Engineering 5812 Assignment 3 **Due: Noon, Thursday, Feb. 9, 2012**

(Electric Field Intensity)

1. A line charge with uniform density ρ_L extends between $(0, 0, -L/2)$ to $(0, 0, L/2)$. Apply Coulomb's law to obtain an expression for the electric field intensity at any point $P(\rho, \phi, 0)$ on the $z = 0$ plane. Show that the result reduces to the appropriate expression for an infinite line of charge as the line becomes infinitely long.

2. Three infinite lines of charge, $\rho_{\ell_1} = 5$ nC/m, $\rho_{\ell_1} = -5$ nC/m, and $\rho_{\ell_1} = 5$ nC/m parallel to the y-axis pass through the respective points $(0, 0, -b)$, $(0, 0, 0)$, and $(0, 0, b)$. Find the electric field at $(a, 0, 0)$. Evaluate the result for $a = 2$ cm and $b = 1$ cm.

3. Starting from first principles show that if an infinite sheet of positive charge with a uniform density ρ_S is confined to the $z = 0$ plane in free space, then at position $(0, 0, -z)$
$$\vec{E} = -\frac{\rho_S}{2\epsilon_0}\hat{z}\ .$$
Be sure to make an appropriate sketch and include all relevant vectors. Also, state your reasons for any simplifications which may be made.

4. A disc $\rho = \rho_0$ containing a uniform positive surface charge density ρ_S is centred on the origin in the x-y plane in free space. Show that the electric field intensity at $P(0, 0, z_0)$, $z_0 > 0$, on the z-axis is given by
$$\vec{E} = \hat{z}\frac{\rho_S}{2\epsilon_0}\left[1 - \frac{z_0}{\sqrt{\rho_0^2 + z_0^2}}\right]\ .$$
and show what this becomes as the disc radius becomes infinite (does this make sense?).

5. The electric field intensity is given as $\vec{E} = 5e^{-2x}(\sin 2y\,\hat{x} - \cos 2y\,\hat{y})$ V/m. (a) Find the equation of the streamline passing through the point $P(0.5, \pi/10, 0)$. (b) Find a unit vector tangent to the streamline at P.

Answers: (1) $\hat{\rho}\frac{\rho_\ell}{2\pi\epsilon_0\rho}\frac{L/2}{\sqrt{\rho^2 + (L/2)^2}}$; (3) Answer supplied in question. (4) Answer supplied in question. (5) (a) $x = 0.5 + \frac{1}{2}\ln\left[\frac{\cos(2y)}{\cos(\pi/5)}\right]$; (b) $\hat{E}_P = \sin(0.2\pi)\hat{x} - \cos(0.2\pi)\hat{y}$

Answer to number (2) to follow.