Engineering 5812 Assignment 2 **Due Mon., Feb.8, 2010**

1. A cube 2 m on a side is located in the first octant in the cartesian coordinate system. One of the corners is at the origin. If the charge density is $\rho_v = xy^2e^{-2z}$ mC/m^3, what is the total charge in the cubical region?

2. A cone described by $r \leq 2$ m and $0 \leq \theta \leq \pi/4$ contains a charge density $\rho_v = 20r^2\cos^2\theta$ mC/m^3. How much charge is in the cone?

3. Find the total charge on a circular disc defined by $\rho \leq a$ and $z = 0$ if $\rho_S = \rho_{S_0}e^{-\rho}\sin^2\phi$ C/m^2. Here ρ_{S_0} is a constant

4. Three point charges, each with $Q = 3$ nC, are located at the corners of a triangle in the x-y plane. One corner is at the origin, another at $(2\text{ cm}, 0, 0)$, and the third at $(0, 2\text{ cm}, 0)$. Find the force acting on the charge at the origin. Give the answer as a magnitude and direction as well as in cartesian form.

5. Charge $Q_1 = 4\ \mu$C is located at $(1\text{ cm}, 1\text{ cm}, 0)$ and charge Q_2 is located at $(0, 0, 4\text{ cm})$. What should Q_2 be so that $\vec{E}$ at $(0, 2\text{ cm}, 0)$ has no y-component?

6. Electric charge is distributed along an arc in the $z = 0$ plane and defined by $\rho = 2$ cm and $0 \leq \phi \leq \pi/4$. If the linear charge density is 5 μC/m, find $\vec{E}$ at $P(0, 0, z)$ and then evaluate it at the origin. Hint: After setting up the integral, write $\hat{\rho}$ in terms of cartesian components.

7. A line charge with uniform density ρ_L extends between $(0, 0, -L/2)$ to $(0, 0, L/2)$. Apply Coulomb's law to obtain an expression for the electric field intensity at any point $P(\rho, \phi, 0)$ on the x-y plane. Show that the result reduces to the appropriate expresson for an infinite line of charge when L is extended to infinity.

8. Three infinite lines of charge, $\rho_{\ell_1} = 5$ nC/m, $\rho_{\ell_2} = -5$ nC/m, and $\rho_{\ell_3} = 5$ nC/m parallel to the z-axis pass through the respective points $(0, -b)$, $(0, 0)$, and $(0, b)$ in the x-y plane. Find the electric field at $(a, 0, 0)$. Evaluate the result for $a = 2$ cm and $b = 1$ cm.

9. Use the method of Section 2.5 of the text to determine that if an infinite sheet of charge with uniform density ρ_S is confined to the $y = 0$ plane in free space, then at position $(0, a, 0)$
$$\vec{E} = \frac{\rho_S}{2\epsilon_0}\hat{y}\ .$$

10. A disc $\rho = a$ containing a uniform positive surface charge density ρ_S is centred on the origin in the x-y plane in free space. Show that the field at $P(0, 0, h)$, $h > 0$, on the z-axis is given by
$$\vec{E} = \hat{z}\frac{\rho_S}{2\epsilon_0}\left[1 - \frac{h}{\sqrt{a^2 + h^2}}\right]$$
and show what this becomes as the disc radius becomes infinite (does this make sense?).

11. The electric field intensity is given as $\vec{E} = 5e^{-2x}(\sin 2y\,\hat{x} - \cos 2y\,\hat{y}$ V/m. (a) Find the equation of the streamline passing through the point $P(0.5, \pi/10, 0)$. (b) Find a unit vector tangent to the streamline at P.