Engineering 9867 Advanced Computing Concepts
Assignment #2

Due: Tuesday, April 2 at 0900

1. [10 points] Consider the following implementation of the palindrome checking problem (question 4 on assignment 1):

```
bool
isPalindrome(const string& s)
{
  bool result = true;
  int size = s.size();
  int i = 0;

  while (result && i < size/2) {
    result = (s[i] == s[size-1-i]);
    i++;
  }
  return result;
}
```

   a) [5 points] Give an expression for the worst case time (note **not** complexity) of this algorithm.

   b) [5 points] Assuming that the strings contain only of the letters “a” and/or “b”, and that all strings are equally likely, what is the average case time for this algorithm?

2. [10 points] Consider the following algorithm that satisfies the specification as follows:

   Pre: $1 \leq \mathtt{m} \leq \mathtt{n}$

   Post: $\mathtt{result} \leq \mathtt{n} - \mathtt{m} \rightarrow (\forall i, 0 \leq i < \mathtt{m} \rightarrow \mathtt{P}[i] = \mathtt{S}[\mathtt{result} + i]) \ \wedge$
   $\mathtt{result} > \mathtt{n} - \mathtt{m} \rightarrow (\neg \exists j, 0 \leq j \leq \mathtt{n} - \mathtt{m} \wedge (\forall i, 0 \leq i < \mathtt{m} \rightarrow \mathtt{P}[i] = \mathtt{S}[j + i]))$

```
result = −1
matched = false
while (result < n − m ∧ ¬matched) do
  result = result + 1
  i = 0
  matched = true
  while (i < m ∧ matched) do
    matched = matched ∧ (P[i] == S[result + i])
    i = i + 1
  end while
end while
if (¬matched) then
  result = result + 1
end if
```

   a) [9 points] Give an expression for the exact worst case (i.e., Θ) complexity for this algorithm. Show your workings.

   b) [1 point] What does this tell us about the complexity of the problem solved by this algorithm?

3. [15 points] Give a (deterministic) finite state automata on the input language $\Sigma = \{0, 1\}$ accepting each of the following languages:

   a) [5 points] The set of all strings ending in 00.

   b) [5 points] The set of all strings with three consecutive 0's.

   c) [5 points] The set of all strings such that every block of five consecutive symbols contains at least two 0's.

4. [15 points] Let G be an undirected graph consisting of a set of nodes, N, and a set of edges $E \subseteq N \times N$. A set of nodes $N' \subseteq N$ is called a *vertex cover* for G, if for every edge in E, at least one of its end-points is in N'. The *vertex cover problem* is, given a graph, G, and a positive integer, K, determine whether there is a vertex cover for G with at most K nodes.

   Show that the vertex cover problem is NP-complete.