ENGI5812 Data and Formulae Sheet

	$\hat{\rho}$	$\hat{\phi}$	$\hat{z}$
$\hat{x}\cdot$	$\cos\phi$	$-\sin\phi$	0
$\hat{y}\cdot$	$\sin\phi$	$\cos\phi$	0
$\hat{z}\cdot$	0	0	1

	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$
$\hat{x}\cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\hat{y}\cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\hat{z}\cdot$	$\cos\theta$	$-\sin\theta$	0

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \;;\; \theta = \cos^{-1}\left(\frac{z}{|\vec{r}|}\right)$$

$$x = r\sin\theta\cos\phi \;;\; y = r\sin\theta\sin\phi \;;\; z = r\cos\theta \;;\; r^2 = x^2+y^2+z^2$$

$$d\vec{S}_r = r^2\sin\theta d\theta d\phi\hat{r} \;;\; d\vec{S}_\theta = r\sin\theta dr d\phi\hat{\theta} \;;\; d\vec{S}_\phi = r dr d\theta\hat{\phi} \;;\; d\vec{S}_\rho = \rho d\phi dz\hat{\rho} \;;\; d\vec{S}_z = \rho d\rho d\phi\hat{z}$$

$$dV = \rho d\rho d\phi dz \quad;\quad dV = r^2\sin\theta dr d\theta d\phi$$

$$\vec{r} = \rho\,\hat{\rho} + z\,\hat{z} \quad;\quad d\vec{L} = d\vec{r} = d\rho\hat{\rho} + \rho d\phi\hat{\phi} + dz\hat{z} \quad;\quad d\vec{r} = dr\hat{r} + r d\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \quad;\quad \cos^2(\theta/2) = \frac{1+\cos\theta}{2} \quad;\quad \sin^2(\theta/2) = \frac{1-\cos\theta}{2}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z} \;;\; \vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(A_\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\vec{\nabla}\varphi = \frac{\partial\varphi}{\partial\rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial\varphi}{\partial\phi}\hat{\phi} + \frac{\partial\varphi}{\partial z}\hat{z} \;;\; \vec{\nabla}\varphi = \frac{\partial\varphi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\varphi}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial\varphi}{\partial\phi}\hat{\phi} \;;\; \oint_S \vec{A}\cdot d\vec{S} = \int_{\text{vol}} \vec{\nabla}\cdot\vec{A}\,dv$$

$$\vec{F}_T = \sum_{m=2}^{n}\frac{Q_1Q_m}{4\pi\epsilon R_{m1}^2}\hat{R}_{m1} \;;\; \vec{E}(\vec{r}) = \frac{Q(\vec{r}-\vec{r}\,')}{4\pi\epsilon|\vec{r}-\vec{r}\,'|^3} \;;\; \vec{E}(\vec{r}) = \sum_{m=1}^{n}\frac{Q_m(\vec{r}-\vec{r}_m)}{4\pi\epsilon|\vec{r}-\vec{r}_m|^3} \;;\; d\vec{E}(\vec{r}) = \frac{dQ(\vec{r}-\vec{r}\,')}{4\pi\epsilon_0|\vec{r}-\vec{r}\,'|^3}$$

$$\epsilon_0 = \frac{10^{-9}}{36\pi} = 8.854\times10^{-12}\ \text{F/m} \;;\; \vec{D} = \epsilon\vec{E} \;;\; \oint_S \vec{D}\cdot d\vec{S} = \int_{\text{vol}}\vec{\nabla}\cdot\vec{D}\,dv = Q \;;\; \vec{\nabla}\cdot\vec{D} = \rho_v$$

$$W = -Q\int_C \vec{E}\cdot d\vec{L} \;;\; V_{AB} = \frac{W}{Q} = -\int_B^A \vec{E}\cdot d\vec{L} \;;\; V(\vec{r}) = \sum_{m=1}^{n}\frac{Q_m}{4\pi\epsilon_0|\vec{r}-\vec{r}_m|}$$

$$V(\vec{r}) = \int_\ell \frac{\rho_L dL'}{4\pi\epsilon_0|\vec{r}-\vec{r}\,'|} \;;\; V(\vec{r}) = \int_S \frac{\rho_S dS'}{4\pi\epsilon_0|\vec{r}-\vec{r}\,'|} \;;\; V(\vec{r}) = \int_{\text{vol}}\frac{\rho_v dv'}{4\pi\epsilon_0|\vec{r}-\vec{r}\,'|}$$

$$\vec{E} = -\vec{\nabla}V \;;\; V = \frac{1}{4\pi\epsilon_0|\vec{r}-\vec{r}\,'|^2}\vec{p}\cdot\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}$$

$$W_E = \frac{1}{2}\sum_{m=1}^{n}Q_mV_m \;;\; W_E = \frac{1}{2}\int_{\text{vol}}\rho_v V dv \;;\; W_E = \frac{1}{2}\int_{\text{vol}}\epsilon_0 E^2 dv$$

$$I = \int_S \vec{J}\cdot d\vec{S} \;;\; \vec{J} = \rho_v\vec{v} \;;\; I = \oint_S \vec{J}\cdot d\vec{S} = -\frac{dQ_i}{dt} \;;\; \vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho_v}{\partial t}$$

$$\vec{F} = Q\vec{E} = -e\vec{E} \;;\; \vec{v}_d = -\mu_e\vec{E} \;;\; \vec{J} = \rho_e\vec{v}_d \;;\; \sigma = -\rho_e\mu_e \;;\; \vec{J} = \sigma\vec{E} \;;\; \sigma = -\rho_e\mu_e + \rho_h\mu_h$$

$$R = \frac{L}{\sigma S} \;;\; R = \frac{V_{ab}}{I} = \frac{-\int_b^a \vec{E}\cdot d\vec{L}}{\int_S \sigma\vec{E}\cdot d\vec{S}} \;;$$

$$\vec{P} = \lim_{\Delta v\to 0}\frac{1}{\Delta v}\sum_{i=1}^{n\Delta v}\vec{p}_i \;;\; Q_b = -\oint \vec{P}\cdot d\vec{S} \;;\; \vec{\nabla}\cdot\vec{P} = -\rho_b \;;\; \vec{\nabla}\cdot\epsilon_0\vec{E} = \rho_T \;;\; \vec{P} = \chi_e\epsilon_0\vec{E} \;;\; \epsilon_r = \chi_e + 1$$

$$C = \frac{Q}{V_0} = \frac{\epsilon S}{d} \;;\; C = \frac{\oint_S \epsilon\vec{E}\cdot d\vec{S}}{-\int_-^+ \vec{E}\cdot d\vec{L}} \;;\; \vec{\nabla}^2 V = -\frac{\rho_v}{\epsilon}$$

$$\vec{\nabla}^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial\phi^2}\right) + \frac{\partial^2 V}{\partial z^2} \;;\; \vec{\nabla}^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

$$\oint_C \vec{H}\cdot d\vec{L} = I_e = \int_S \vec{J}\cdot d\vec{S} \quad;\quad \vec{H} = \oint_C \frac{Id\vec{L}\,'\times\hat{R}}{4\pi R^2}$$