Fall 2009

Engineering 1040
Mechanisms and Electric Circuits

Electric Circuits Module

by
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This module deals with the Electric Circuits portion of the Engineering 1040 course. The early sections of the module are designed to briefly review some of the concepts encountered in various science courses in secondary school (high school), inasmuch as these are important in providing a basic background to elementary ideas in electric circuits. Many of the topics covered here will be dealt with in much greater detail in more senior courses. Furthermore, some of the fundamental **equations which appear in this module in elementary form will need modification** before they can be applied to a more general treatment of the material. This will be an extremely important point to remember, as you progress through your engineering program.

For the most part, the elementary circuit theory that you encounter here will not be different than that encountered in high school, but in some cases the approach may be a little more general and the problems may be slightly more challenging. Wherever feasible, we will seek to 'prove' any of the formulations that we use which may be founded in various axioms or laws.

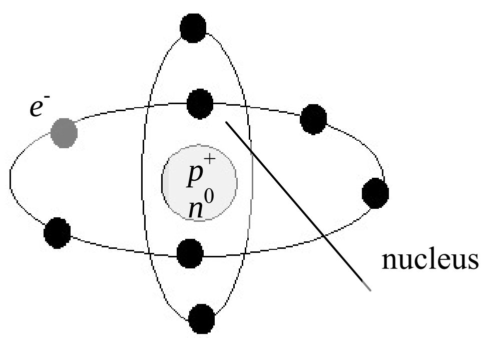
Unit E1: Basic Concepts

You will almost certainly have met much of what we consider here in your earlier studies. It is intended only as a brief review.

E1.1 Electric Charge and Materials

E1.1.1 Charge

The atom, the elementary unit of a material which contains the properties of that material, may be thought of in a simplistic way as consisting of *negatively charged* ***electrons*** orbiting a *positively charged* **nucleus**, which itself consists of *positively charged* ***protons*** and *neutrally charged* ***neutrons***. This arbitrary assigning of *positive* and *negative* can be attributed to terminology used by Benjamin Franklin in the eighteenth century.



electrons – e^-

protons – p^+

neutrons – n^0

Figure E1.1 The atom.

The **unit of electric charge** is the **coulomb**, symbolized by C. The quantity of charge, in general, is often symbolized by *q* (often reserved for time-varying charge) or *Q* (for a fixed amount of charge). A single electron has a charge of -1.60×10^{-19} C, and the proton which is equally, but oppositely charged carries a charge of $+1.60 \times 10^{-19}$ C. Thus, there are 6.25×10^{18} electrons or protons in a charge whose size is 1 C.

When a material has more electrons than protons then it will be *negatively charged*. On the other hand, if more protons than electrons exist in a body, then it will be *positively charged*.

It is an observed feature of charge that if two charges have the *same polarity* (i.e. both are negative or both are positive) then the charges will *repel* each other. However, two charges of *opposite polarity* (i.e. one positive and one negative) will *attract* each other.

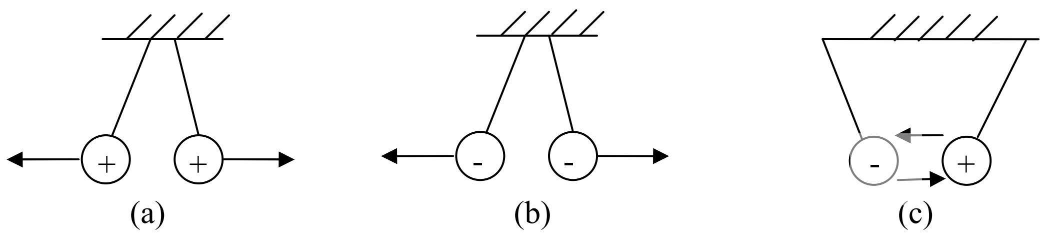


Figure E1.2 Like charges (a) and (b) repel; unlike charges (c) attract.

The **force** (F) measured in **newtons** (N) between two charges Q_1 and Q_2 which are separated by a **distance** (d) in **metres** (m) is *proportional to* the product $Q_1 Q_2$ of the charges and is *inversely proportional to* the separation between them. This statement is known as Coulomb's law and using k as the proportionality constant we write

$$\boxed{F = k\frac{Q_1 Q_2}{d^2}} \quad \ldots\ldots\ldots \qquad \text{(E1.1)}$$

where it is known that $k = 8.99 \times 10^9\ \mathrm{N{\cdot}m^2/C^2}$ if there is empty space (vacuum) or, to a good approximation, air between the charges. You may recall that a **newton** is the size of the *net force* required to accelerate (or decelerate) a mass of 1 kilogram at a rate of 1 metre per second per second (i.e the velocity of the mass changes by 1 m/s each second and the acceleration is 1 m/s^2).

With these notions in mind, it is convenient to consider the concept of an ***electric field***; that is, a region where an electric charge brought into that region may experience a force being exerted upon it. By convention, the *direction* of an electric field is the direction of the force exerted on a *small positive* ***test charge*** brought into that field. Using this convention, a field caused by a negative charge points toward that charge while the field due to a positive charge points away from the charge as seen in Figure E1.3. Notice, then, that the force fields of the 'point' charges are distributed *radially* around the charges. The charges themselves are *sources* of electric fields.

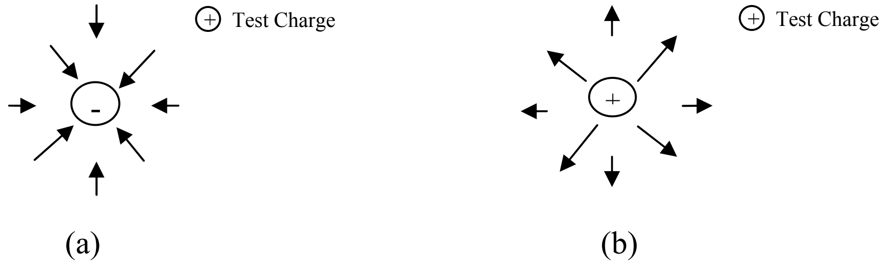
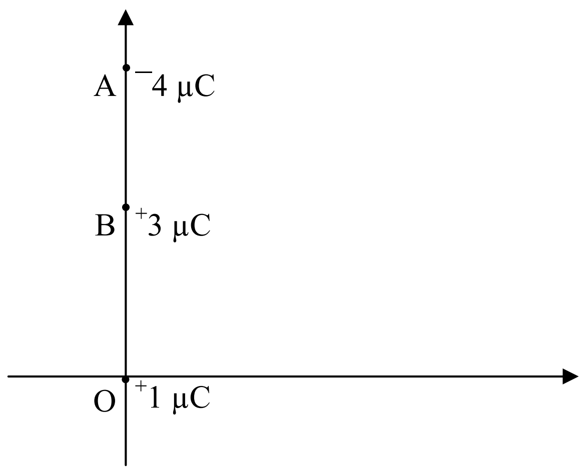


Figure E1.3 The electric field points (a) towards a negative charge and (b) away from a positive charge.

Example: A charge of +3 μC is located on the y-axis at (0, 1 cm) and a charge of –4 μC is located at (0, 2 cm) on the *x*-axis. Determine the size and direction of the total force that these two charges exert on a +1 μC charge residing at the origin (O).



E1.1.2 Materials – Conductors, Insulators and Semiconductors

In some materials the outer (or *valence*) electrons of the atoms may move freely from atom to atom. Copper is such a material (along with many others). Such materials are said to be good ***conductors***. That is, an ***electric current*** – defined as the time rate of flow of electric charge – may be set up easily in such materials (more notes shortly on this).

In other materials even the most energetic electrons are bound very tightly to their nuclei and it is extremely difficult to cause the electrons to move, in a bulk sense, from one position in the material to another. Such materials are good ***insulators.*** They do not conduct electric current easily and ideally they don't conduct at all. These materials, which may *store* charge, but **not** *conduct* charge are also referred to as ***dielectrics***. Glass, plastic, and air are a few such examples. While you may not know why, you do know from observation that even such materials can be made to conduct under extreme circumstances: think, for example, of lightening travelling between clouds or between clouds and the earth through the air. Under what we might vaguely say are 'normal' conditions, however, insulators do not conduct appreciably. Thus, for example, *copper wire conductors* used in house wiring are wrapped in *plastic insulation*.

An intermediate group of materials that can be classed *neither* as good conductors nor good insulators is referred to as ***semiconductors***. In such materials, which include carbon, silicon, and germanium, the numbers of electrons free to move around from atom to atom are far fewer than in conductors, but significantly more than in insulators. We will not consider these directly in this course, but many of the lab components and electronic devices that will be encountered will consist of such materials.

E1.2 Concepts and Definitions Associated with a Simple Electric Circuit

An ***electric circuit*** is a path around which electric charge may flow. What causes such an electric current to flow and the quantities, along with their symbols and units, which are basic to a description of this flow, will be briefly considered in this section. The most fundamental relationship between these quantities will appear in Section E1.2.4.

E1.2.1 Electric Current

As noted already, an ***electric current*** is defined as *the time rate of flow of electric charge* pass a given point. In terms of this course, the point will be in a conductor. Using the symbol *i* for current and *t* for time, the simplest formulation of this definition, which is valid when the *rate of charge flow is a constant*, may be given as

$$\boxed{i = \frac{q}{t}} \quad \ldots\ldots\ldots \qquad \text{(E1.2)}$$

Here, as usual, q is measured in coulombs (C), while time is measured in seconds (s). Thus, current is measured in coulombs per second (C/s), which is given the special name of **amperes** (A). That is, when 1 C of charge passes a point in 1 s, the current is 1 A. Notice that for a constant value of i, say I, flowing for a time $t = T$, the total charge passing a particular point is simply the area 'under' the "i versus t" graph as shown in the sketch below:

Example: The current in a circuit is 2 A. How many electrons pass a given point in the circuit in 1.5 ms?

Solution:

E1.2.2 Work, Potential and Potential Difference

Work and Energy

When a constant **force** (*F*) acts (or is exerted) through a **distance** (*d*) which is in the direction of the force, the **work** (*W*) done by the force is simply the product of the two quantities. That is, for this *special situation*,

Work = Force · distance .

Symbolically,

$$\boxed{W = F \cdot d} \quad .... \qquad \text{(E1.3)}$$

The **unit** for work is the **joule** (J) and 1 J equals the amount of work done when a force of 1 N is exerted through a distance of 1 m. Notice that for a constant force acting through a distance *D*, the work done is simply the area 'under' the "*force* versus *distance*" graph as shown in the sketch below:

As is the case for the current-time situation discussed in section E1.2.1, these graphical ideas can be extended to more general cases when you have encountered the concept of *integration* in calculus.

Finally we note that ***energy*** is the ability to do work and, in particular, ***potential energy***, is the energy stored in an object (or a charge) due to its *position* in a force field.

Example: 100 J of work is expended in moving a charge a distance of 50 cm. What was the average force required to do this? What was the change in the potential energy of the charge? **(Assume no change in the velocity of the charge.)**

Potential and Potential Difference

Consider again, a positive charge Q which, of course, has an associated force field as shown by the arrows in Figure E1.4. Into this field a small positive charge q is brought

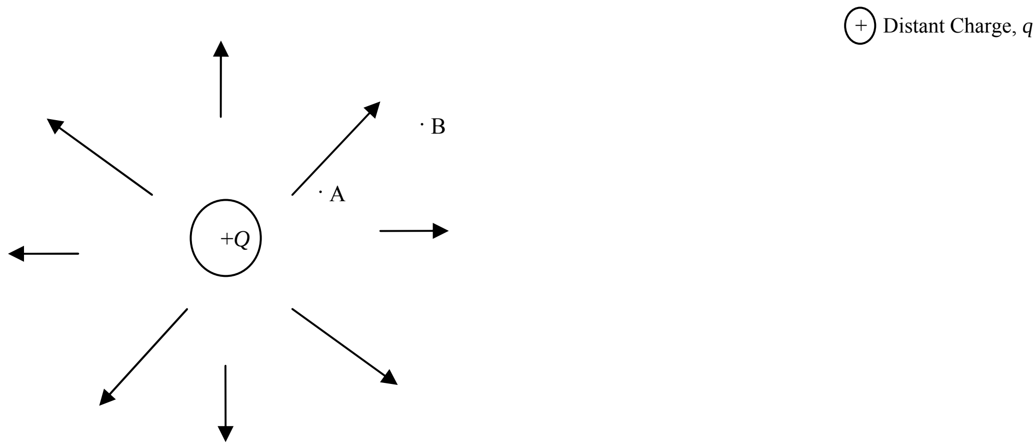


Figure E1.4 Illustration for work done on a charge q to bring it into the force field of Q.

from a very long distance away (ideally, infinity). Here, infinity is simply an appropriate reference from which to begin. The more general ideas may be encountered in further courses.

The ***potential*** at point B in the force field of Q is, by definition, the *work per unit charge* done on q to bring it to point B from very far away. Of course, to bring q to point A, which is even closer to Q, requires more work, since, for one thing, the average force required will be bigger in this case (can you see this from equations (E1.1) and (E1.3)?). Thus, at point A, the ***potential*** is greater (i.e. higher) than at point B. From the definition, we see that the **unit** for potential will be the unit for work divided by the unit for charge – i.e. **joule/coulomb** (J/C), which is given the special name of **volt** (V).

In view of the above discussion, a ***potential difference*** exists between points A and B in the force field of Q. Let v_A denote the potential at A, v_B the potential at B and v_{AB} the potential difference between points A and B. Then, in units of volts,

$$\boxed{v_{AB} = v_A - v_B} \quad \text{……..} \quad \text{(E1.4)}$$

Note, again, that if q is at point B, then more work must be done on q to move it to point A, a position of higher potential. Conversely, if q is released from point A, then it will be forced to move further from Q and toward B – i.e., *the tendency of the charge is to move to a place in the* *field where the potential is lower*. In summary, *the* ***potential difference*** *between two points is equivalent to* ***the work per unit charge*** *done to move a charge between the two points*. Another way of stating this is that, since the *work done* in moving the charge equals the *change in potential energy*, the potential difference is equal to the difference in potential energy per unit charge.

In a simple electric circuit, potential differences are established by means of a **source** which forces charges out one **terminal** of the source and into another which is at a lower potential and connected to the first by a conducting path. In doing so, the charges may pass through various kinds of circuit elements. The energy expended per unit charge (which is the same as the work done *on* the charge) in moving it from one terminal to another *inside* the source is referred to as the ***electromotive force*** (***emf***). This is equal in size to the potential difference between the terminals if the source – which we will, for the present, refer to as a *voltage* source – is not connected to a circuit. In fact, if the voltage source is *ideal*, it will by definition present the same potential difference to any circuit to which it is connected. That is, the same potential difference will be measured across the terminals of an ideal voltage source no matter how much current flows between the terminals. Of course, in reality, there is ***no such device*** as an *ideal voltage source* and we shall address this fact later.

The source of dc voltages may be provided by many different devices. For example, we are all familiar with the common chemical cells used in flashlights and cameras etc.. A combination of such cells is referred to as a *battery*. See Figure E1.5 which also shows the symbols for cells and batteries often used in circuit diagrams.

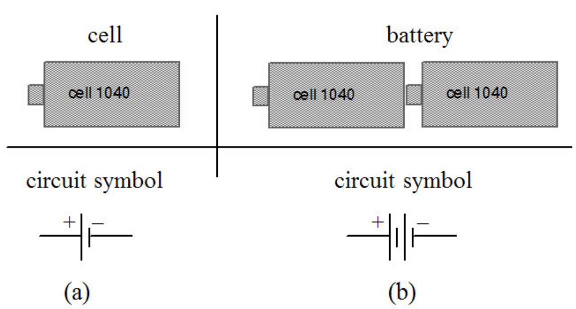


Figure E1.5 (a) A cell and (b) a battery and their circuit symbols. Notice the *polarities* shown on the symbols.

In Figure E1.5(b) the two cells, connected so that the negative terminal of one is joined to the positive terminal of the other, are said to be connected *in series*. The **total voltage** of the battery for the *series combination* will be the *sum of the two individual cell voltages*. It is also possible to connect similar cells such that two positive terminals and the two negative terminals are connected to each other in a so-called *parallel* connection (see Figure E1.6). The **total voltage** of the battery for the *parallel combination* is that of a *single cell*.

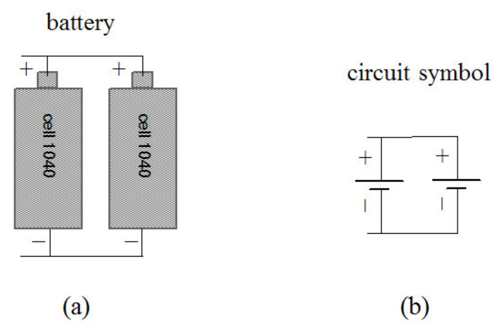


Figure E1.6 (a) A battery formed by connecting two cells *in parallel* and (b) the circuit symbol.

However, in the *parallel combination* of identical cells it is possible to show that when this battery forms a part of a circuit the **total current** output is the *sum of the outputs* from the individual cells, while in the *series combination* the **total current** output is that of *one cell*.

E1.2.3 A Simple Electric Circuit and Conventional Current

Having discussed the ideas of charge, current, and potential difference, we are now in a position to consider an electric circuit in which the current flows in only one direction – such a circuit is referred to as a ***direct current*** circuit or dc circuit. Such a circuit might be depicted in block form as in Figure E1.7. The load could be any simple or complicated combination of circuit elements which is energized by the source. In order for a current (i) to flow, the switch must be flipped to position A to form a *closed circuit*. The figure actually shows an *open circuit because* the switch is in the *open* position. In this case $i = 0$ (at least if the switch has been open for a long time). Notice that the direction we have chosen for the current is *opposite to the direction in which electrons would flow* when the switch is closed. This is the so called ***conventional current***. Instead of thinking of negative charges flowing out of the negative terminal of the source, we consider that an equal amount of positive charge flows out of the positive terminal and therefore, of course, in the direction which is opposite the electron flow. In the context of the

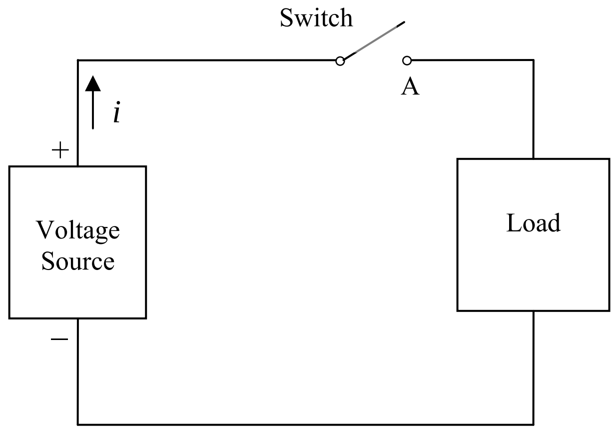


Figure E1.7 A block diagram of a circuit showing a switch and conventional current *i*.

previous section, these 'positive charges' flow from a region of high positive potential to one of lower positive potential (i.e. toward the negative terminal of the source). In so doing, they transfer energy to the **load** – this process may be *observed*, for example, as heat, light, or sound, or some combination of these forms of energy depending on the nature of the load (many examples may be considered, such as electric heaters, light bulbs, or any other electrical device). The transfer of energy to the load causes a decrease or *drop* in the voltage across the load. If the current flows in the direction of the voltage *drop*, then the current is considered to be *positive*. In any expression relating voltage and current we use a positive sign/negative sign whenever the reference direction for the current is in the direction of the reference voltage drop/voltage rise. This is the so-called **passive sign convention** (it is illustrated in Section E1.2.5 in our initial consideration of power). **This is the convention that will be used for current flow throughout this course** – it was a convention adopted before the electron was discovered!

E1.2.4 Resistance and Ohm's Law

Resistance

The **load** in an electric circuit may, of course, be very complicated electrically. In the context in which it appears in the previous section it would consist of ***passive circuit elements*** which *consume* electrical energy. (Note: the source voltage is an ***active elemen***t since it *supplies* energy.) In any real circuit there will be always be some *opposition* to the flow of charge as the moving charges collide with the atoms of the materials making up the circuit. The load, the connecting conducting wires, and even the materials of the source will provide some opposition to the current. This opposition to the flow of current is termed ***resistance*** (*R*) and its **unit** is the **ohm** (Ω). As you will learn elsewhere, the resistance of a conductor is dependent on (1) its length, (2) its cross sectional area and (3) the type of material and may also significantly depend on temperature. In this course, we will assume that the resistance in a circuit is a constant for that particular circuit under all operating conditions.

Let's reconsider Figure E1.7 with the blocks replaced by explicit elements including a voltage source *v* and a load resistance *R* as shown in Figure E1.8. Here we have *modeled* the original circuit by using the standard electrical symbols for a battery as the voltage source and a resistance symbol for the load (note a load could be a light bulb or any other actual device having a resistance – as long as the load is purely resistive, it is modeled by the symbol shown in the figure). Notice that the polarity on the resistance is *chosen* to correspond to that on the source. The direction of the current through the resistance is *chosen* to be in the direction of the *assigned* voltage drop. In keeping with the passive sign convention, we shall again see in Section E1.2.5 that this choice of signs allows the power *absorbed* by the resistor to maintain a positive value. We note that for *ideal* connecting wires (which have no resistance) all points along the wire are at the same potential since no energy is expended in moving charges along such a path. However, once a resistance is encountered, energy must be used to push the charges across such an element – that is, there is a potential difference between the ends of the resistor. This potential difference has been denoted by v_R, which in this case is equal to the voltage *v* of the source. Also, in this special case, the current i_R through the resistive element equals the current *i* of the source. These latter two facts will be explored in detail in Section E2.2.

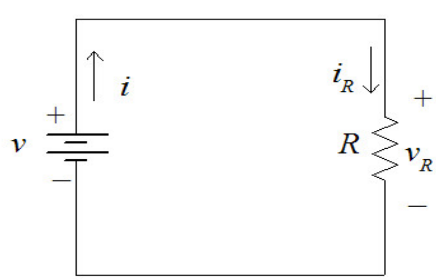


Figure E1.8 A schematic diagram of a simple circuit containing a source voltage and a load resistance.

Ohm's Law

In 1827, Georg Simon Ohm discovered that the current *i* developed in a circuit like that shown in Figure E1.8 was proportional to the voltage *v* supplied by the source. Another way of stating this is to say that the ratio of the voltage to the current is constant:

$$\frac{\text{voltage}}{\text{current}} = \text{constant}$$

Clearly, for a fixed value of source voltage, the constant increases as the current decreases. Thus, the ratio is indicative of an opposition to the flow of charges in the circuit. This is precisely the property of *resistance*. Often, as temperature changes the resistance of the load resistor changes. However, this ratio is a constant if the load is a **linear resistor,** and such elements, which maintain essentially constant value of resistance over a wide range of temperatures, are common.

We write Ohm's Law for the resistor symbolically as

$$\frac{v_R}{i_R} = R$$

or

$$\boxed{v_R = i_R R}$$

In general, Ohm's Law is written as

Ohm's Law $\boxed{v = iR}$ ….. (E1.5)

where *v* and *i* are the voltage and current, respectively, associated with the resistance *R*.
Equation (E1.5) holds for *any* resistance in a circuit, even if the resistance is **nonlinear**. A graph indicating the typical ***voltage-current characteristics*** of both linear and a nonlinear resistors is shown in Figure E1.9.

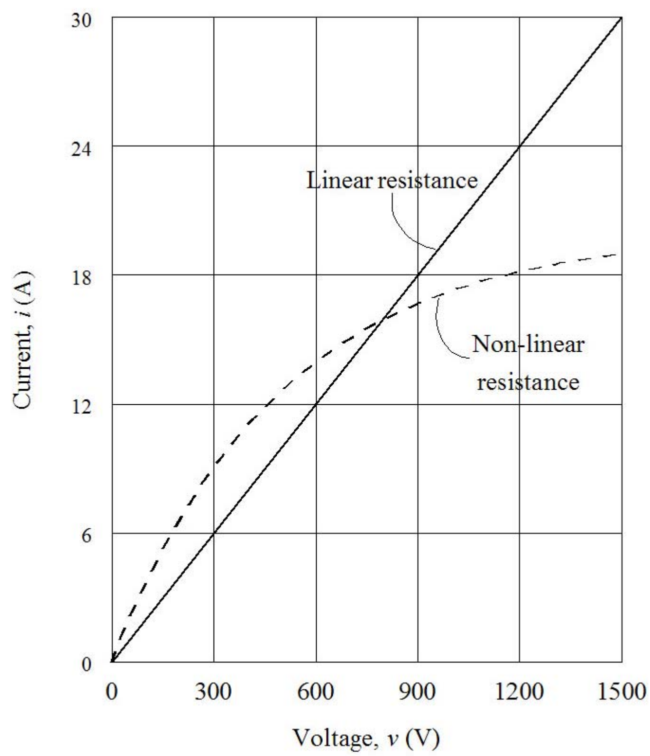


Figure E1.9 Voltage-current characteristics for a linear and non-linear resistor.

Example: Determine the resistance of the linear resistor whose voltage-current characteristic is shown in Figure E1.9. Can you suggest how one might determine the *dynamic* value of the changing, non-linear resistance for a particular voltage?

The circuits encountered later in this course will deal only with linear resistors. However, it is possible to construct linear resistors whose values may be manually changed, as happens for example in some dimmer switches used to control a light or in the volume control of an audio device or in the speed control of a motor. Physically this may be done by adjusting the 'amount' of the resistor which actually appears in the circuit. Consider, for example, Figure E1.10(a). Here the conducting 'arrow' may be moved along the resistor to put a variable amount of resistance into the circuit. Types of variable resistors that may be encountered are ***rheostats*** and ***potentiometers***, which differ from each other mainly in the number of connections that they make within the circuit.

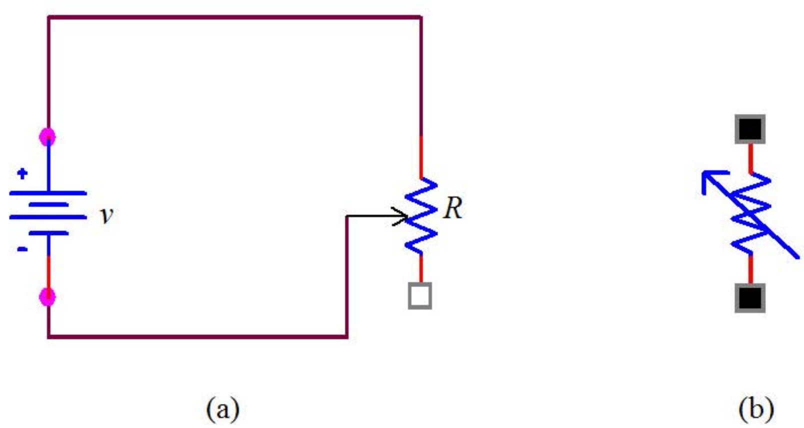


Figure E1.10 Figure (a) shows how a variable resistor may be introduced into a circuit and (b) shows the symbol that is often used for such an element when appearing in a schematic diagram of the circuit.

Example: If the voltage source in Figure E1.10 has a value of 5.0 V, what range of values must the variable resistance be able to take in order that the current may be varied from 2.0 to 10 mA?

E1.2.5 Electrical Power

Power (P) is the *time rate of doing work*, or alternatively, the rate at which energy is expended or absorbed.

$$\text{power} = \frac{\text{work}}{\text{time}}$$

Symbolically,

$$\boxed{P = \frac{W}{t}} \quad \text{.........} \qquad \text{(E1.6)}$$

Since work is measured in joules (J) and time is in second (s), the power **unit** is the joule per second (J/s) which is given the name **watt** (W). Note that in equation (E1.6), W may be interpreted as either **work** or **energy**.

From our discussion in Section 1.2.2, we may concluded that the potential difference between two points in a circuit equals the work done per unit charge in moving the charge between the points. That is,

$$\text{potential difference} = \frac{\text{work}}{\text{charge}}$$

or

$$\boxed{v = \frac{W}{q}} \quad \text{.........} \qquad \text{(E1.7)}$$

From equation (E1.2) we note that $q = it$. Substituting this into (E1.7) gives

Solving this for W we see that

Now we may substitute this result for W into equation (E1.6):

$$P =$$

From which we finally have

$$\boxed{P = vi} \qquad \text{..........} \text{ (E1.8)}$$

Since v is measured in **volts** and i is measured in **amperes**, 1 **watt** equals 1 **volt·ampere**. Remember that here we are discussing dc circuits and furthermore only circuits whose loads are purely resistive. In more advanced circuits, the **watt** and the **volt·ampere** may have to be distinguished from each other.

We may also write equation (E1.8) in terms of current and resistance. From Ohm's Law given in equation (E1.5), $v = iR$ and substituting this into equation (E1.8) in place of v we get

$$P =$$

$$\boxed{P = i^2 R} \quad \ldots\ldots\ldots \quad \text{(E1.9)}$$

Example: A potential difference of 3.0 volts exists across a 300 kΩ resistor for 5.0 s. (a) What power is consumed by the resistor? (b) What energy is dissipated over the stated time interval?

Given: $v = 3.0$ V
$R = 300$ kΩ
$t = 5.0$ s

To find: (a) P
(b) W

Solution:

(a) We first note that equation (E1.9) may be modified to incorporate voltage explicitly by recalling from Ohm's Law that $i = v/R$. Putting this into equation (E1.8),

$$P =$$

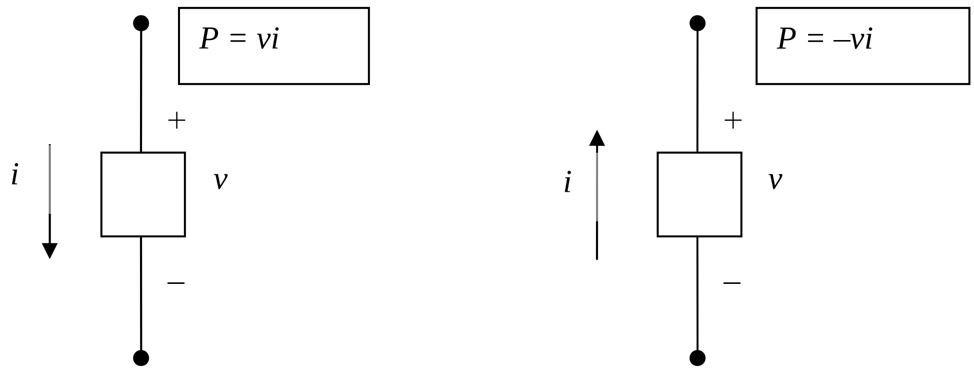
or

$$\boxed{P = \frac{v^2}{R}} \quad \ldots\ldots\ldots \quad \text{(E1.10)}$$

(b)

Power and Sign Conventions

We have intimated that power may be *delivered* by a source or *absorbed* by a load. Consider a general circuit element with the polarities given in Figure E1.11 and current direction as indicated.

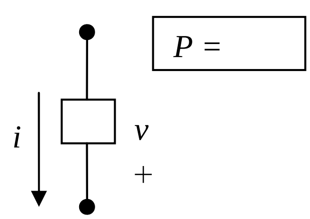


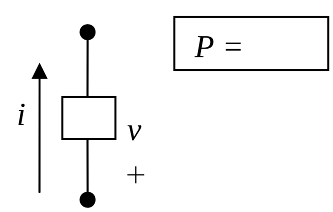
(a) (b)

Figure E1.11 The so-called passive sign convention showing (a) power absorbed and (b) power delivered by a circuit element.

In Figure E1.11(a), the current flows from a region of *high to low* potential, which we earlier decided from our definition of conventional current flow to mean that positive charge flows in this direction. Figure E1.11(a) may be thought of as corresponding to the situation for the resistive load in Figure E1.8 – the resistor absorbs power and the convention is that *positive power* means *power absorption*. However, Figure E1.11(b) corresponds to the source situation in Figure E1.8. Inside the source, the conventional current flows from *low to high* potential (i.e. in the negative sense). The source, of course, *delivers* power so that *negative power* means *power delivery*. Another way of stating these two facts is (a) if power is delivered **to** an element, then it is positive power and (b) if power is delivered **by** an element then it is negative power. This is the **convention** used throughout this course.

Exercise: Complete the power expressions below.





Unit E2: Basic Circuits, Kirchhoff's Laws and Applications

Having considered that a closed electric circuit contains a source and a load *across* which a potential difference (or *voltage*) exists and *through* which a *current* may flow, we are ready to consider circuits containing loads of varying complexity. In fact many circuits will have multiple sources also, but we will not consider those in any detail in this course.

E2.1 Nodes, Loops and Combinations of Circuit Elements

Nodes and Loops

A ***node*** is a point in a circuit where two or more elements meet. Consider the circuit depicted in Figure E2.1. It is important to realize that if we consider that the connecting wires are perfect

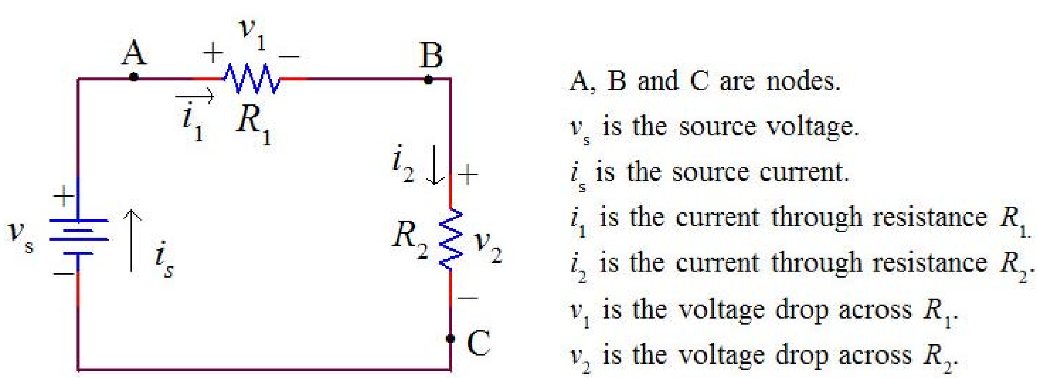


Figure E2.1 A simple circuit containing three nodes and one loop.

conductors, then if no circuit element is encountered as we trace between two elements, we are at the same node. Thus, for example, between the source and R_1 there is only one node as we trace the circuit in the direction of i_1. In total there are only three nodes in this circuit.

With reference to Figure E2.1, it may be observed that there is only one **closed path** or **loop** around which a current may flow. A closed path or loop is traced from any node in a circuit through selected circuit elements, returning to the original node in a manner such that any node in between is crossed only once. For example, if we start at node B, cross R_2, pass node C, cross v_s, pass node A, cross R_1 and return to B, this trace fits the definition of a loop. Furthermore, in this circuit, if we start at any other node and return to it, we trace exactly the same loop.

Consider, next, Figure E2.2. While there are several connection points in this figure, there are really only **two nodes** (A and B). Remember that if you can trace along a connector without crossing a circuit element then you are at the same node. While there are only two nodes, there are **six loops**: one contains the source and R_1, one the source and R_2, a third the source and R_3; a fourth loop contains R_1 and R_2, a fifth R_1 and R_3, and a final loop contains R_2 and R_3.

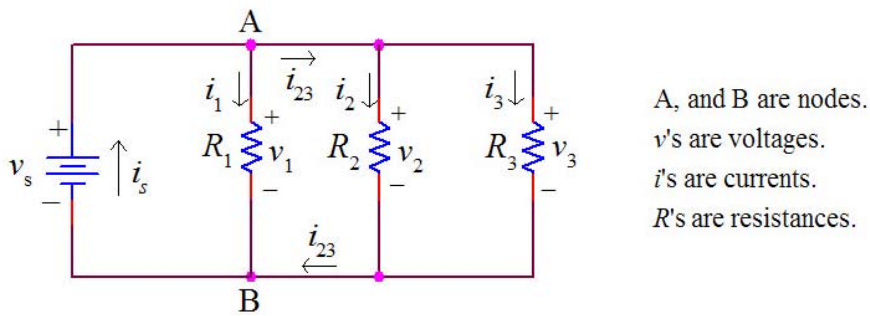


Figure E2.2 A circuit with two nodes but several loops.

<u>Series Combination of Elements:</u>

When two circuit elements connect at just a *single* node they are said to be ***in series***. Notice that for the circuit in Figure E2.1 this is true of any two of the elements: v_s and R_1 connect only at node A, R_1 and R_2 connect only at node B, and R_2 and v_s connect only at node C. Thus in this circuit all three elements are connected **in series** and the combination is referred to as a **series circuit.**

<u>Parallel Combination of Elements:</u>

When two circuit elements connect at a single *pair* of nodes they are said to be ***in parallel***. Notice that in Figure E2.2 this is true of any two elements: for example, R_1 and R_2 are connected at the AB pair of nodes. The identical statement can be made of any other pair of elements in this circuit. That is, all elements, including the source, are connected **in parallel** and the circuit itself may be referred to as a **parallel circuit**. This has absolutely nothing to do with how the circuit diagram appears – i.e. with the shape of the connecting wires. For example, the circuit in Figure E2.3 is exactly the same as that in Figure E2.2.

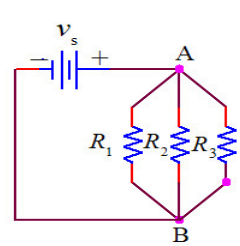


Figure E2.3 Same as Figure E2.2.

Series-Parallel Combination of Elements:

A single circuit need not be limited to series or parallel connection of elements. Often both types of connections may be found together. See, for example, Figure E2.4.

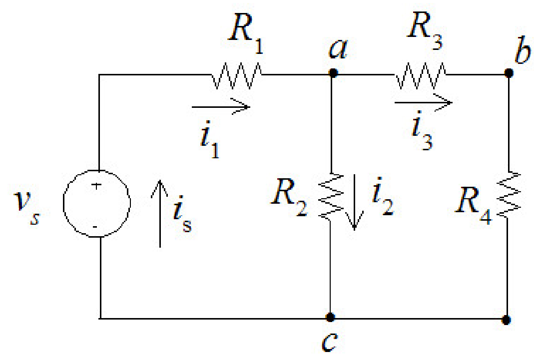


Figure E2.4 A series-parallel circuit.

Notice that R_3 and R_4 share only a single node (b) in common – this indicates they are in series. However, the R_3 and R_4 taken as a combination share the node pair a and c in common with R_2. Thus, this series combination is in parallel with R_2. Resistance R_1 is in series with the parallel combination of R_2 and the series combination of R_3 and R_4. Note that there is a single unlabeled node shared by v_s and R_1 – these two elements are in series. While we haven't yet proven it, notice that for elements in series, the same current flows through them. On the other hand, when a current reaches a parallel combination it divides. Also, we can expect the same voltages across parallel elements. For now these are simply observations, but we will soon deal with such issues in detail.

E2.2 Kirchhoff's Laws

To completely analyze a circuit it is required that the voltage *across* and current *through* each element be determined. For example, suppose that in the simple circuit of Figure E2.5, which is a repetition of that in Figure E2.1, the source voltage v_s and resistance values are known. There remain to be determined the voltage across each resistance and the current through it. It thus appears initially that there are 5 unknowns, namely, i_s, i_1, i_2, v_1, and v_2. Strictly speaking, 5 independent equations are needed to obtain the solution. The tools needed for the analysis were proposed in 1845 by Gustav Kirchhoff and are commonly referred to as Kirchhoff's laws.

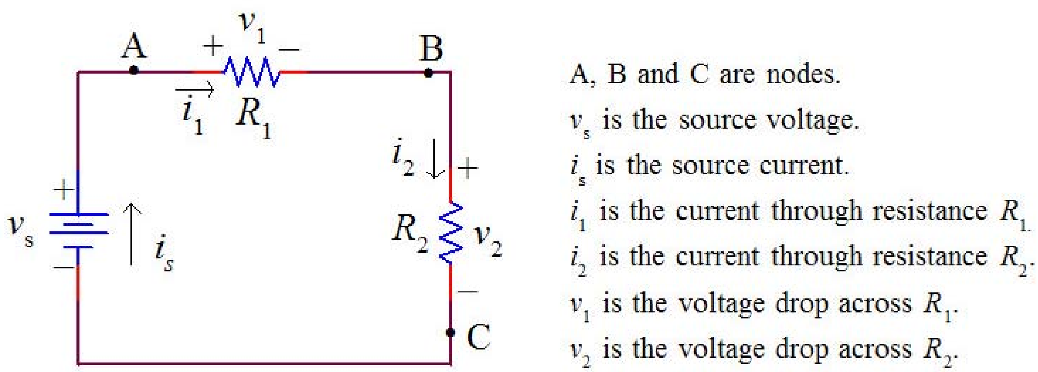


Figure E2.5 The circuit of Figure E2.4 used for illustrating Kirchhoff's Laws.

__**Kirchhoff's Current Law:**__

Kirchhoff's ***current law*** states that *the algebraic sum of all the currents at any node in a circuit equals zero*. We use the convention that currents *entering* a node are *negative* and those *leaving* a node are *positive*. Consider that the current labels in Figure E2.5 represent the *size* of the currents. Applying Kirchhoff's current law at nodes A, B and C yields:

Node A: $i_1 - i_s = 0$ ….. (I)

Node B: $i_2 - i_1 = 0$ ….. (II)

Node C: $i_s - i_2 = 0$ ….. (III)

Only equations (I) and (III) are required to see that $i_1 = i_s = i_2$. That is, there is really only one current in the loop of this series circuit and that current is the source current i_s – this is a general property indicating that the current through any elements connected in series is the same. This certainly seems to be 'common sense' if we believe in the concept of *continuity of current* (i.e. what goes into a node, must come out of the node). Notice too that the system of equations does *not* form an *independent* set (any one equation can be derived from the other two, so there are only two independent equations here). In fact, in any circuit with *n* nodes, only *n* –1 independent equations can be formed using Kirchhoff's current law.

__**Kirchhoff's Voltage Law:**__

Kirchhoff's ***voltage law*** states that *the algebraic sum of all voltages around any closed path in a circuit equals zero*. We will by convention assign a *positive* sign to voltage *drops* and a *negative* sign to voltages *rises* as a loop is traversed in a fixed direction. Applying this to the circuit in Figure E2.5 (see direction of arrows in the above circuit),

$$-v_s + v_1 + v_2 = 0 \;\; \ldots\ldots \;\; \text{(IV)}$$

Clearly this could be restated as the "sum of the voltage rises equals the sum of the voltage drops" around any closed loop and we may write (IV) above in a different form as

$$v_s = v_1 + v_2 \;\; \ldots\ldots \; \text{(IV)}$$

We have previously stated that Ohm's law may be written for each resistance in the circuit. Thus,

$$v_1 = i_1 R_1 \;\; \ldots\ldots \; \text{(V)} \qquad \text{and} \qquad v_2 = i_2 R_2 \;\; \ldots\ldots \; \text{(VI)}$$

Thus, using Kirchhoff's laws and Ohm's law, equations (I) to (VI) give a set of 5 independent equations which can be used to solve for the 5 unknowns. For such a simple circuit, this approach is a bit of an 'overkill', but it will be important to understand the concepts for more complicated cases.

Example: In Figure E2.5, the source voltage is 20.0 V, R_1 = 12 Ω and R_2 = 8.0 Ω. Determine the source current and the voltage drops across R_1 and R_2 using Kirchhoff's laws and Ohm's law.

Given: v_s = 20.0 V

R_1 = 12 Ω

R_2 = 8.0 Ω

To find: (i) i_s = ?

(ii) v_1 =? and v_2 = ?

Solution: (i) We know from Kirchhoff's current law in (I) – (III) that i_s is the only current in the circuit. Thus, from the Ohm's law equations (V) and (VI),

$$v_1 = 12 i_s$$

$$v_2 = 8.0 i_s$$

Then, from Kirchhoff's voltage law in equation (IV),

$$v_s = v_1 + v_2$$

or

$$v_s = 12 i_s + 8.0 i_s = 20 i_s$$

$$\rightarrow \qquad i_s = v_s/20 =$$

(ii)

E2.3 Resistors in Series

Consider, the series circuit in Figure E2.6 where now we have not specified the precise number of resistors connected in series (there are *n* resistors, the dashed lines indicating that other resistors are in the circuit, but not shown). The *n* voltage drops are shown across the *n* resistors. We wish to find **an expression for the total resistance** in the circuit.

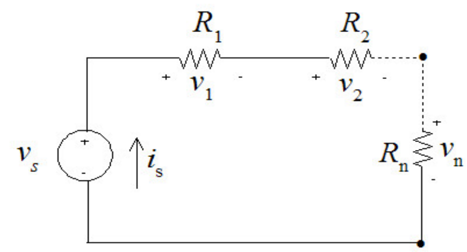


Figure E2.6 Series circuit with *n* resistors.

We know from the previous example and **Kirchhoff's current law** that the current *through* each resistor is i_s. Thus, writing **Ohm's law** for each resistor we have

$$v_1 = i_s R_1$$

$$v_2 = i_s R_2$$

$$\dots \quad \dots$$

$$v_n = i_s R_n$$

(Aside: For a series circuit, it is seen from these expressions the ratio of the voltage drops across any two resistors equals the ratio of their resistances.)

Now, using **Kirchhoff's voltage law** for the single loop circuit, we have

$$-v_s + v_1 + v_2 + \cdots + v_n = 0$$

or

$$v_s = v_1 + v_2 + \cdots + v_n$$

Substituting the Ohm's law expressions for each resistor into this last equation gives

$$v_s =$$

or

$$v_s = i_s(R_1 + R_2 + \cdots + R_n)$$

Now, for the whole circuit, the total or *equivalent* resistance (R_{eq}) is the ratio v_s/i_s, so that for the **total series resistance** we have

$$\boxed{R_{eq} = R_1 + R_2 + \cdots + R_n} \quad \ldots\ldots \text{(E2.1)}$$

or

$$\boxed{R_{eq} = \sum_{i=1}^{n} R_i} \quad \ldots\ldots \quad \text{(E2.2)}$$

where $\sum$ is a *summation symbol* meaning here that we allow the subscript on *R* to vary from 1 to *n* and then add the result. For example, $\sum_{i=1}^{3} R_i = R_1 + R_2 + R_3$.

Example: A circuit similar to that in Figure E2.6 contains 3 identical 10-Ω resistors in series. The current in the circuit is 600 mA. (i) What is the value of the source voltage? (ii) What is the voltage drop across each of the resistors?

E2.4 Resistors in Parallel

Consider, the parallel circuit in Figure E2.7 where again we have not specified the precise number of resistors connected in parallel (there are *n* resistors, the dashed lines indicating that other resistors are in the circuit, but not shown). The *n* voltage drops are shown across the *n* resistors. We wish to find **an expression for the total resistance** in the circuit and along the way we will observe other characteristics.

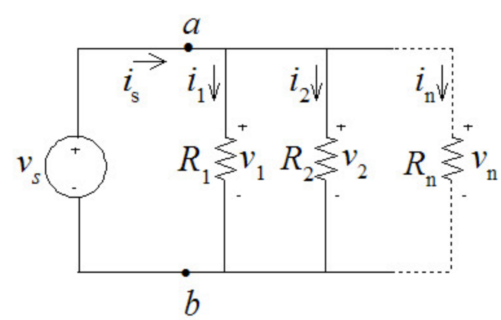


Figure E2.7 Parallel circuit with n resistors.

From what we know about potential difference, it can be seen that the voltage between nodes *a* and *b* must be the same across each of the resistors and is equal to the source voltage v_s. This, in fact, can be easily seen from Kirchhoff's voltage law.

Consider the loop containing only the source and R_1: $-v_s + v_1 = 0$.

Consider the loop containing only the source and R_2: $-v_s + v_2 = 0$.

Consider the loop containing only the source and R_n: $-v_s + v_n = 0$.

Clearly,

$$\boxed{v_s = v_1 = v_2 = \cdots = v_n} \quad \text{..... (E2.3)}$$

It is one of the distinguishing features of **elements connected in parallel that the voltage drop across each must be the same**.

From Kirchhoff's current law we can see (by looking at node *a,* for example) that

$$\boxed{i_s = i_1 + i_2 + \cdots + i_n} \quad \ldots.. \text{ (E2.4)}$$

Writing Ohm's law for each resistance, we have

$$i_1 = v_1/R_1, i_2 = v_2/R_2, \ldots. , i_n = v_n/R_n,$$

which from equation (E2.3) means that

$$i_1 = v_s/R_1, i_2 = v_s/R_2, \ldots. , i_n = v_s/R_n \, .$$

Substituting these expressions for the currents into equation (E2.4) we get

or

$$\frac{i_s}{v_s} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \; ,$$

but from Ohm's law,

$$\frac{i_s}{v_s} = \frac{1}{R_{eq}}$$

where R_{eq} is the total or equivalent resistance in parallel. Therefore, for **resistors in parallel,**

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}} \quad \ldots.. \quad \text{(E2.5)}$$

or

$$\boxed{\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}} \quad \ldots.. \quad \text{(E2.6)}$$

Example: A circuit like that in Figure E2.7 has only two resistors (R_1 and R_2) in parallel. Find an expression for the equivalent resistance.

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}} \quad \ldots.. \quad \text{(E2.7)}$$

<u>Note on Conductance:</u>

Sometimes for parallel combinations of circuit elements it is convenient to use **conductance** (G) which is defined as *reciprocal of resistance*. That is

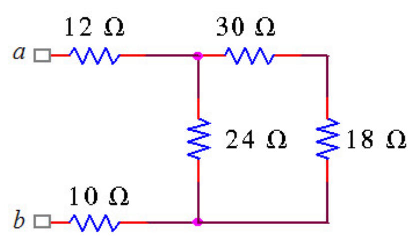
$$\boxed{G = \frac{1}{R}} \quad \text{..... (E2.8)}$$

Conductance is measured in **siemens** (S). Many times in the literature, the **mho** (which is ohm spelled backward) is used in place of the siemen. The symbol used for this is an inverted Ω: ℧ .
Thus, for example, a 10 Ω resistor may be equally well described as having conductance of 0.1 mho or 0.1 S.

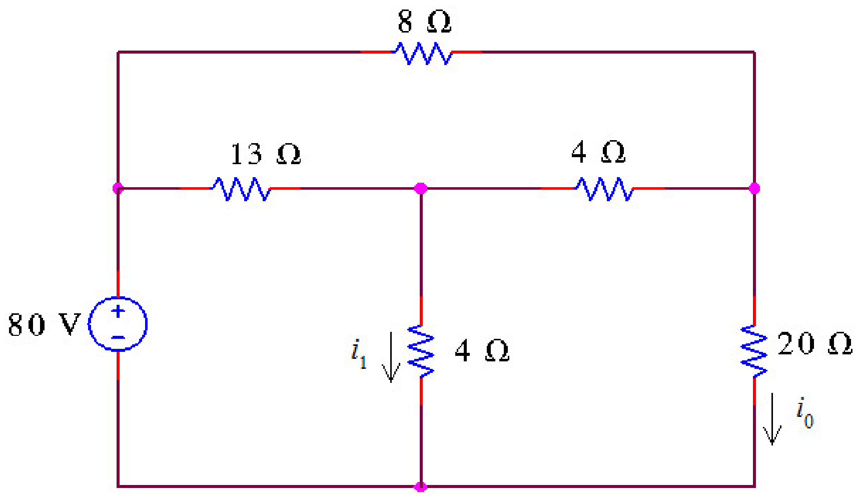
With this definition in place, equations (E2.5 and E2.6) may be written in terms of conductance as

$$\boxed{G_{eq} = \sum_{i=1}^{n} G_i = G_1 + G_2 + ... + G_n} \quad \text{..... (E2.9)}$$

Example: (a) Find the equivalent resistance at terminals *a* - *b* for the series-parallel combination shown below. (b) What is the equivalent conductance of the parallel section? (c) If a 19 V dc source was connected across the terminals *a* - *b*, what power would be absorbed by the 30 Ω resistor?



Summary example of Kirchhoff's laws: For the circuit below, $i_0 = 2.0$ A. (a) Find i_1. (b) Determine the power dissipated by each resistor and (c) verify that the total power *dissipated* in the circuit resistors equals the total power *delivered* by the source. (d) Next, suppose the 20 Ω resistor is short circuited and find the new i_0 and i_1.



E2.5 Applications

E2.5.1Voltage-Divider Circuit

It is often a very useful thing to be able to produce two or more voltage levels from a single source. One method of accomplishing this involves a **voltage-divider circuit.**

As an initial approach, consider the series circuit in Figure E2.8 (it's really the same circuit as in Figure E2.1). We wish to find the expression for the voltages v_1 and v_2 in terms of the source voltage v_s and the resistances R_1 and R_2. Applying Kirchhoff's **voltage law:**

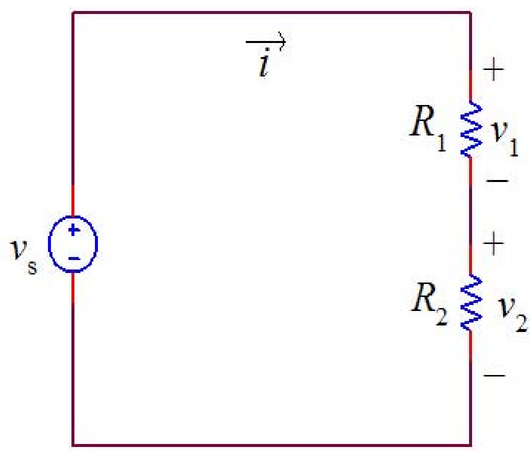


Figure E2.8 Voltage divider circuit.

$$v_s = i\ R_1 + i\ R_2$$

$$\Rightarrow i = \frac{v_s}{(R_1 + R_2)} \quad \ldots\ldots \text{ (E2.10)}$$

Using **Ohm's law** again for the voltage across each resistor we write

$$v_1 = iR_1 = v_s\left[\frac{R_1}{R_1 + R_2}\right] \quad \ldots\ldots \text{ (E2.11)}$$

and

$$v_2 = iR_2 = v_s\left[\frac{R_2}{R_1 + R_2}\right] \quad \ldots\ldots \text{ (E2.12)}$$

We have written equations (E2.11) and (E2.12) in such a way as to emphasize that the each of v_1 and v_2 are fractions of v_s. Notice that the fraction of the voltage which is dropped across each resistance is the ratio of that resistance to the total resistance in the circuit – this is true for any number of series connected resistors. So, for, a series connection of n resistors, the voltage across the j^{th} resistor is

$$\boxed{v_j = v_s\left[\frac{R_j}{R_{\text{eq}}}\right]} \quad \ldots\ldots \text{ (E2.13)}$$

where R_{eq} is given in equation (E2.1). Clearly, each of the individual voltages is less than the total source voltage since the ratio referred to is always less than unity (i.e. less than 1).

Of course, if other circuit elements are connected in parallel with the resistances R_1 or R_2, the voltages v_1 and v_2 will change from their values calculated in equations (E2.11) and (E2.12). Ideally, we would like for the voltage provided by the dividing circuit to NOT DEPEND on the elements (i.e. the load) that the voltage is across. To illustrate this concept, consider such a load resistance R_L connected across (i.e. 'in parallel with') R_2 as in Figure E2.9. We are particularly interested in the voltage v_o.

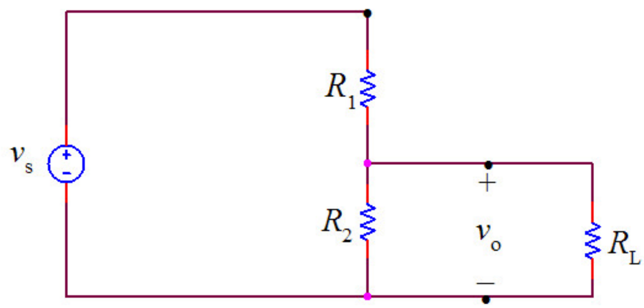


Figure E2.9 Voltage divider connected to a load resistance R_L.

We note that the equivalent resistance of the parallel section across which v_0 exists may be found from equation (E2.7) as

$$R_{eq} = \frac{R_2 R_L}{R_2 + R_L} \quad \text{..... (E2.14)}$$

R_1 is in series with this R_{eq} and therefore from Equation (E2.12),

$$v_0 = v_s \left[\frac{R_{eq}}{R_1 + R_{eq}} \right] =$$

$$=$$

$$=$$

$$\Rightarrow \boxed{v_0 = v_s \left[\frac{R_2}{R_1[1 + (R_2 / R_L)] + R_2} \right]} \quad \text{..... (E2.15)}$$

Now, if R_L>>R_2, then R_2/R_L<<1 and equation (E2.15) reduces, to a good approximation, to equation (E2.12) as expected:

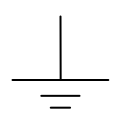
$$\boxed{v_0 = v_s\left[\frac{R_2}{R_1 + R_2}\right]}$$

This shows that if R_L>>R_2, then the voltage division is essentially independent of the load resistance R_L – i.e. the output voltage is unaffected by the load. In this case, since R_2<<R_L, most of the current would pass through R_2 and for this reason R_2 is sometimes referred to as a ***bleeder*** resistor (sometimes symbolized as R_B).

ALTERNATE REPRESENTATION OF THE VOLTAGE DIVIDER CIRCUITS

The following, which uses the previous voltage divider circuits as a point of discussion, may be useful in many circuit representations (or *schematic* diagrams).

Before considering an alternate diagrammatic representation of the circuit, we briefly note the concept of ***electric ground***. One definition of an *electric ground* is the "reference point" in a circuit from which other voltages are measured, the "ground" being considered to be at a reference potential, usually taken to be zero. Ideally, an electrical ground can absorb an unlimited amount of current without changing its potential. A ground can also be viewed as a common return path by which all currents in a circuit return to the source. In our simple circuits with ideal sources, the negative terminal of the source may be considered to act as a ground. The symbol for an electrical ground is



No-load Voltage Divider Equivalent Circuit Diagram

In view of the above discussion of electrical ground, Figure E2.8 may be redrawn as shown. It may be noted that, in this simple format, R_2 is sometimes referred to as the load resistor (R_L) and R_1 is called the *series dropping* resistor.

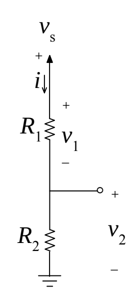


Figure E2.10 Alternate representation of Figure E2.8.

Loaded Voltage Divider Equivalent Circuit Diagram

Now, Figure E2.9 may be redrawn as shown in Figure E2.9. As noted previously, for the loaded voltage divider, R_2 is sometimes referred to as the *bleeder* resistor (R_B). Also, R_1 may be referred to as the *series-dropping* resistor (R_s). See Figure E2.11.

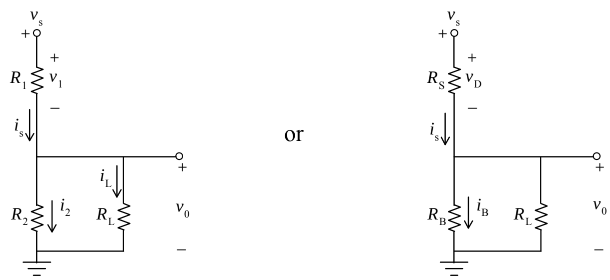


Figure E2.11 Alternate representation of Figure E2.9.

With reference to Figure E2.11, we note that if $R_B << R_L$, then $i_B >> i_L$ and

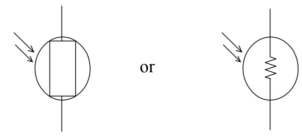
$$v_0 = v_s\left[\frac{R_B}{R_S + R_B}\right] \quad \text{….. (E2.16)}$$

To add a realistic property of resistors to the discussion, we consider the idea of ***tolerance***. Depending on the quality of the resistors, a manufacturer will guarantee that the stated value of a resistance is within a certain range of possible values. This range is denoted as the *tolerance*. For example a 100 Ω resistor which has a tolerance of ±10% has a possible range of values from 90 Ω to 110 Ω.

Example: In Figure E2.8 suppose that R_1 = 25 kΩ and R_2 = 100 kΩ (these are referred to as the *nominal* values) and that the tolerance on each is ±10%. (a) If v_s = 100 V and find value of v_2 if the current *i* has the maximum possible value or the given tolerances. (b) What is the percent difference between this value of v_2 and that if the nominal resistance values were used?

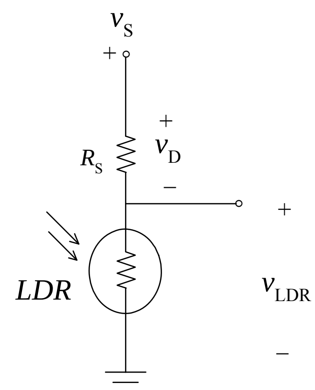
Example: Voltage Division Application

As its name suggests, a **light dependent resistor** or **LDR** (sometimes called a photoconductive cell), is an electrical element whose resistance depends on the intensity of the light falling upon it. The circuit symbol for such a resistance is shown below:



The light-sensitive part of the LDR is a wavy track of cadmium sulphide. Light energy triggers the release of extra charge carriers in this material, so that its *resistance falls as the level of illumination increases*.

A **light sensor** uses an LDR as part of a voltage divider. Consider that the second resistor of Figure E2.10 is replaced by an LDR and that the top resistor (the so-called *series dropping* resistor) R_S has a value of 10 kΩ. The source voltage is 9 V. Suppose the LDR has a resistance of 500 Ω, in bright light, and 200 kΩ in the shade (these values are reasonable).



When the LDR is in bright light, V_{LDR} will be:

In the shade, V_{LDR} will be:

The voltage divider circuit gives an output *voltage* which changes with illumination. In other words, this circuit gives a LOW voltage when the LDR is in the light and a HIGH voltage when the LDR is in the shade. A sensor subsystem which functions like this could be thought of as a '**dark sensor**' and could be used to control lighting circuits which are switched on automatically in the evening – i.e. the 8.57 V could be used to activate a flood light circuit – and off in the morning when the activating voltage drops to 0.43 V.

E2.5.2 Current-Divider Circuit

Consider the circuit in Figure E2.12 which contains a *current source* i_s connected to a parallel combination of two resistors.

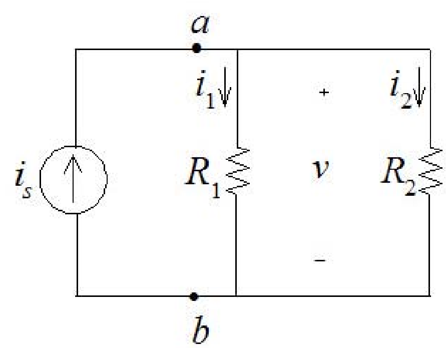


Figure E2.12 A current-divider circuit containing two resistors.

From Ohm's law applied to the **whole circuit** we know that

$$v = i_s R_{eq} = i_s \frac{R_1 R_2}{R_1 + R_2} \quad \text{….. (E2.17)}$$

Applying Ohm's law to R_1 and R_2 individually gives:

$$v = i_1 R_1 = i_s \frac{R_1 R_2}{R_1 + R_2}$$

$$\Rightarrow \boxed{i_1 = \frac{R_2}{R_1 + R_2} i_s} \quad \text{….. (E2.18)}$$

and

$$v = i_2 R_2 = i_s \frac{R_1 R_2}{R_1 + R_2}$$

$$\Rightarrow \boxed{i_2 = \frac{R_1}{R_1 + R_2} i_s} \quad \text{….. (E2.19)}$$

Notice from equations (E2.18) and (E2.19) that the fraction of the total current in one resistor is found by dividing the other resistance by the sum of the resistances. However, notice also that the equations immediately preceding (E2.18) and (E2.19) may be written as

$$i_1 = \qquad \text{and} \qquad i_2 =$$

or

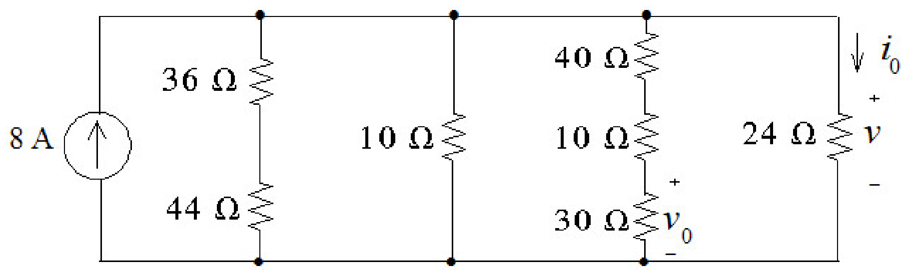
$$\boxed{i_1 = \frac{R_{eq}}{R_1} i_s \quad \text{and} \quad i_2 = \frac{R_{eq}}{R_2} i_s} \quad \text{….. (E2.20)}$$

Carrying this idea to the case of dividing the current through *n* resistors connected in parallel (as in Figure E2.7), we can easily argue that the current in the branch of the circuit containing the *j*th resistor is

$$\boxed{i_j = \frac{R_{\text{eq}}}{R_j} i_s} \quad \text{.....} \ (\text{E2.21})$$

where i_s is the total current entering the parallel equivalent resistance R_{eq}. Notice from equation (E2.20) or (E2.21) that, in a parallel circuit, *the ratio between any two branch currents equals the inverse ratio of the resistances through which they flow*.

Example: (a) Use the current-division method to find the current i_0 in the circuit below. (b) Use voltage division to determine the voltage v_0.



E2.5.3 Measuring Current and Voltage

<u>The Galvanometer</u>

It is now common to measure values of electrical circuit parameters (voltage, current, resistance) using ***digital*** devices whose principles of operation require an understanding of solid state electronics involving, for example, transistors. Digital meters such as the *digital multimeter* (DMM) that you will encounter in the laboratory *display information as numbers*. However, the underlying ideas involved in measuring the electrical properties existing in a circuit may be better explored using a *moving coil detector* called a ***galvanometer***. This is an ***analog*** device, meaning that it displays readings on *a continuous scale* (see Figure E2.13).

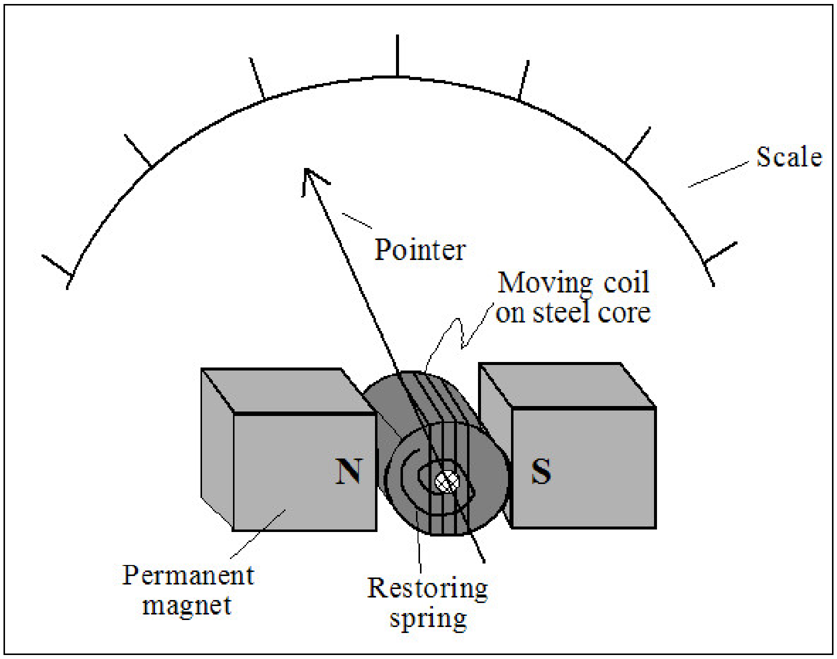


Figure E2.13 A simple illustration of a moving coil detector or galvanometer.

The operation of a galvanometer is based on the fact that a current carrying wire loop in a magnetic field experiences a turning effect or *torque*. An energized coil, constructed in this case by wrapping several loops of wire around a cylindrical core (see figure), will turn in the permanent magnetic field (see N/S poles in figure) until the torque it experiences is counterbalanced by that provided by a restoring torque in the spring. The galvanometer construction is such that the angle through which the spring and the attached pointer turn is directly proportional to the current flow. This moving-coil configuration is referred to as a *d'Arsonval movement*.

Of course, the galvanometer coil will have some resistance, say R_G and there is usually a *calibrating resistor* (sometimes referred to as a *swamping* resistor) R_c placed in series with this to bring the total resistance to some number for which the pointer deflection will be full-scale. **By way of example:** Suppose a full-scale deflection is caused by a 1 mA current in the coil (this is referred to as the *sensitivity* of the galvanometer) which flows when there is a 50 mV voltage drop (v_g) across the coil and its calibrating resistor. For this to happen the combined resistance ($R_G + R_c$) must be, from Ohm's law, (50 mV/1 mA) = 50 Ω. Thus, in this case, $R_c = 50\ \Omega - R_G$. The equivalent circuit which depicts this situation is given in Figure E2.14. A typical coil might have a resistance of

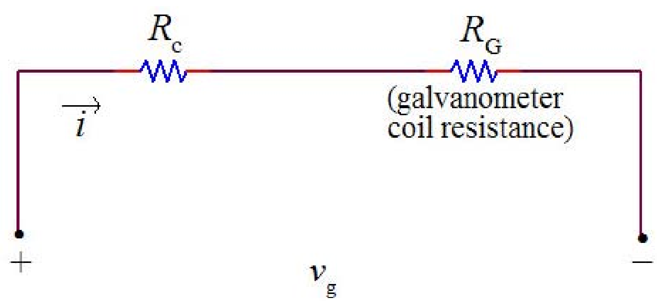


Figure E2.14 Circuit representation of galvanometer.

$R_G = 27\ \Omega$. In this case, for a galvanometer sensitivity of 1 mA when $v_g = 50$ mV, the calibrating resistor would be $R_C = 23\ \Omega$.

The Ammeter

We next consider how the galvanometer might be used to design a device to measure larger currents – such a device is called an ***ammeter*** (short for ampere meter). Since ammeters must measure the current *through* a circuit element, they have to be connected *in series* with the element. At the same time, we don't want the ammeter to introduce an appreciable resistance into the circuit – that is, we don't want the device to significantly affect the characteristics of the circuit itself.

In accomplishing an appropriate design, we recall that the total resistance of a parallel combination of resistors is always less than the smallest resistor. Also, most of the current will flow through the branch of the circuit with the smallest resistance. Consider Figure E2.15 in which a resistor R_{sh}, typically referred to as a ***shunt***, is placed in parallel with the galvanometer for which the total resistance (coil plus calibrating resistance) is $R_T = R_c + R_G$. We know that the same voltage exists across each of the

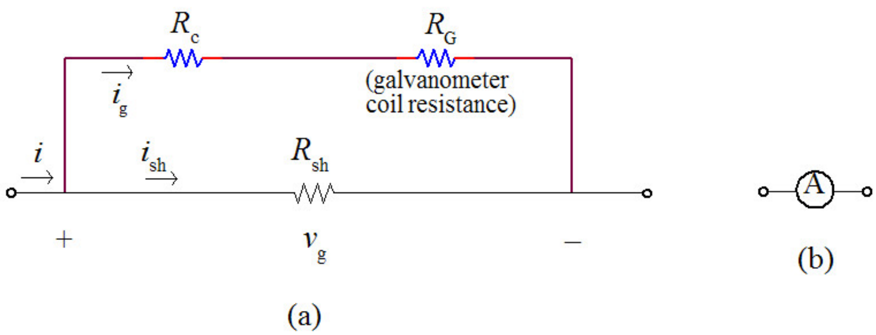


Figure E2.15 (a) Circuit diagram for an analog ammeter. (b) Ammeter circuit symbol.

parallel branches of the ammeter circuit. Therefore,

$$v_g = i_g R_T$$

and

$$v_g = i_{sh} R_{sh}$$

Therefore,

$$i_{sh} R_{sh} = i_g R_T$$

or

$$\boxed{R_{sh} = \left(\frac{i_g}{i_{sh}}\right) R_T} \quad \ldots\ldots \text{(E2.22)}$$

Note that equation (E2.22) could have been written down directly from our knowledge that *the ratio between any two branch currents equals the inverse ratio of the resistances through which they flow*. The shunt is so named because it shunts (or diverts) a large portion of the current away from the sensitive galvanometer coil. As we shall see in an example, the shunt resistor determines the full-scale reading of the ammeter.

Example: A 50mV, 1 mA moving-coil galvanometer is to be used in an ammeter with a full-scale reading of 10 mA. (a) Determine the required value of the shunt resistor. (b) Repeat (a) for a full-scale reading of 1 A. (c) How much resistance does the 10 mA ammeter add to a circuit in which it is inserted? (d) Repeat (c) for the ammeter in (b).

Solution: See Figure E2.15;

The Voltmeter

We next consider how the galvanometer might be used to design a device to measure voltage – such a device is called a ***voltmeter***. Since voltmeters must measure the voltage drop *across* a circuit element, they have to be connected *in parallel* with the element. As with the ammeter, we don't want the voltmeter to introduce an appreciable resistance into the circuit – that is, as before, we don't want the device to significantly affect the characteristics of the circuit itself. A way of accomplishing this is to connect a resistance R_M, called a ***multiplier resistance***, in series with the galvanometer resistance R_T with the combination being connected *across* a circuit element – i.e. *in parallel* with it – to determine the voltage drop across it. A typical voltmeter configuration is depicted in Figure E2.16. Of course, because the

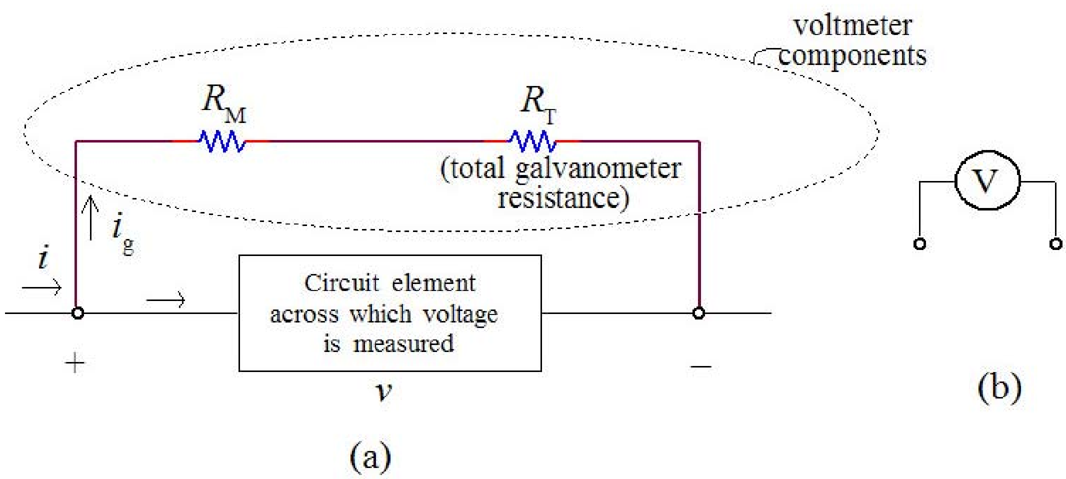


Figure E2.16 (a) Circuit diagram for an analog voltmeter. (b) Voltmeter circuit symbol.

voltmeter and circuit element (resistance in our case) are in parallel, the voltage v is the same across each. For the configuration shown,

$$\boxed{v = i_g (R_M + R_T)} \quad \text{..... (E2.23)}$$

where i_g is the current through the voltmeter; therefore,

$$\boxed{R_M = \left(\frac{v}{i_g}\right) - R_T} \quad \text{..... (E2.24)}$$

As we shall see in an example, the value R_M of the multiplier resistor determines the full-scale reading of the voltmeter.

Example: A 50mV, 1 mA moving-coil galvanometer is to be used in a voltmeter with a full-scale reading of 150 V. (a) Determine the required value of the multiplier resistor. (b) Repeat (a) for a full-scale reading of 5 V. (c) How much resistance does the 150 V meter add to a circuit in which it is inserted? (d) Repeat (c) for the voltmeter in (b).

Solution:

E2.5.4 Measuring Resistance

The Wheatstone Bridge

A variety of circuit configurations may be used to measure resistance. Of these, we consider the ***Wheatstone bridge***. It may typically be used to measure resistance in the range from 1 Ω to 1 MΩ with accuracies within about ±0.1%. As shown in Figure E2.17, the circuit consists of four resistors, one (R_x) which is unknown or the resistor *under test*, one (R_3) which is known but variable (see Figure E1.10), and two (R_1 and R_2) which have fixed values. The circuit also contains a voltage source v and a galvanometer, indicated by the circle with an arrow, between nodes a and b as shown. We use the term ***balanced*** to mean that the current in the galvanometer branch is zero ($i_g = 0$) or, equivalently, the potential difference between nodes a and b is zero.

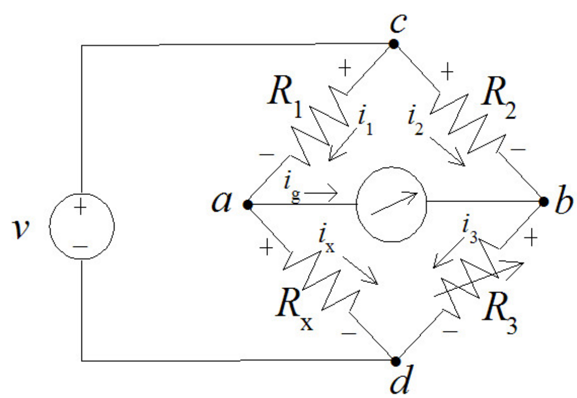


Figure E2.17 The Wheatstone bridge circuit used to measure R_x.

Consider that the bridge shown is balanced so that $i_g = 0$ and the voltage drop v_{ab} between nodes a and b is also zero. Applying Kirchhoff's voltage law (KVL) to loop *cbac* gives

$$i_2 R_2 + 0 - i_1 R_1 = 0$$

or

$$i_1 R_1 = i_2 R_2 \quad \text{.....} \ (A)$$

Again, applying KVL to loop *abda* gives

$$0 + i_3 R_3 - i_x R_x = 0$$

or

$$i_x R_x = i_3 R_3 \quad \text{.....} \ (B)$$

Applying Kirchhoff's current law (KCL) to node a (since $i_g = 0$) gives

$$i_1 = i_x \quad \text{.....} \ (C)$$

and applying KCL to node b gives

$$i_2 = i_3 \quad ..... \text{(D)}$$

Using (C) and (D) in (B) allows us to write

$$i_1 R_x = i_2 R_3 \quad ..... \text{(E)}$$

Now divide (E) by (A)

and rearrange to get

$$\boxed{R_x = \left(\frac{R_1}{R_2}\right) R_3} \quad ..... \text{(E2.25)}$$

Equation (E2.25) shows that if R_3 is adjusted to make the galvanometer current zero, then the known resistances can be used to determine unknown resistance R_x. Notice that the size of the source voltage does not have an effect on the final result. The source voltage simply causes a deflection of the galvanometer pointer if the bridge is not balanced.

Example: Find the value to which R_3 must be adjusted to balance a Wheatstone bridge if the fixed bridge resistors have identical values and the resistor under test is 100 Ω.

Solution: