Engineering 5812 Assignment 1 (Review)
Due: Noon, Wednesday, January 18, 2012

1. Let $\vec{V} = \hat{x} + 2\hat{y} + 3\hat{z}$ and $\vec{U} = -2\hat{x} + 3\hat{y} - \hat{z}$. Find (a) $\vec{V} + \vec{U}$, (b) $\vec{V} - \vec{U}$, (c) $\vec{V} \cdot \vec{U}$, and (d) $\vec{V} \times \vec{U}$.

2. Find the angle between $\vec{V}$ and $\vec{U}$ given in Problem 1.

3. Let $\vec{V} = \hat{x} + b\hat{y} + c\hat{z}$ and $\vec{U} = -\hat{x} + 3\hat{y} - 8\hat{z}$. (a) What values of b and c will make $\vec{V} \parallel \vec{U}$? (b) What values of b and c will make $\vec{V} \perp \vec{U}$?

4. *Prove* that vector $\vec{V}$ is always perpendicular $\vec{V} \times \vec{U}$. In doing, so let $\vec{V} = V_x\hat{x} + V_y\hat{y} + V_z\hat{z}$ and analogously for $\vec{U}$.

5. For vectors $\vec{A} = \hat{x} - \hat{y} + 2\hat{z}$, $\vec{B} = \hat{y} + \hat{z}$, and $\vec{C} = -2\hat{x} + 3\hat{z}$, find the vector triple products $(\vec{A} \times \vec{B}) \times \vec{C}$ and $\vec{A} \times (\vec{B} \times \vec{C})$. What does this indicate about vector triple products?

6. *Prove* that a vector $\vec{C}$ that is parallel to $\vec{A} = 5\hat{x} - 8\hat{y} + 2\hat{z}$ and has a magnitude of unity is equal to $\pm\hat{a}$ where $\hat{a}$ is the unit vector associated with $\vec{A}$. Start by letting $\vec{C} = x\hat{x} + y\hat{y} + z\hat{z}$.

7. Find an expression for a unit vector directed toward point P located on the z-axis at height h above the x-y plane from an arbitrary point $Q(x, y, 2)$ on the $z = 2$ plane.

8. A given line is described by $x + 2y = 8$. If vector $\vec{A}$ starts at the origin and ends at point P on the line such that $\vec{A}$ is orthogonal to the line, find $\vec{A}$.

9. Determine the distance between (a) $P_1(1, 2, 3)$ and $P_2(-2, -3, 4)$ where the points are in Cartesian coordinates and between (b) $P_3(2, \pi/2, 1)$ and $P_4(4, \pi/2, 0)$ where the points are given in cylindrical coordinates.

10. A certain vector field is given in Cartesian coordinates as $\vec{C} = [y^2/(x^2 + y^2)]\hat{x} - [x^2/(x^2 + y^2)]\hat{y} + 4\hat{z}$. Transform the field to cylindrical coordinates and evaluate it at the point $P(1, -1, 2)$.

11. Show that the distance, d, between the points $P_1(r_1, \theta_1, \phi_1)$ and $P_2(r_2, \theta_2, \phi_2)$ which are specified in spherical coordinates is given by

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\left[\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\phi_2 - \phi_1)\right]}\,.$$

12. Use integration to find the area of the surface specified by $2 \leq \rho \leq 5$, $\phi = \pi/4$, $-2 \leq z \leq 2$. Provide a sketch showing the appropriate differential area that is required for the integration.

13. Find the volume and the surface area surrounding it for the region described by $0 \leq r \leq 4$, $0 \leq \theta \leq \pi/3$, $0^\circ \leq \phi \leq \pi$. Sketch the region.