Engineering 9867 Advanced Computing Concepts  
Assignment #1  
Due: Tuesday, March 12 at 0900

1. [10 points] Express the following in predicate logic, using the given predicate symbols 
   and types.

   a) [3 points] There is a smallest integer.
      Predicates: ≤
      Types: Integer

   b) [3 points] The array \( A[N] \) is bitonic. (An array is said to be bitonic iff the 
      elements are in non-decreasing order in some initial portion of the array, and in non-
      increasing order for the remainder. For example, \([1, 1, 2, 3, 4, 4, 2, 1]\) is bitonic, but 
      \([1, 1, 2, 3, 4, 3, 4, 2, 1]\) is not.)
      Predicates: <, ≤, >, ≥

   c) [4 points] The definition of “\( \lim_{x \to a} f(x) = L \)”. (Hint: Quantify variables \( x, \epsilon \) and \( \delta \) 
      over Real and relate \( |f(x) - L| \) to \( \epsilon \) and \( |x - a| \) to \( \delta \).)
      Predicates: <, ≤
      Types: Real

2. [10 points] A permutation of an array is an array containing exactly the same values in 
   another order, i.e.,
   \[
   \text{permutation}(a, b) \triangleq \begin{cases} 
   \text{length}(a) = \text{length}(b) \land \\
   \forall i, (0 \leq i < \text{length}(a) \rightarrow (\text{card}(\{ j | 0 \leq j < \text{length}(b) \land a[i] = b[j] \}) = \text{card}(\{ j | 0 \leq j < \text{length}(a) \land a[i] = a[j] \})) 
   \end{cases}
   \]
   Prove that the number of permutations of an array of length \( N \) is \( N! \).

3. [15 points] In this question you are to reason about a C++ function \texttt{int gcd(int x, int y)} 
   which returns the greatest common divisor of the natural numbers \( x \) and \( y \).

   a) [5 points] Give the specification for this function. You may find it helpful to recall 
      that any common divisor, \( d \), of natural numbers \( x \) and \( y \), will also be a divisor of the 
      GCD of \( x \) and \( y \). You may use the following predicate in your specification:
      \[
      \text{divisor}(d, x) \triangleq (\exists q : \text{int}, 0 < q \land x = d \times q)
      \]

   b) [10 points] Implement the function in C++ and add comments to your implementation 
      to reason, as formally as possible, that it is correct. You may find it helpful to recall 
      the property of natural numbers, that
      \[
      \forall x, y : \text{int}, (0 \leq x \land 0 \leq y) \rightarrow \gcd(x, y) = \gcd(y, x \% y)
      \]
4. [15 points] A palindrome is a string that is the same when read forward and backward. Some examples of palindromes are “ABBA”, “radar” and “20022022002”. In this question you are to reason about a C++ function `bool isPalindrome(const string& s)`, which returns `true` if `s` is a palindrome and `false` otherwise.

a) [5 points] Give the specification for this function.

b) [10 points] Implement the function in C++ and add comments to your implementation to reason, as formally as possible, that it is correct.