Engineering 5812 Assignment 1 Due: Monday, January 25, 2010

1. If $\vec{A} = 2\hat{x} - 3\hat{y} + \hat{z}$, $\vec{B} = 2\hat{x} - \hat{y} + 3\hat{z}$, and $\vec{C} = 4\hat{x} + 2\hat{y} - 2\hat{z}$, show that $\vec{C}$ is (a) perpendicular to both $\vec{A}$ and $\vec{B}$ and (b) that it is in a direction which is opposite to that of $\vec{A} \times \vec{B}$.

2. If $\vec{A} = 2\hat{x} - 3\hat{y} + \hat{z}$ and $\vec{B} = B_x\hat{x} + 2\hat{y} + B_z\hat{z}$ find (a) B_x and B_z if $\vec{A} \parallel \vec{B}$ and (b) a relation between B_x and B_z if $\vec{A} \perp \vec{B}$.

3. Given $\vec{A} = \hat{x} + 2\hat{y} - 3\hat{z}$, $\vec{B} = 3\hat{x} - 4\hat{y}$ and $\vec{C} = 3\hat{y} - 4\hat{z}$, find (a) $\vec{A} \cdot (\vec{B} \times \vec{C})$ and (b) $\vec{A} \times (\vec{B} \times \vec{C})$.

4. Use cartesian components (i.e. let $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$, and so on for $\vec{B}$ and $\vec{C}$) to show that $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$ and thus the vector triple product is not generally associative.

5. Two lines in the x-y plane are described by (line 1) $x + 2y = -6$ and (line 2) $3x + 4y = 8$. Use vector algebra to find the smaller angle between the lines at their intersection point.

6. A particular vector field is specified as

$$\vec{E} = \hat{\rho}\rho\cos\phi + \hat{\phi}\rho\sin\phi + \hat{z}z^2 \ .$$

   The point $P(4, \pi, 2)$ is located on the surface of a cylinder $\rho = 4$. At point P, determine:

   (a) the vector component of $\vec{E}$ perpendicular to the cylinder.

   (b) the vector component of $\vec{E}$ tangent to the cylinder.

7. Determine the distance between $P_1(2, \pi/3, 1)$ and $P_2(4, \pi/2, 0)$ which are specified in cylindrical coordinates. Assume that the angles are in radians and the ρ and z coordinates have units of metres.

8. Transform the following vectors into spherical coordinates *first*, and then evaluate them at the indicated points:

   (a) $\vec{A} = y^2\hat{x} + xz\hat{y} + 4\hat{z}$ at $P(1, -1, 2)$.

   (b) $\vec{C} = \cos\phi\hat{\rho} - \sin\phi\hat{\phi} + \cos\phi\sin\phi\hat{z}$ at $P(2, \pi/4, 2)$.

9. Use integration to find the area of the surface specified by $2 \le \rho \le 5$, $\pi/2 \le \phi \le \pi$, $z = 0$. Provide a sketch showing the appropriate diffrential area that is required for the integration.

10. A section of a sphere is described by $0 \le r \le 2$, $0 \le \theta \le 90°$, $30° \le \phi \le 90°$. Sketch the outline of the section and use integration to find the following:

    (a) the surface area of the spherical section.

    (b) the enclosed volume.