Engineering 6813 Assignment 1 **Due: Mon., Sept. 21, 2009**

(Mostly Electrostatics Review)

1. An electric flux density is given by

$$\vec{D} = y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z} \text{ C/m}^2 .$$

(i) What is the charge density associated with this field at the points $(1, 1, z)$?
(ii) Check that both sides of the divergence theorem evaluate to 2 C for $\vec{D}$ using the region $0 \leq x, y, z \leq 1$.

2. A uniformly distributed charge of total value Q exists in the region $r \leq R$.
(i) Determine the electric field in the regions $r > R$ and $r < R$.
(ii) How does your answer for the $r > R$ region compare with what you get if Q was all concentrated at the centre of the sphere instead of being distributed?

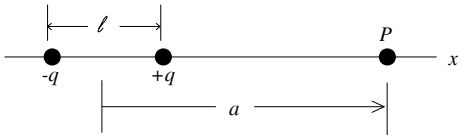
3. A uniform spherical surface charge distribution exists at $r = R$. The total charge on the surface is Q.
(i) Determine the electric field inside and outside the surface.
(ii) Use infinity as the zero reference and determine the potential outside and inside the surface.

4. The capacitance between two conductors, one which has a charge of $+Q$ and the other $-Q$, is given by

$$C = \frac{Q}{V}$$

where V is the potential difference between them. Now, two concentric spherical metal shells of radii a and b, with $a < b$ and centred on the origin, carry charges $+Q$ and $-Q$, respectively.
(i) Determine the electric field between the shells.
(ii) Determine the potential difference V_{ab} and note that $V_{\text{ab}} \propto Q$.
(iii) Find the capacitance.

5. A dipole of charge $\pm q$ and separation ℓ is placed on the x-axis as shown. Show that the potential at point P is given by $V_P = \dfrac{p}{4\pi\epsilon_0\left(a^2 - \frac{\ell^2}{4}\right)}$ where $p = q\ell$ is referred to as the dipole moment and a is the distance from P to the centre of the dipole.



6. A straight conductor of circular cross section has a diameter d and is centred along the z-axis. The current density in the wire is given by $\vec{J} = k\rho\hat{z}$ where k is a proportionality constant and ρ is the distance from the wire centre.
(i) Find the total current I in the wire.
(ii) Find the volume charge density ρ_v.