used, i.e. a Pressure/Area relationship in which the averaged instantaneous contact pressure is a function of total apparent contact area. This is discussed in more detail at Section 3.4.

This energy-derived solution for the oblique collision then becomes (see Appendix A for derivation):

$$F_n = (3 + 2 \cdot ex)^{\frac{2+2 \cdot ex}{3+2 \cdot ex}} \cdot P_o^{\frac{1}{3+2 \cdot ex}} \cdot \left(\frac{\tan(\phi/2)}{\sin(\beta') \cdot \cos^2(\beta')} \right)^{\frac{1+ex}{3+2 \cdot ex}} \cdot \left(\frac{1}{2} \Delta_n \cdot V_n^2 \right)^{\frac{2+2 \cdot ex}{3+2 \cdot ex}} \quad (1)$$

where

P_o : ice pressure (at 1 m²)

ex : pressure-area exponent

ϕ : ice edge opening angle

β : normal frame angle (see Figure 2)

Δ_n : normalized mass

V_n : normalized velocity

3.4 Force/Penetration

Interaction models must have a way to determine the overall loads as the ship penetrates into the ice, i.e. - force at any penetration distance, or time. As force is the product of average pressure and area it has been found useful to use representations of these two parameters to model many types of loading. Empirical evidence has shown that pressure is a function of area. Pressure/Area curves are thus used in much western work. A number of these have been derived from different experimental programs at a range of configurations, ice strengths, scales and strain-rates in Canada, the US, Finland and elsewhere.

Pressure/Area curves are normally expressed in the general form:

$$p_{av} = CA^e$$

In this model, the curve is described as:

$$P = P_o.A^{ex} \quad (2)$$

and the values of P_o are class dependent, while ex always takes the value of -0.1. This exponent has a smaller absolute value than many which have been used in previous design studies, or derived from experimental programs. However, it is important to note that this is only one of several types of pressure/area relationship which have been applied, as there is often considerable confusion as to the terminology [8]. The force/indentation P/A curve applied up to this point in the current approach only aims to describe the average pressure over a geometrically calculated apparent area. Other relationships may be derived to characterize the distribution of pressure over some or all of the active contact area. Care must always be taken in comparing P/A curves derived from overall measurements (and apparent areas) with those derived from analyses of strain gauge arrays (which will normally deal with 'true' contact areas). The local design pressure/area curves implied in this approach are discussed in Section 4.1

3.5 Parametric Representation

For these proposals, a number of assumptions and simplifications have been made. The P/A curve has been discussed at Section 3.4. The normalized values of mass and velocity depend on the hull angles and the position. As noted earlier, all angle effects can be condensed into an equation that approximates the angle influences (which has been set to a constant for now, while work continues). The resulting force equation thus becomes:

$$F_n = f_a P_o^{0.36} D^{0.64} V^{1.28} [MN] \quad (3)$$

where Δ is the displacement in kT

V is ship velocity in m/s

fa is a function of the hull angles (α , β)

(Note: fa has been set to .400, subject to revision)

α is the waterline angle at the stem (btw. water line and fwd.) [deg]

γ is the shear (buttock) angle at the stem (btw. shear line and vertical)

[deg]

3.6 Class Relationships

The relationships of structural capability between the new Polar Classes are defined by the values of the ice and velocity parameters selected for each. General spreads in load-carrying capability were discussed at the SWG meetings in Oslo in February 1998. It was agreed that these would broadly follow the ratios proposed in the 3rd Edition of the Russian proposals [9], adjusted to reflect line loads rather than particular combinations of pressure and load height in that document [10]. This provided the general spread shown in the final column of Table 3.1 below, normalized to show all values for PC 7 as 1. The actual values used in constructing the table are drawn from the 4th Edition [4], which are very similar to those of the 3rd Edition. All numbers have been rounded off for ease of comparison.

Table 3.1 - Class Load Ratios

Polar Class	Pressure index	Load ht index	Line Load index
1	5.7	2	11.7
2	4.5	1.8	8.0
3	3.4	1.6	5.3
4	2.5	1.4	3.5
5	1.8	1.3	2.2
6	1.3	1.1	1.4
7	1	1	1

As a result of more recent discussion in the SWG, this approach has been modified somewhat in the proposals presented below. In particular, an effort has been made to differentiate between the 'seasonal ships' (PCs 6 and 7) and the other classes, in order to help align them with the Baltic classes and also to provide information which may be helpful to future operational decisions. This affects the ice strengths and thus (to some extent) the load patch shapes.

Table 3.2 shows the values of class parameters which are proposed for use with equations (1) - (3) above. It should be noted that these are preliminary values, which may change somewhat to reflect the final version of the angle formulation, or to achieve a better fit

between certain existing ship designs and particular classes. However, they are considered to be reasonable representations of 'design' interactions for each of the polar classes.

Table 3.2: Proposed Class Coefficients

Class	V (m/s)	Po (MPa)	h_ice (m)	Sig_f (MPa)
1	5.70	6.10	7.0	1.40
2	4.60	4.90	6.0	1.30
3	3.50	4.00	5.0	1.20
4	2.75	3.50	4.0	1.10
5	2.25	3.00	3.0	1.00
6	2.00	1.50	2.5	0.70
7	1.50	1.25	2.0	0.65

The values of ice thickness and flexural strength affect the limiting loads for the classes; i.e. the loads at which the ice will break in bending rather than by more local crushing.

$$\text{Load Limit, } L_{max} = c_f \cdot \sigma_f (h_{ice})^2 \quad (4)$$

which in the model used here is modified for use with the calculated glancing impact force as follows:

$$\text{Flexible Limit Force, } F_f = 1.2 \cdot \sigma_f (h_{ice})^2 / \sin \beta' \quad (5)$$

where β' is the normal frame angle (see Figure 2)

Following recent discussions in the SWG, it has been agreed that the limit for the PC6 and 7 classes should be set in the 30-40,000 tonne displacement range, above which capability trends similar to those found in the Baltic Rules will be applied. Values for σ_f , the flexural strength, have been adjusted to achieve this. The Baltic Rules do not follow a rigorous load model in this area, and none is available for the Polar scenario. However, applying what is effectively a ship size-related safety factor appears reasonable and prudent. Therefore, using a consistent approach both simplifies the task of aligning rule requirements and also focuses attention on the need for future work to explore this issue. It is considered that the Baltic Rules give conservative results for larger ships.

3.7 Outstanding Issues

As noted earlier, there are a number of issues regarding this model which have not been fully resolved, and here work is continuing within the bilateral project. The most

important is the angle dependency. Since, under agreements to date in the SWG, the bow loads are factored to give the loads on the rest of the body, a strong angle dependency will lead to considerable variability for scantlings throughout the hull due to bow shape. There is limited justification for making this link, and so there is an argument for having little or no angle dependency in the general force formulae.

Alternatively, it may be possible to define a standard value for the angle coefficient, for use with the other hull areas, and a variable value for the bow (and perhaps the stern). Using similar logic, the aspect ratio for the loads elsewhere in the hull could also be fixed. This is discussed in more detail at section 4.1

4. LOAD IDEALIZATION

4.1 Load Patch Definition

The interaction model can be used to derive an apparent contact area, over which an average pressure can be considered to apply. In order to produce a set of structural design loads, this nominal patch must be adjusted for size, shape, and pressure distribution. This is done in both the existing Canadian and Russian rule approaches, in somewhat different ways.

The geometrical shape of the contact area (given the ice edge shape assumed) will be roughly triangular, with (usually) edge rounding due to the hull shape. However, for the model, the bow load patch is assumed to be rectangular with a nominal aspect ratio (AR) dependent on the normal frame angle β' . As discussed above, the collision is assumed to occur on an angular edge as shown in Figure 4.1. The rectangular nominal and design patch is sketched in Figure 4.2. The aspect ratio is determined from the angular collision. The exact aspect ratio is:

$$AR = 2 * \tan(\phi/2) \sin(\beta') \quad (6)$$

This has been simplified (see Figure 4.3) to the formula;

$$AR = 2 + 0.07 \beta' \quad (7)$$

Where β' is the normal frame angle [deg]
 (Note: $\tan(\beta) = \tan(\gamma) \tan(\alpha)$, $\tan(\beta') = \tan(\beta) \cos(\alpha)$)

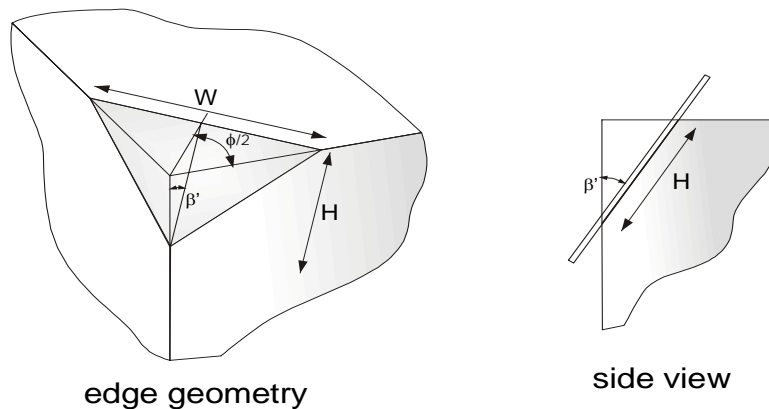


Figure 4.1: Ice Edge Geometry

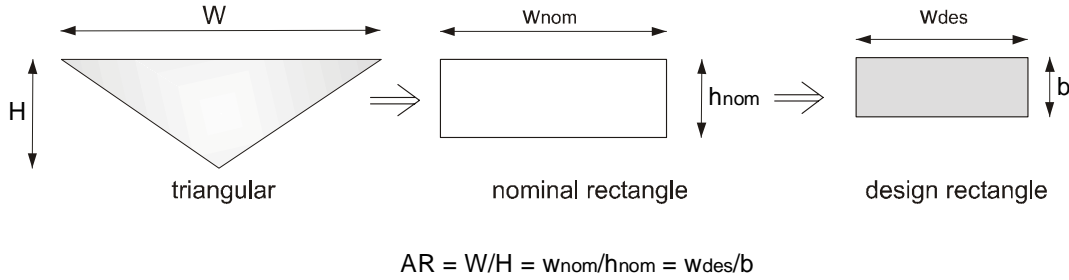


Figure 4.2: Design Patch Idealization

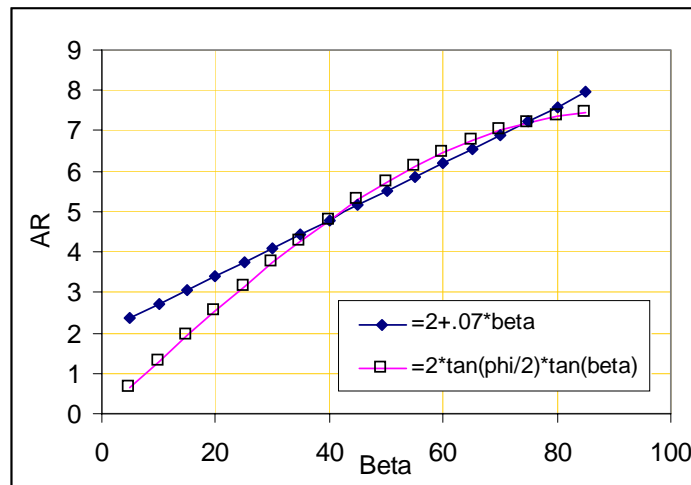


Figure 4.3: Aspect Ratio Dependency

The simplified formula is both easy to use and also prevents the aspect ratio from becoming 'too small' at low values of β' . Experimental results from a range of ships suggests that a minimum aspect ratio of 2:1 is a reasonable lower bound. As discussed at Section 3.7, it may also be desirable to set a fixed value of aspect ratio for loads on the midbody (and possibly other values for loads elsewhere). In the midbody, as β' tends to zero, this would be expected to be towards the lower end of the values shown in Figure 5; possibly at the 2:1 level. If the line load, Q is selected to be at the bow value multiplied by the hull area factor (always < 1), this will ensure that the impact forces on other areas of the hull are always smaller than the bow forces. More work is needed to investigate the implications of midbody patch options, and a discussion paper will be generated by the bilateral project prior to the next SWG meeting.

The nominal area A_{nom} is;

$$A_{nom} = W_{nom} h_{nom} = h_{nom}^2 AR \quad (8)$$

with the force being;

$$Fn = P_{avg} A_{nom} = Po A_{nom}^{(1+ex)} \quad (9)$$

values for Po and ex being selected as discussed at Section 3.

Form the above equations the nominal load height and width can be found;

$$h_{nom} = (Fn / Po / AR^{(1+ex)})^{(1/(2+2*ex))} \quad (10)$$

$$w_{nom} = h_{nom} AR \quad (11)$$

It can be seen that Po has to be used to derive h_{nom} . This effectively means that, if class factors are used to define loads, two factors are necessary; the first linking V and Po and the second involving only Po.

It is generally acknowledged that the apparent geometric contact area is larger than the actual area over which significant loads are carried. At the edge of the apparent area, pieces of ice flake and spall away. Few attempts have been made to define this relationship, but it is necessary to do so in order to derive the desired load model for the URs.

Three main options might be considered for the area reduction, each with potential variants. Most simply, it could be assumed that the edge loss always involves pieces of the same size, dictated only by the properties of the ice. This would yield a relationship of the form:

$$I_{actual} = I_{apparent} - x, \text{ where } x \text{ is the characteristic dimension.}$$

However, it has also been observed that the interaction itself will influence the spalling process, with larger pieces resulting from bigger impacts into the same ice. This could be represented either by:

$$I_{actual} = C * I_{apparent}^{ex}, \text{ or by}$$

$$I_{actual} = I_{apparent}^{ex}$$

The last of these options has been assumed in the proposed load model, based on research reported in [11]. This leads to the set of equations provided below.

The design load height (b) and width (w_{des}) are found as follows;

$$w_{des} = w_{nom}^{w_{ex}} \quad (12)$$

$$b = h_{nom}^{w_{ex}} AR^{w_{ex}-1} \quad (13)$$

where: $w_{ex} = 0.7$

Thus, the design load length w_{des} is reduced from the nominal, and the design load height b is reduced to maintain the aspect ratio. This appears consistent with results observed during field test programs on ships such as the LOUIS, ODEN, and POLAR SEA, where in no cases does there appear to be a significant change in aspect ratio during the course of an impact.

This set of equations is only valid where w_{nom} is greater than 1m in length, and although this is almost always the case for a Polar Class ship), a more robust representation will be developed to avoid problems of application at small dimensions.

The use of these relationships has the effect of generating a second pressure/area curve for the actual design loads. Equation (2) can be rewritten:

$$P_{av_nom} = P_o . A_{nom}^{ex}$$

but as the effective contact area, A_{eff} can be found to be

$$A_{eff} = A_{nom}^{w_ex} . AR^{w_ex-1}$$

thus the effective average pressure is

4.2 Load Distribution

From the design patch dimensions it is easy to derive a line load $Q = F_n/w_{des}$, while the average pressure Pde in the patch is Q/b .

It is generally acknowledged that ice loads are not uniformly distributed over the instantaneous contact area. The load patch applied to any structural component therefore needs to modify the average pressure which could be calculated from the force and contact area.

For the most local structural elements (plating and first level framing) a load concentration factor is thus applied to the average pressure. This has been linked to the characteristic horizontal dimension of transverse frame spacing, S , through the formula:

$$\begin{aligned} loc_f &= 1.8 - S; S \leq 0.6 \\ &= 1.2 \quad S > 0.6 \end{aligned} \quad (11)$$

As explained in [12], the resulting values are reasonably consistent with the ASPPR and with the Russian Rules over a frame spacings up to 0.6m. Beyond this, under the Canadian approach, they are somewhat more conservative. Experience with using the

Canadian approach has suggested that this becomes non-conservative at large spacings, and thus the change is in the right direction.

The same initial average pressure, and the same concentration factors are assumed for both framing and plating. Under several existing rule sets (including the Baltic Rules and ASPPR) there is a modification to pressure values between plate and framing to account for bridging effects, the plate pressures being reduced compared to those seen by framing. There is no explicit acknowledgement of this effect in the current proposals, though it could be argued that the resulting scantlings, being chosen for consistency with existing 'successful practice', may implicitly account for this in the selection of values for various coefficients.

4.3 Outstanding Issues

Two issues which have not been addressed fully in the material above are the variation in load idealizations with hull area (if any), and the overall representation of any and all load patches for use in direct design (or grillage analysis). The first of these will be treated in the discussion paper mentioned in Section 3.7. The second will need to be discussed as part of the grillage analysis project agreed in New York; and will be reported in the materials which that exercise generates.

5. SCANTLING REQUIREMENTS

5.1 Overview

In order to illustrate the workings of the proposed load model, it is necessary to show the actual scantlings which result from its use. Accordingly, the spreadsheet attached (entitled LMPSA98 for 'Load Model Proposal Spreadsheet; July 1998) performs all calculations needed to generate 'as-built' plating and framing scantlings which will just withstand the design loads derived above.

The as-built values add a wear allowance to the ice scantlings. The values of wear accord with those tabled prior to the New York meetings and are subject to modification in light of the discussions there [13].

5.2 Scantling Formulae

The first level plating and framing structural requirements are based on those given in the 4^d issue of the Krylov proposals for the Unified Requirements [4], which in turn are explained by supplementary documents tabled in Oslo and New York. No attempt has yet been made to include second-level structure (stringers and web frames) in the spreadsheet calculations, as the approach to grillage design remains under discussion.

The proposed plating formula is;

$$t_p = CTL \cdot 500 \cdot S \cdot \sqrt{\frac{loc_f \cdot Pde}{FY}} \cdot \frac{1}{1 + \frac{S}{2b}} + \Delta t \quad (12)$$

where

t_p : plate thickness

CTL : orientation factor

S : frame spacing

loc_f : pressure localization factor

Pde : design pressure

FY : yield strength

$1/(1+s/(2b))$ is a load height factor

Δt : corrosion allowance

It should be noted that the corrosion allowance which is included here has been set at 4mm for all classes and hull areas, rather than at the values previously proposed. This reflects the discussions held in New York, where the need to provide an adequate operational margin for internal and external effects was highlighted. In recognition of the fact that current norms allow approximately 25% wastage of Baltic class ships, where typical plating thicknesses are in the 16mm range, the 4mm allowance permits reasonable alignment of polar 'net scantling' thicknesses with current practice. No wastage has been assumed in any of the other scantling formulae, except in noting that plating net values should be used in modulus and shear area calculations

The shear area formula is;

$$SA = \frac{Ks \cdot Pde \cdot S \cdot b}{FY} \quad (13)$$

where

SA : shear area

Ks : represents several coefficients, defined in the 4th Edition

The section modulus formula is;

$$PM = \frac{Km \cdot Pde \cdot S \cdot b \cdot LF}{FY} \quad (14)$$

where

PM : plastic section modulus

LF : frame span

Km : represents several coefficients, defined in the 4th Edition

All of the above represent plastic limit states (under assumed uniformly distributed loading) in relatively rigorous analytical solutions.

It should be noted that the proposed stability criteria have not yet been implemented in the spreadsheet, in order to facilitate the identifications of basic trends. It is therefore assumed (for example) that the calculated versions of the frame area and modulus can be satisfied exactly simultaneously, without concerns over slenderness or other ratios.

5.3 Spreadsheet Calculations

The various parameters and formulae used in the spreadsheet are listed below. The formulae are identical to those described above. Parameter values conform to those listed for the class, and to the ship under analysis.

5.3.1 Class Factors

The values in the Class Table correspond to the class parameters listed at Table 3.2. Two class-dependent items are added, the wear allowance (WA) and the midship hull area coefficient (CAPm). The spreadsheet will automatically select all class values for the

class specified as input, and will factor the line load according to selection of Bow or Mid as an option in the Constants Table. At this time, constants for other hull areas are not included in the spreadsheet.

Table 5.1 - Class Factor Inputs

Class Table						
Class	V (m/s)	Po (MPa)	h_ice (m)	Sig_f (MPa)	WA (mm)	CAPm
1	5.70	4.00	7.0	1.40	2.50	0.7
2	4.60	3.20	6.0	1.30	2.25	0.6
3	3.70	2.40	5.0	1.20	2.00	0.5
4	2.90	1.80	4.0	1.10	1.75	0.5
5	2.30	1.50	3.0	1.00	1.50	0.47
6	1.90	1.30	2.5	0.90	1.25	0.45
7	1.50	1.00	2.0	0.80	1.00	0.4

5.3.2 Constants

The calculation constants are given in the constants table, which includes the following important elements:

Table 5.2 - Calculation Constants

Constants Table		
Item	Name	Value
Hull Region	Hregion	bow
Intercostal Stringer Type	Intercostal	none
Yield Strength (Mpa)	FY	360
exponent on mass	m_ex	0.64
exponent on velocity	V_ex	1.285
exponent on pressure	P_ex	.357
exponent on Area	Ex	-0.1
exponent on width	w_ex	0.7

The default exponent values shown are those calculated as explained in Sections 3 and 4. Yield strength can be selected as desired, and in this version is assumed to be identical for shell and frames. The spreadsheet will calculate scantlings for grillages with or without the intercostal supporting stringers used in much Russian practice.

5.3.3 Vessel Parameters

The main vessel parameters are given below. The values in *italics* are not used directly, but have been included for other comparative purposes. As noted, the hull angles are at present only used for the aspect ratio calculation. Angle data is used in the comparative calculations under the earlier Russian rule proposals.

The default ship information shown in the spreadsheet is drawn from the 'grid' vessels tabled by the Russian side as the basis for comparative calculations.

Table 5.3 - Ship Data

Vessel Particulars			
<u>Parameter</u>	<u>Name</u>	<u>Formula</u>	<u>Comment</u>
Ice Class	Class	ipc1	ipc1 -ipc7
Ship Displacement [kT]	Δ	5	the force depends on the displacement and velocity
<i>Ship Length [m]</i>	<i>Length</i>	<i>92.7</i>	
<i>Ship Draft [m]</i>	<i>Draft</i>	<i>5.3</i>	
<i>Ship Beam [m]</i>	<i>Beam</i>	<i>32</i>	
waterline angle at bow	alf	28	angles at stem are assumed to be representative of the hull form
frame angle at bow	beta	=ATAN(TAN(gama)*TAN(alf))	
buttock angle at bow	gama	59	
normal frame angle	betap	=ATAN(TAN(beta)*COS(alf))	
velocity	V	=LOOKUP in Class Table	

5.3.4 Ice Load Parameters

The ice load values are given below. The formulae are described in Sections 3 and 4. Although it is intended to use the flexural limit as a true limit for the lower classes, at present the values of class coefficients for bending failure do not show this, to facilitate identification of trends and comparisons with Russian practice.

Table 5.4 - Ice Load Data

General Design Pressure Coefficients			
<u>Parameter</u>	<u>Name</u>	<u>Formula</u>	<u>Comment</u>
Ice Pressure Constant	Po	= LOOKUP in Class Table	This is comparable to the observed annual maximum pressure on a 1m ² panel.
Ice Pressure Exponent	ex	= LOOKUP in Class Table	
alfa factor	a_f	= .400	To be revised
gama factor	g_f	=	To be determined
Ice Thickness	h_ice	= LOOKUP in Class Table	For flexural force
Flex Strength	sig_f	= LOOKUP in Class Table	For flexural force
Flex Limit Force	F_f	=(1.2*sig_f*h_ice^2)/SIN(betap)	Flexural force
Total Force	F_n	=MIN(a_f*V^v_ex*D^m_ex,E41)	

Hull Area Factor	HAF	=IF("bow",1,IF("mid", LOOKUP in Class Table))	Taken from Krylov approach
aspect ratio	AR	=2+0.07*betap	For simple wedge collision
nominal load height	h_nom	=(F_n/Po/AR^(1+ex))^(1/(2+2*ex))	
nominal load length	w_nom	=h_nom*AR	
design load height	b	=h_nom^w_ex*AR^(w_ex-1)	Accounts for flaking
design load length	W_des	=(w_nom)^w_ex	
line load	Q	=F_n/W_des	
design pressure	Pde	=Q/b*HAF	

5.3.5 Scantling Formulae

The structural scantlings are calculated as described in Section 5.2. Each of the formulae quoted is used in the spreadsheet in its full form, with the coefficients and constants defined in the 4th Edition [4]. There are two exceptions to this - to simplify the calculation process, it is assumed that one of the section modulus coefficients, a_1 in the 4th Edition (which relates minimum to as-fitted shear area) takes a value of 0.85. This assumption is used in all the calculations and thus does not affect the comparative results, though it should be understood that different values could be used in service for a variety of practical reasons. Similarly, the frame attachment parameter, j , is initially set at 4, which assumes end fixity. Actual designs may have different fixity levels and the spreadsheet can be adjusted to reflect this.

All the characteristic dimensions can be changed to allow investigation of scantlings with different configurations (span, spacing, orientation). The default configuration of the spreadsheet is designed to investigate transverse framing, but it can be changed to look at longitudinals.

Table 5.5 - Scantling Formulae and Input Data

Structural Calculations (Transversely Framed Ship)			
<u>Parameter</u>	<u>Name</u>	<u>Formula</u>	<u>Comment</u>
Frame Span	LF	2	
Frame spacing	S	0.4	
Wear Allowance	ΔT	= LOOKUP in Class Table	
Frame Orientation Angle	Ω	=0	Default value - can be adjusted to analyze longitudinal frames.
Frame Orientation Coefficient	CTL	=IF('Ω'<=20,1,IF('Ω'>=70,1+S/3.5,1+S/3.5*SIN(Ω*PI()/180)))	
Shell Plating Thickness	t_net_pro	=CTLt*500*S*SQRT(loc_f*Pde/FY)*1/(1+S/2/b)+DT	
nom Plate Pressure	p_pl	=(E58-E53)/(500*E52))^2*FY	Used to determine the equivalent uniform pressure strength of the plate (for comparison)
localization	loc_f	=MAX(1.8-S,1.2)	
Shear Area Coefficient	BetaFact	=MIN(1,E47/E51)	

Shear Area Coefficient	Y	=1-0.5*BetaFact
Shear Area Coefficient	Z	=1+0.25*(S^2/(LF^2*BetaFact))
Shear Area Coefficient	KT	=(2*Y)/(Z+SQRT(Z^2-2*Y*BetaFact))
Intercostal Stringer Coefficient	KS	=IF(intercostal="none",1,IF(intercostal="discontinuous",0.9,IF(intercostal="continuous",0.8,"error"))))
Shear Area Coefficient	SAI0	=(8700*Pde*b*S*KT*KS)/FY
Intercostal Stringer Coefficient	KL1	=IF(intercostal="none",MAX(1.8-S,1.2),IF(intercostal="discontinuous",MAX(1.6-S,1),IF(intercostal="continuous",1.2,"error"))))
Frame Shear Area	SAI	=SAI0*KL1
Web Plate Angle	theta	15
Web Plate Angle Coefficient	Ka	=IF(theta>15,1/COS(theta),1)
Section Modulus Coefficient	a	0.85
Frame Attachment Parameter	j	4
Section Modulus Coefficient	Afact	=1/(1+0.25*j*SQRT(1-a^2))
Section Modulus Coefficient	PMI0	=(250000*Pde*b*S*LF*Y*Ka)/FY
Plastic Section Modulus	PMI	=PMI0*KL1*Afact

5.4 Comparisons

The spreadsheet can be used to generate results for load and scantlings which can be compared with other rule systems. A set of comparisons with the Canadian ASPPR and the latest Russian Register requirements have been undertaken, and these have also considered Baltic classes at the lower end of the class spectrum. Some examples are presented below in Figures 5.1 - 5.3 for bow and 5.4 - 5.6 for midbody scantlings. All comparisons use a frame spacing of 0.35m and a span of 2m; and the same hull form. It is intended that a full spreadsheet incorporating all these systems and proposals will be circulated in due course, once it has been configured to automatically track any changes to input parameters and ensure consistent results.

The Russian Rules provide the only equally comprehensive system to the new proposals. It can be seen that the L7 class (similar to the old ULA) tracks most scantlings of the PC3 quite closely, generally being a few percent below the new proposals in the bow area and above in the midbody. At the upper end of the range, the PC1 is similar to the L9. Direct comparisons between classes are less close at the lower end of the range, where PC7 has been aligned more with the Baltic 1A (for plating). It is between Russian L3 and L4 for plating and somewhat above L4 for frame parameters; with the caveat that limiting values are expected to lead to lower requirements for larger ships.

Against the CAC classes of the ASPPR, the most dramatic difference in the new system is the absence of minimum requirements, which for smaller ships of the two lower classes dominated the scantlings. The trend lines without these minima have also been plotted to illustrate their effects. In general, the trend of the new proposals is to require less frame shear area than ASPPR, but more section modulus.

Plate thicknesses for the low classes are comparable to the Baltic classes, but the frame requirements are more stringent. This is not unexpected, and is (in part) related to the different loading assumptions. More detailed comparisons with the Baltic requirements will be the subject of a forthcoming paper.

All of the above comparisons will change slightly with different selections of frame spacing and span. In general, the trends in variation with spacing (for transverse framing) are between those in the ASPPR and the Russian Rules (see figure 5.7). Trends with span are similar to those in both ASPPR and the Russian Rules (figure 5.8). The frame requirements for longitudinally framed ships show significantly different trends from any of the existing systems, due to the different load patch shape.

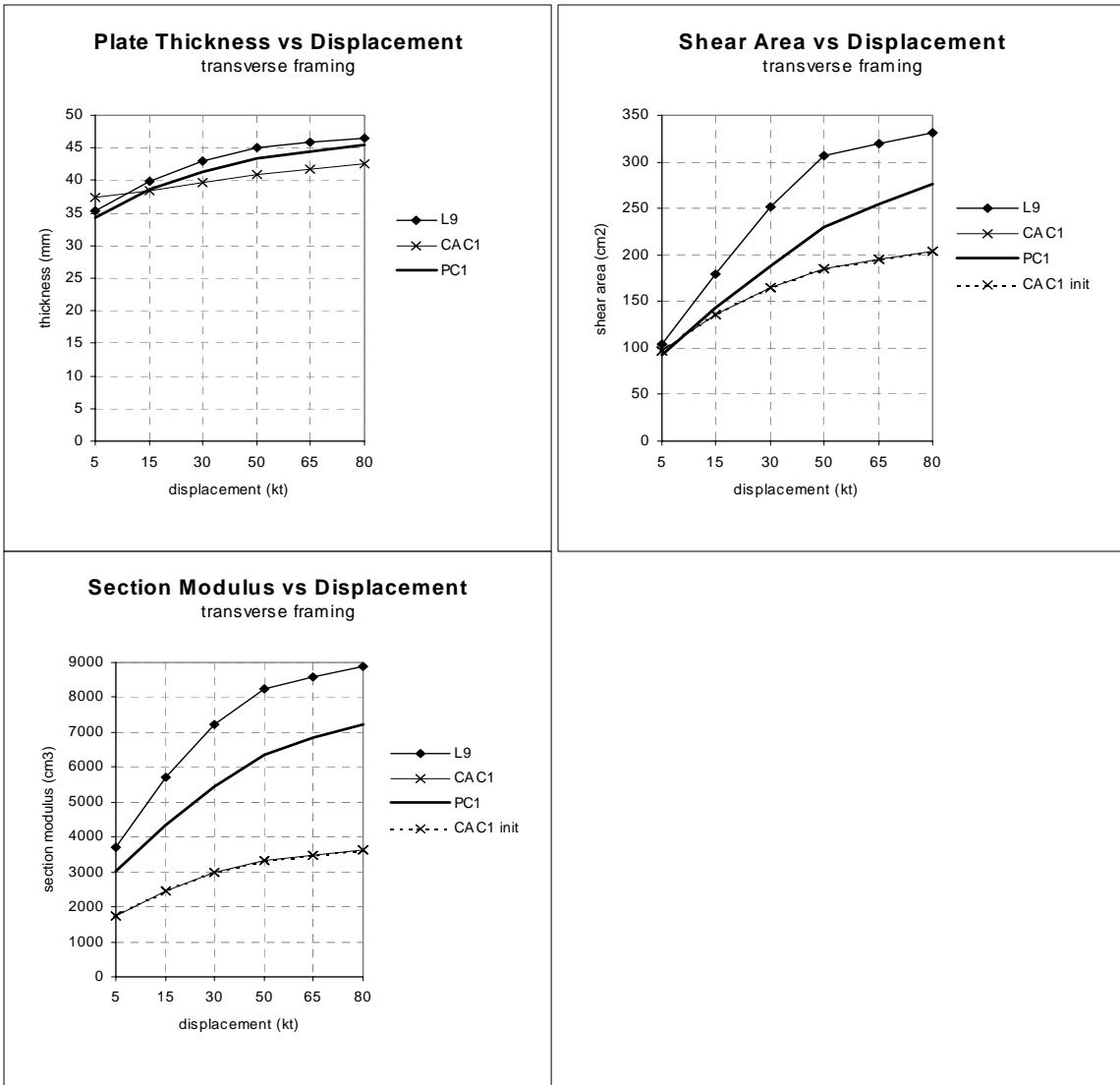


Figure 5.1: Scantlings versus Displacement, highest Polar class, Bow

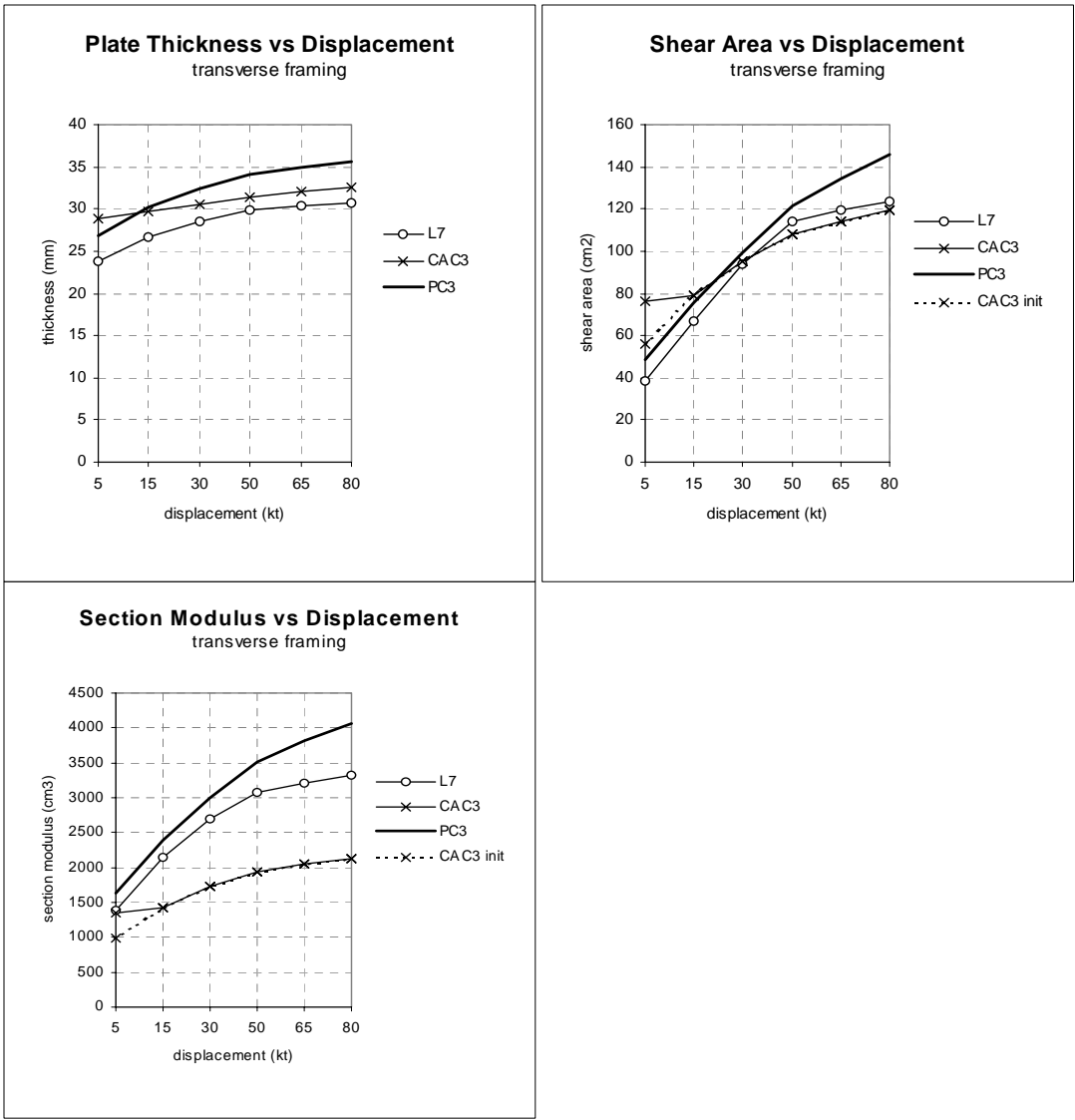


Figure 5.2: Scantlings versus Displacement, medium Polar class, Bow

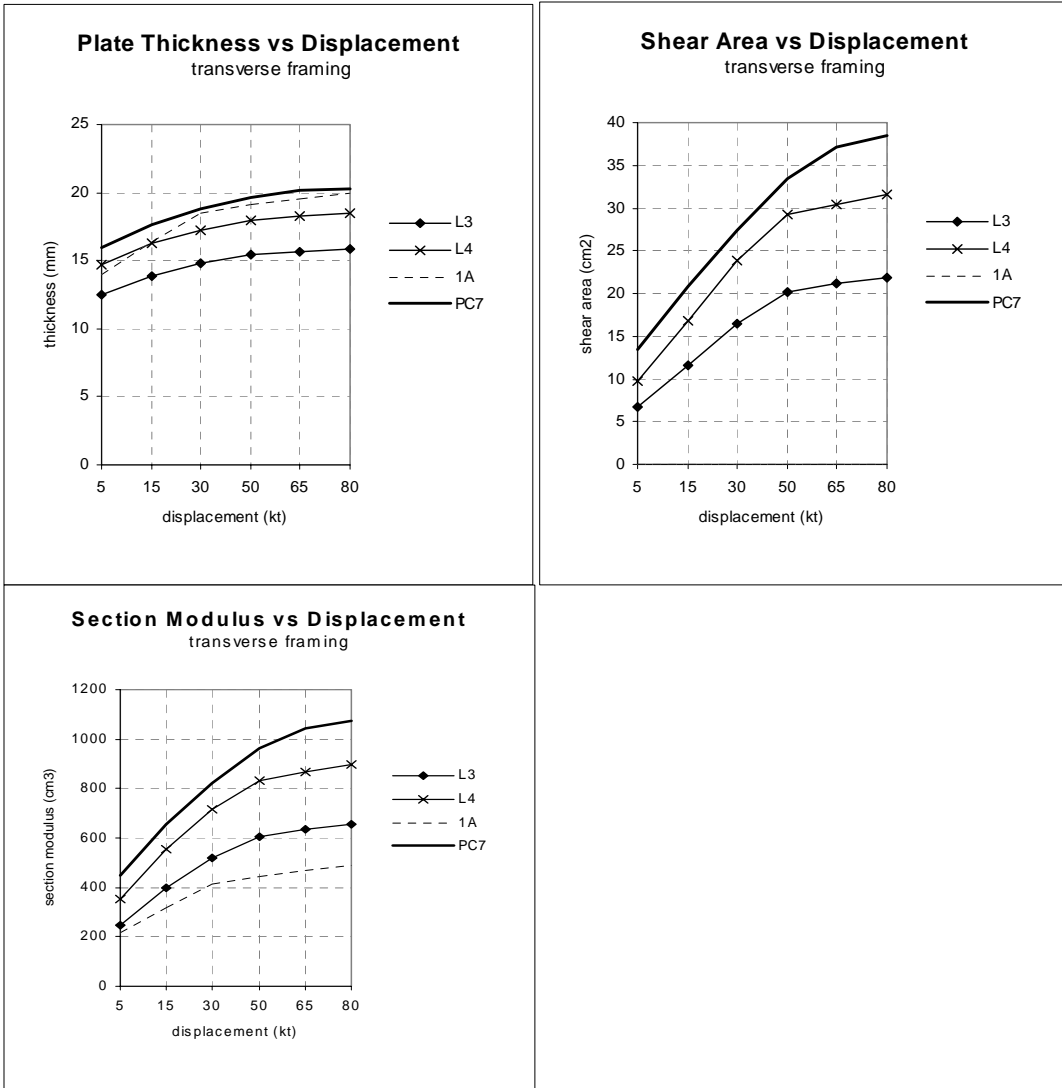


Figure 5.3: Scantlings versus Displacement, lowest Polar class, Bow

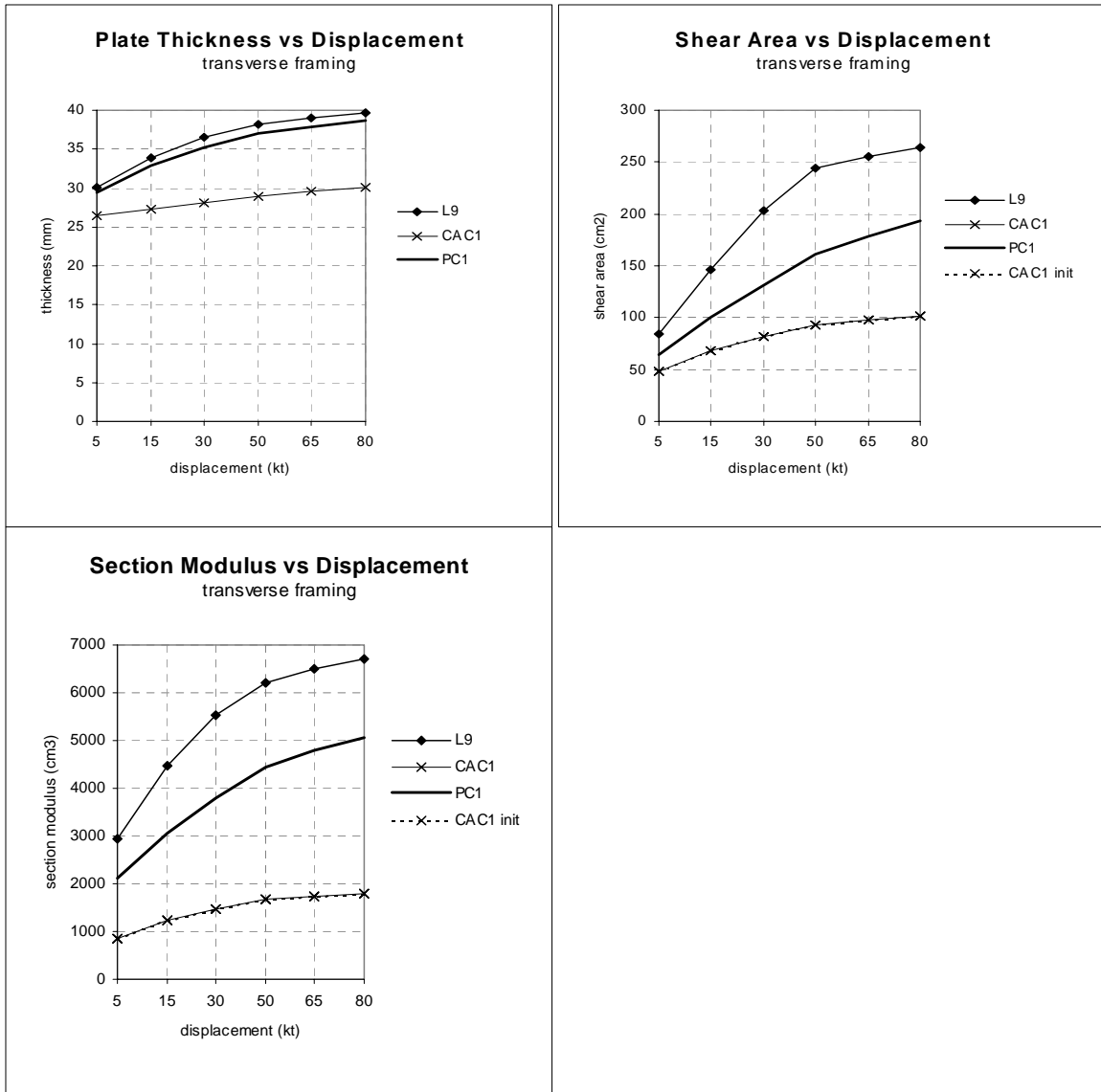


Figure 5.4: Scantlings versus Displacement, highest Polar class, Midbody

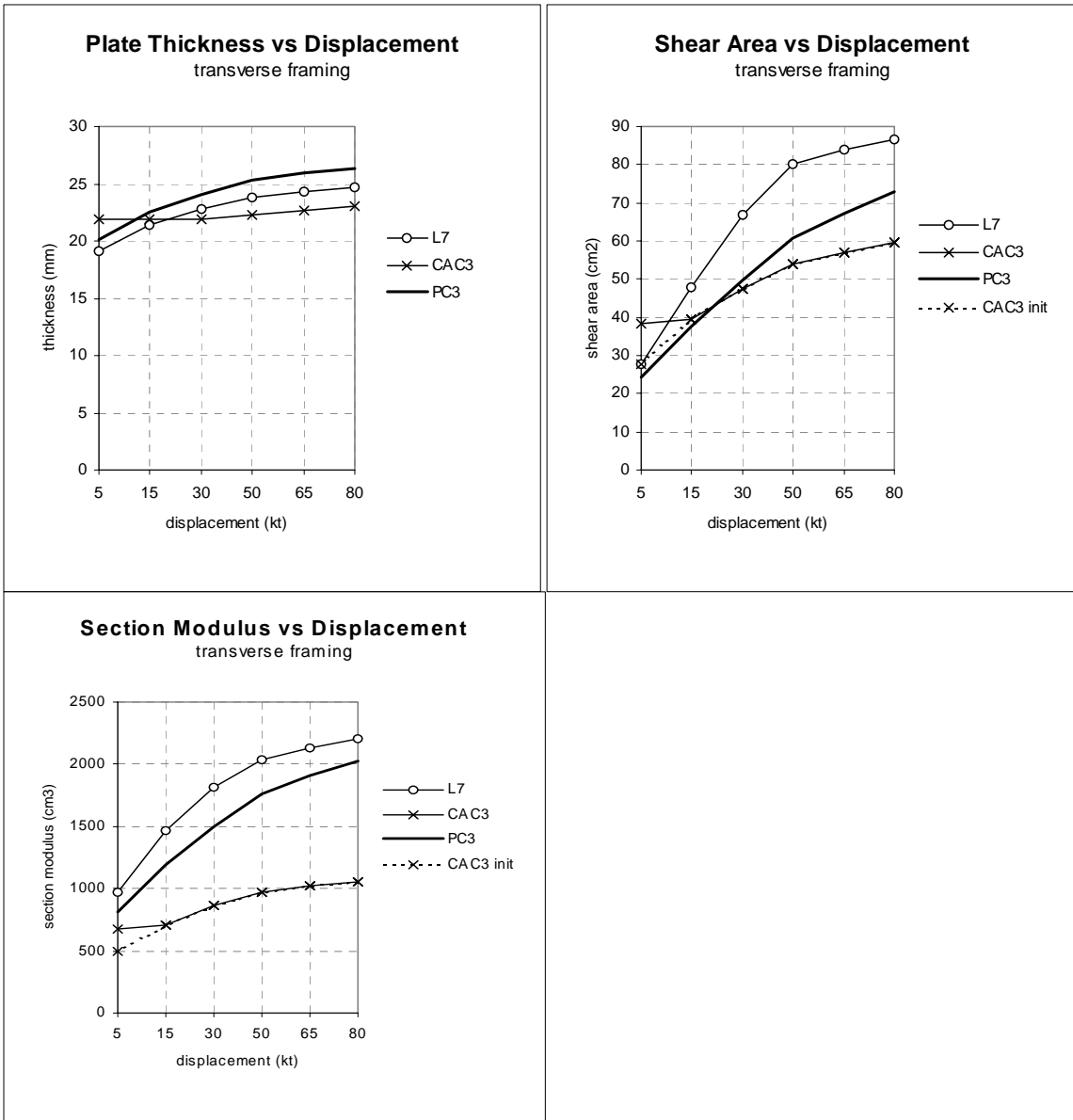


Figure 5.5: Scantlings versus Displacement, medium Polar class, Midbody

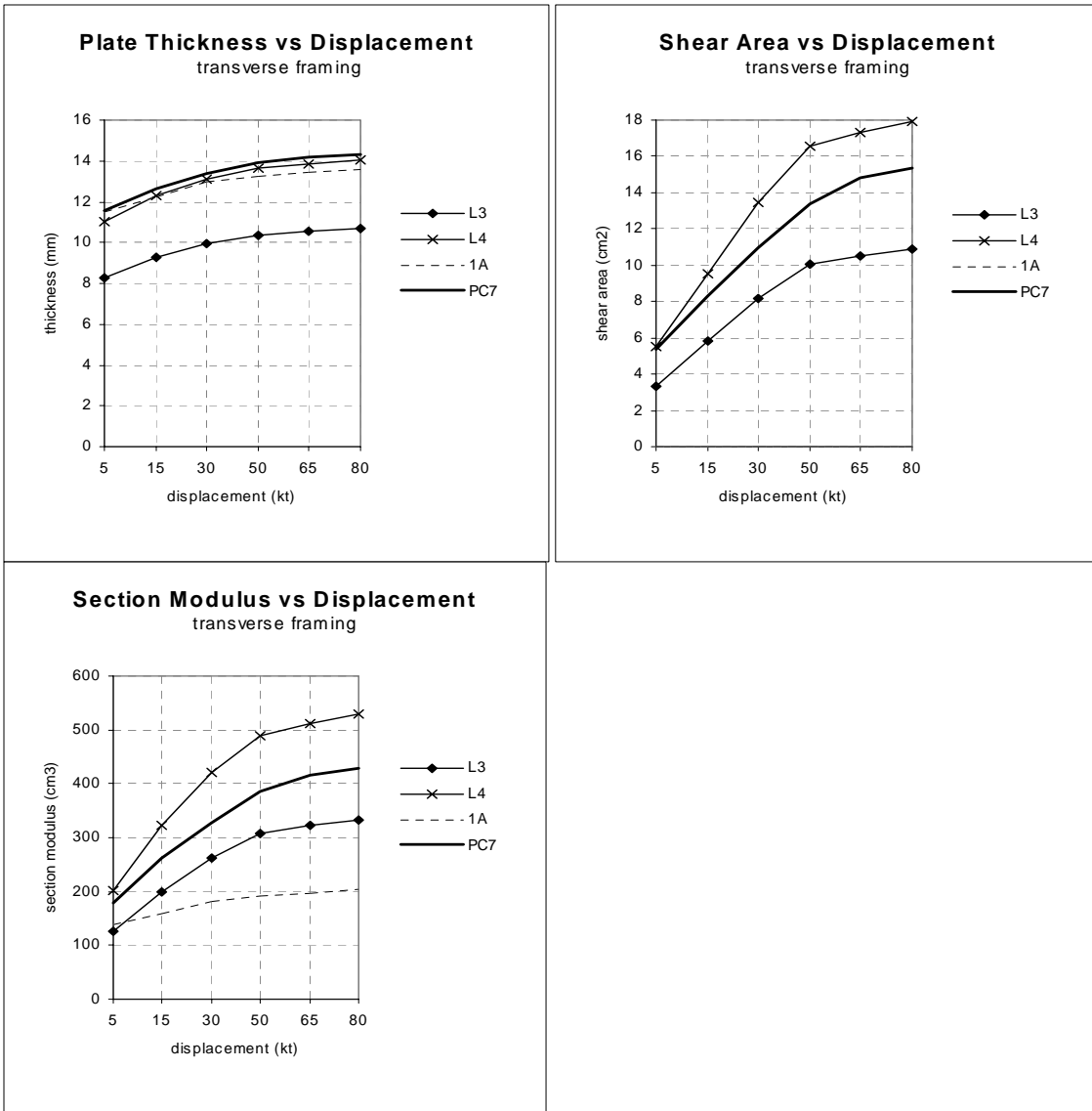


Figure 5.6: Scantlings versus Displacement, Lowest Polar Class, Midbody

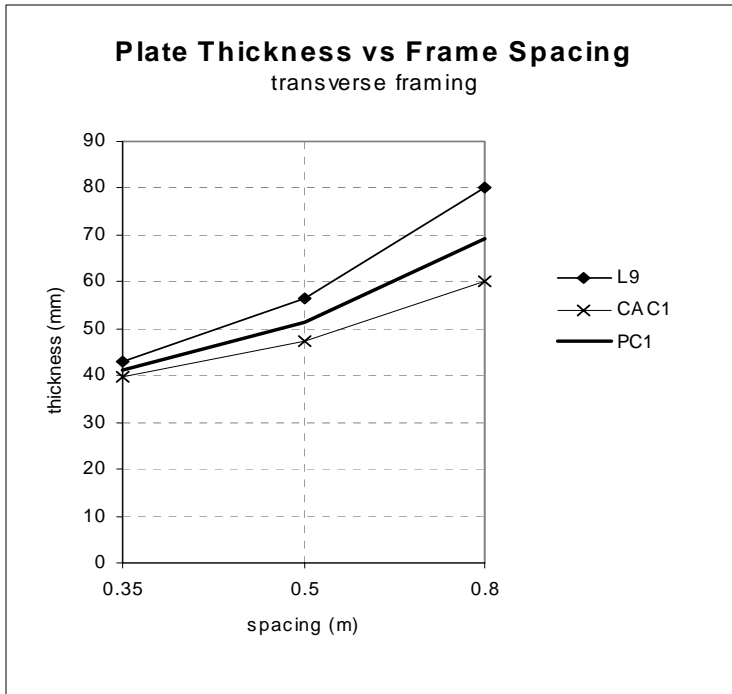


Figure 5.7: Plate Thickness vs Frame Spacing (example for highest Polar Class)

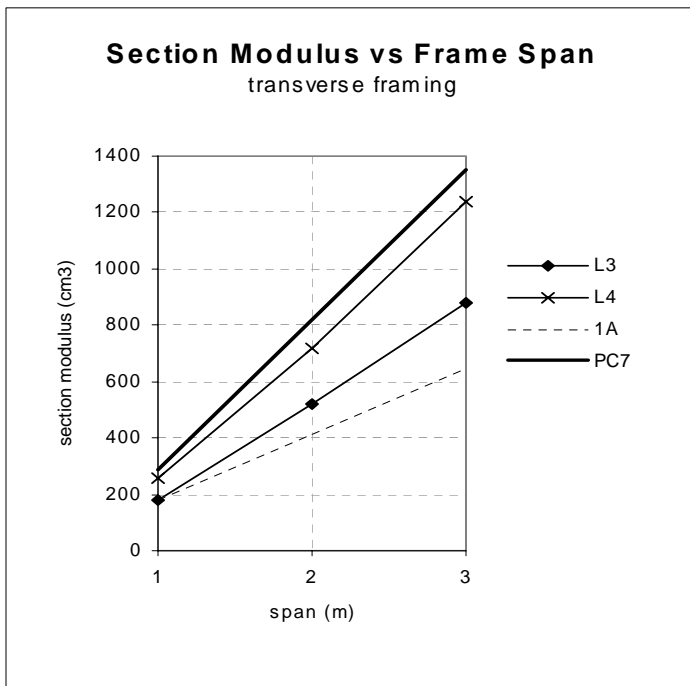
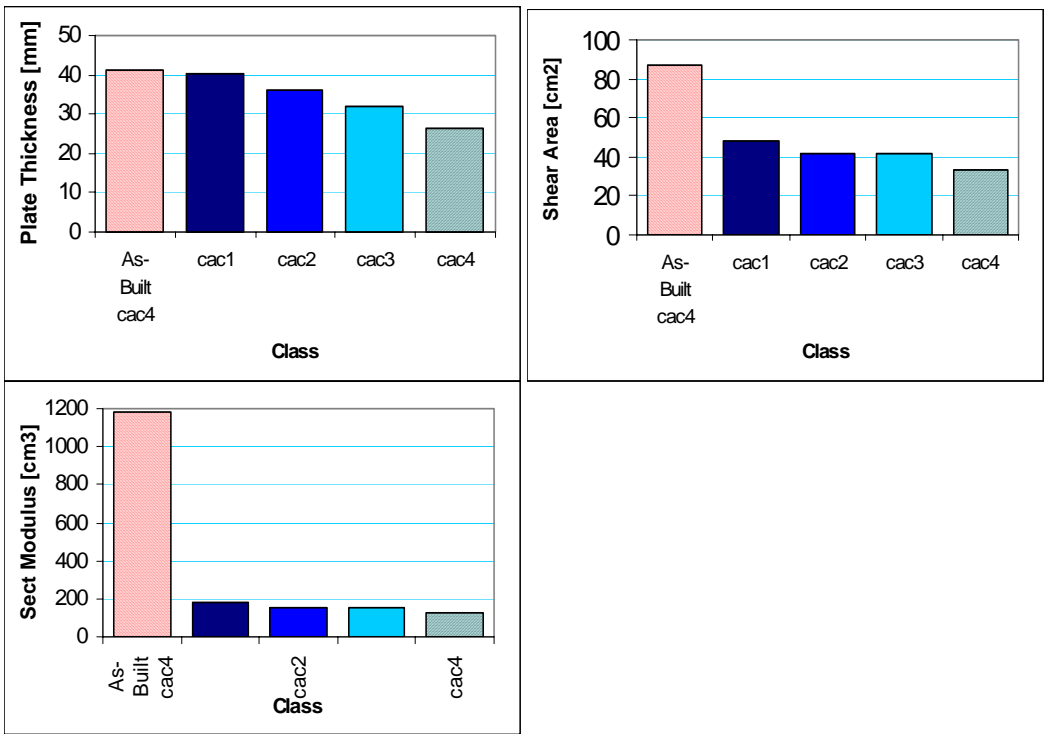


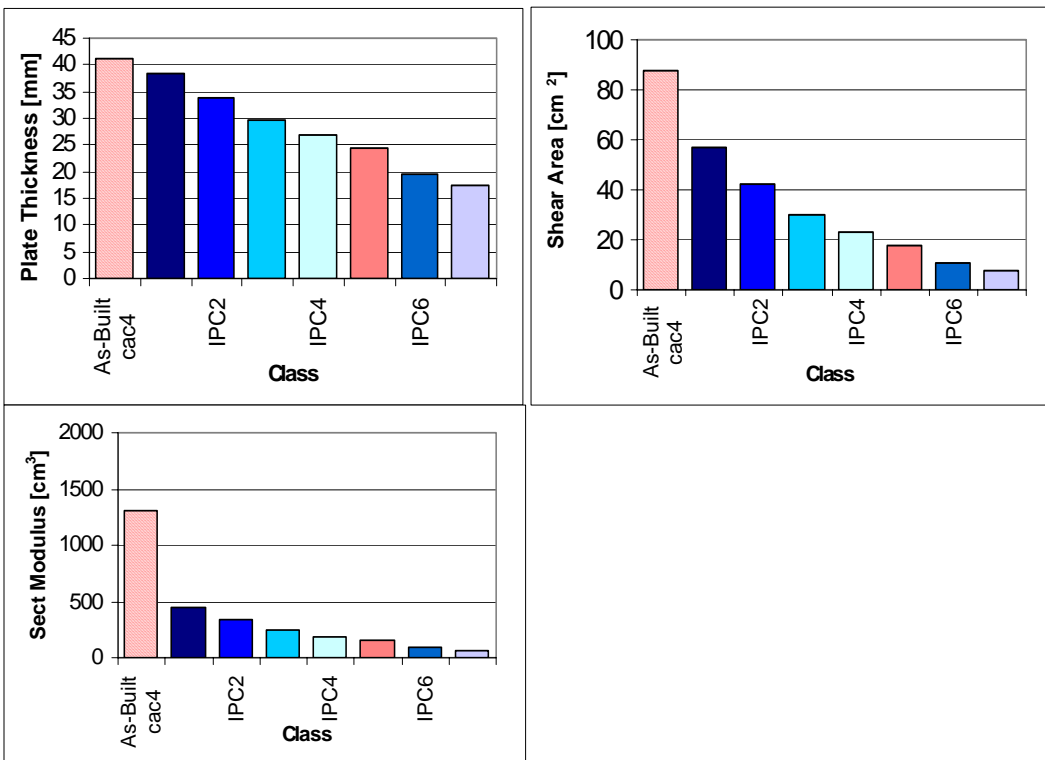
Figure 5.8: Section Modulus vs Frame Span (Example for Lowest Polar Class)

Comparisons with existing ships can also be made, but their interpretation is not straightforward. No existing ships are designed exactly to any set of rules, and almost no 'polar class' vessel has been built under any of the current rule systems. There is also limited agreement on where any existing ship 'should' fall under the new system. With these caveats, figures 5.9 and 5.10 below show where certain as-built scantlings fall for a high ice class (CAC 1/2) icebreaker and a Baltic 1A cargo ship respectively. The former, successful ship would comply in all respects with the new proposals, and would be somewhat better balanced against them than against the CAC system. The latter ship would be compliant with the PC7 requirements, and again would match these somewhat more closely than its nominal Baltic class. This ship has suffered some damage during Baltic service, but as a result of structural instability of deep members, which is not covered under the Baltic rules.

Further comparisons with both successful and less successful existing ships are under way, and will be more fully reported at a later date. At present, no cases have been identified where a satisfactory design would need additional steelweight to comply with its new (broadly) equivalent class.

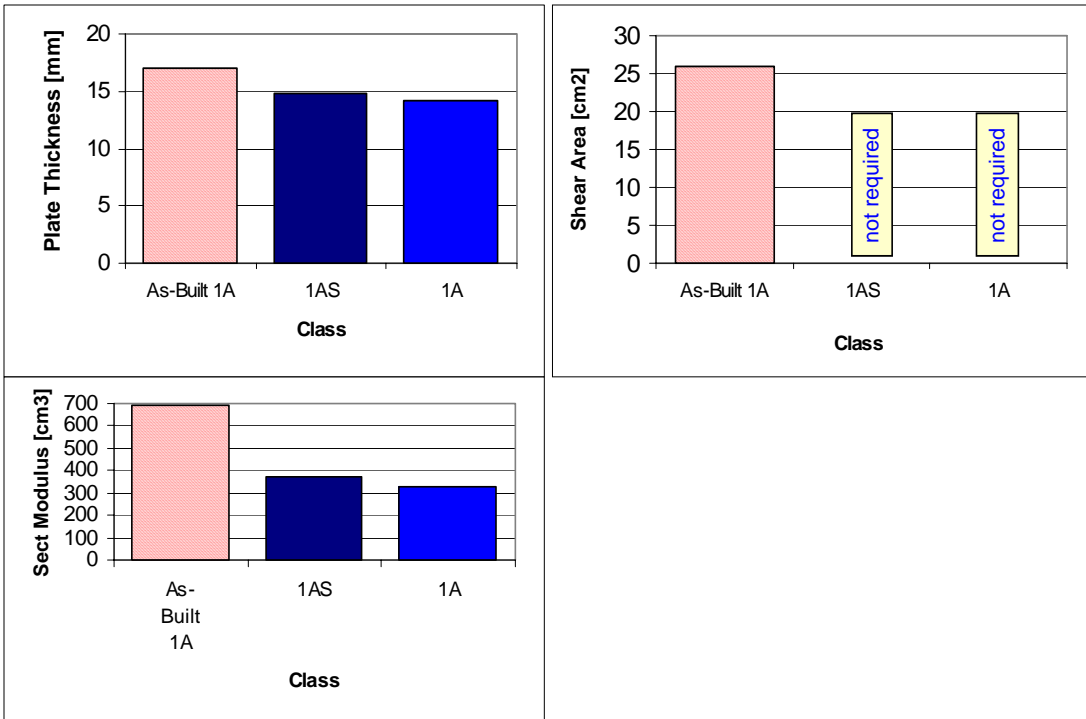


Ship 1, Bow Area; Comparison with CAC

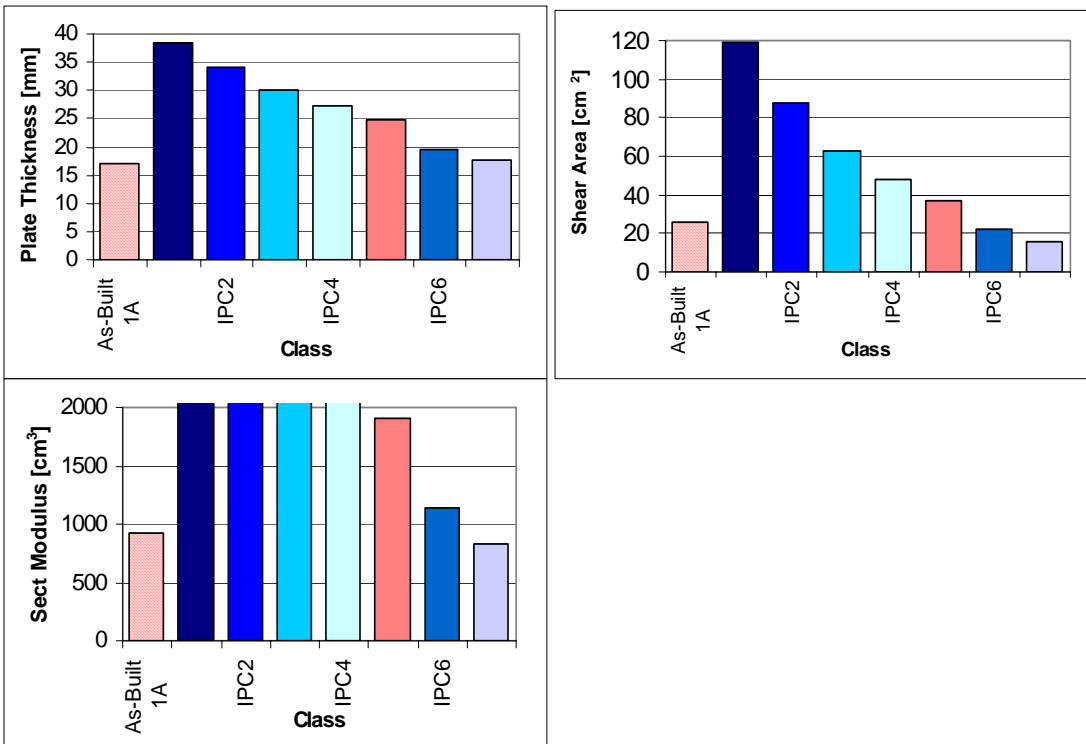


Ship 1, Bow Area, comparison with Proposals

Figure 5.9: As-Built, Icebreaker



Ship 6, Bow Area; Comparison with Baltic Rules



Ship 6, Bow Area; Comparison with Proposals

Figure 10: As-Built, Cargo Vessel

5.5 Outstanding Issues

The spreadsheet has aimed to align Polar and Baltic requirements in the midbody plating of ships of roughly 30,000 tonnes (PC7, Baltic 1A). The formulae have not been adjusted to provide the displacement effects found in the Baltic Rules above the ice flexural limits, and no additional measures have been taken to improve alignment between the rule systems. This subject will be addressed in more depth in a later paper.

The scantling formulae used are acceptable to the bilateral project team, but have not yet been reviewed fully by all other SWG participants. If modifications are introduced, it can be anticipated that other class factors and coefficients will change to maintain (roughly) the same scantling requirements.

6. CONCLUSIONS

The proposed load model described above is acceptable in principle to all participants in the bilateral project. It takes an approach which is reasonably consistent with the theories preferred by the Western and Russian sides, and which is amenable to modification once better data and theoretical models become available. The form of the model is also suitable for use in navigation control applications, which is important to other aspects of the Polar Code system.

A spread of capabilities covering the desired range for Polar Classes can be provided by the model, using class parameters which appear reasonable in practical terms. The lower end of the class range can be matched to the Baltic Classes quite closely by applying similar cut-offs to displacement dependencies. The separations between classes can be adjusted if desired to provide more exact matches to certain existing ships.

More work needs to be done to establish angle dependencies in the bow area, and to ensure that these do not have excessive implications for the remainder of the ship. Additional proposals and discussion papers on these issues will be tabled as soon as they can be generated and tested.

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- [12] Krylov Shipbuilding Research Institute Russian paper on load concentration, Oslo
- [13] Minutes of New York meetings of IACS SWG

APPENDIX A : Derivation of the Oblique Collision Force

In the following material, the force that results from a ship striking and ice edge is derived. The mechanics are based on the Popov collision, modified to include a wedge shaped ice edge and a pressure/area ice indentation model.

The force is found by equating the normal kinetic energy with the ice crushing energy;

$$KE_N = E_{\text{crush}}$$

$$\frac{1}{2} M_e V_N^2 = \int_0^{\delta} F_N(\delta) d\delta$$

where δ = normal ice penetration

F_N = normal force

$M_e = M/C_0$

$V_N = V_{\text{ship}} \cos \alpha$

The ice penetration equation is found by determining the nominal area that results from a penetration δ .

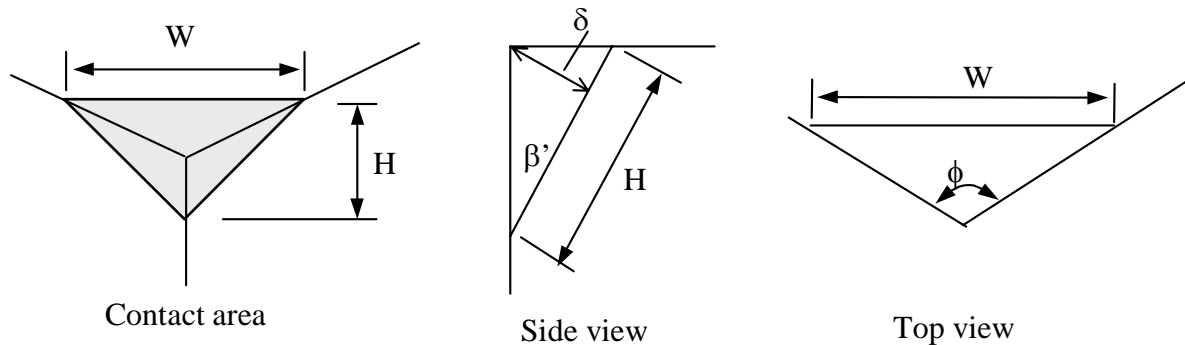


Figure A: 1 Nominal Contact Geometry

The nominal contact area is;

$$\text{Area} = W/2 \times H$$

The width (W) and height (H) of the nominal contact area can be determined by the normal penetration depth (δ) along with the normal frame angle (β') and the ice edge angle (ϕ).

$$W = 2 \delta \tan(\phi/2) / \cos(\beta')$$

$$H = \delta / (\sin(\beta') \cos(\beta'))$$

Hence the area is ;

$$A = \delta^2 \tan(\phi/2) / (\cos^2(\beta') \sin(\beta'))$$

The average pressure is;

$$P = P_o A^{ex}$$

The normal force is;

$$F_N(\delta) = P A = P_o A^{1+ex}$$

Substituting the expression for area gives;

$$\begin{aligned} F_N(\delta) &= P_o (\delta^2 \tan(\phi/2) / (\cos^2(\beta') \sin(\beta')))^{1+ex} \\ &= P_o fa^{1+ex} \delta^{2+2ex} \end{aligned}$$

where

$$fa = \tan(\phi/2) / (\cos^2(\beta') \sin(\beta'))$$

We can now solve the energy balance equation to find the maximum penetration.

$$\begin{aligned} \frac{1}{2} M_e V_N^2 &= \int_0^\delta F_N(\delta) d\delta \\ &= P_o fa^{1+ex} \int_0^\delta \delta^{2+2ex} d\delta \end{aligned}$$

We can extract the maximum penetration;

$$\delta = (\frac{1}{2} M_e V_N^2 (3+2ex) / (P_o fa^{1+ex}))^{1/(3+2ex)}$$

This Can in turn be substituted into the expression for force to give;

$$F_{max} = P_o fa^{1+ex} (\frac{1}{2} M_e V_N^2 (3+2ex) / (P_o fa^{1+ex}))^{(2+2ex)/(3+2ex)}$$

This can be somewhat simplified to give;

$$F_{max} = P_o^{1/(3+2ex)} fa^{(1+ex)/(3+2ex)} (\frac{1}{2} M_e V_N^2 (3+2ex))^{(2+2ex)/(3+2ex)}$$

Which for $ex = -0.1$ gives;

$$F_{max} = P_o^{0.357} fa^{0.321} 1.4 M_e^{0.643} V_N^{1.286}$$

APPENDIX B: Description of the Mass Reduction Coefficient C_o

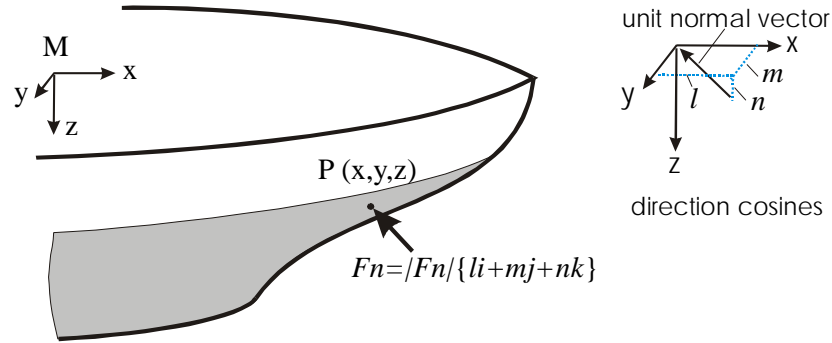


Figure B: 1 Collision point geometry

A collision taking place at point 'P' (see Figure B: 1), will result in a normal force F_n . Point P will accelerate, and a component of the acceleration will be along the normal vector, with a magnitude a_n . The collision can be modeled as if point P were a single mass (a 1 degree of freedom system) with an equivalent mass Me of;

$$Me = F_n/a_n$$

The equivalent mass is a function of the inertial properties (mass, radii of gyration, hull angles and moment arms) of the ship. The equivalent mass is linearly proportional to the mass (displacement) of the vessel, and can be expressed as;

$$Me = M/C_o$$

where C_o is the mass reduction coefficient. This approach was first developed by Popov (1972).

The inertial properties of the vessel are as follows;

Hull angles at point P:

α : waterline angle

β : frame angle

β' : normal frame angle

γ : sheer angle

The various angles are related as follows:

$$\tan(\beta) = \tan(\alpha) \tan(\gamma)$$

$$\tan(\beta') = \tan(\beta) \tan(\alpha)$$

Based on these angles, the direction cosines, l, m, n are

$$\begin{aligned}l &= \sin(\alpha) \cos(\beta') \\m &= \cos(\alpha) \cos(\beta') \\n &= \sin(\beta')\end{aligned}$$

and the moment arms are;

$$\begin{aligned}\lambda_1 &= ny - mz \quad (\text{roll moment arm}) \\ \mu_1 &= lz - nx \quad (\text{pitch moment arm}) \\ \eta_1 &= mx - ly \quad (\text{yaw moment arm})\end{aligned}$$

The added mass terms are as follows (from Popov);

$$\begin{aligned}\text{AM}_x &= \text{added mass factor in surge} = 0 \\ \text{AM}_y &= \text{added mass factor in sway} = 2 T/B \\ \text{AM}_z &= \text{added mass factor in heave} = 2/3 (B C_{wp}^2)/(T(C_b(1+C_{wp}))) \\ \text{AM}_{rol} &= \text{added mass factor in roll} = 0.25 \\ \text{AM}_{pit} &= \text{added mass factor in pitch} = B/((T(3-2C_{wp})(3-C_{wp})) \\ \text{AM}_{yaw} &= \text{added mass factor in yaw} = 0.3 + 0.05 L/B\end{aligned}$$

The mass radii of gyration (squared) are;

$$\begin{aligned}r_x^2 &= C_{wp} B^2/(11.4 C_m) + H^2/12 \quad (\text{roll}) \\ r_y^2 &= 0.07 C_{wp} L^2 \quad (\text{pitch}) \\ r_z^2 &= L^2/16 \quad (\text{yaw})\end{aligned}$$

With the above quantities defined, the mass reduction coefficient is;

$$\begin{aligned}C_o &= l^2/(1+\text{AM}_x) + m^2/(1+\text{AM}_y) + n^2/(1+\text{AM}_z) \\ &\quad + \lambda_1^2/(r_x^2(1+\text{AM}_{rol}) + \mu_1^2/(r_y^2 (1+\text{AM}_{pit})) + \eta_1^2/(r_z^2 (1+\text{AM}_{yaw}))\end{aligned}$$