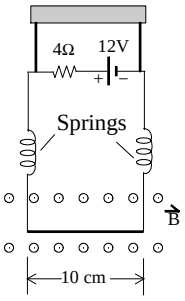
Engineering 6813 Assignment 4 **Due: Wed., Nov. 4, 2009**

1. The plane boundary defined by $z = 0$ separates air ($z \geq 0$) from a block of iron ($z \leq 0$). If the magnetic field in the air is given by

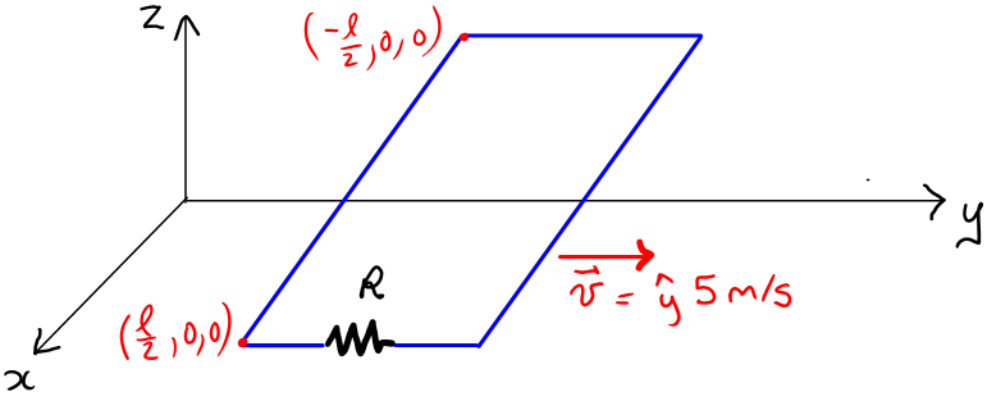
$$\vec{B}_1 = 4\hat{x} - 6\hat{y} + 8\hat{z} \text{ T}$$

and the relative permeability of iron is 5000, find $\vec{B}_2$ in the iron.

2. The circuit shown in the figure below uses two identical springs to support a 10-cm-long horizontal wire with a mass of 5 g. In the absence of a magnetic field, the weight of the wire causes the springs to stretch a distance of 0.2 cm each. When a uniform magentic field is turned on in the region containing the horizontal wire the springs are observed to stretch an additional 0.5 cm. What is the magnitude of the magnetic flux density $\vec{B}$?



3. The rectangular loop, whose long side is 2 m and whose short side is 0.5 m, shown below is situated in the $z = 0$ plane and moves away from the origin with a velocity of $\vec{v} = 5\,\hat{y}$ m/s in a steady magnetic field $\vec{B} = \hat{z}\,0.2e^{-0.1y}$ T. If $R = 5\ \Omega$, determine the size and direction of the current I when the left side of the loop reaches $y = 2$ m.

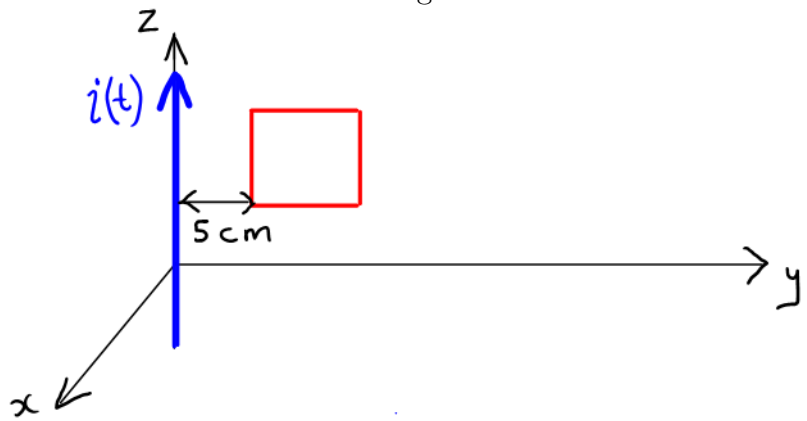


4. (a) Suppose that an initial charge density ρ_{v0} is introduced inside an originally neutral conductor. We know that this charge density will cause free charge in the material to migrate to the surface until the conductor is again essentially neutral on the inside. Show that the charge density $\rho_v(t)$ inside the conductor at time t after the initial charge density is introduced is given by

$$\rho_v(t) = \rho_{v0}e^{-t/t_r}$$

where $t_r = \epsilon/\sigma$ with ϵ being the permittivity of the conductor and σ being its conductivity.
(b) The quantity t_r in part (a) is referred to as the *relaxation time constant* and is clearly the time necessary for the initial charge density to be reduced to $1/e$ of its initial value. Given that the relative permittivity of quartz is 5 and that the conductivity is 10^{-17} S/m, determine (i) the relaxation time constant for quartz and (ii) the time it would take for a charge density to decrease to 1% of its initial value in this material.

5. A current of $i(t) = 2.5\cos 2\pi \times 10^4 t$ A flows along the z-axis in the vicintiy of a planar square loop in the $x = 0$ plane. The loop is 10 cm on an edge and the left side of the loop is 5 cm from the z axis. (a) Determine the emf induced across a small gap in the loop. (b) If the loop has an internal reistance of $1\,\Omega$ what current would flow through a $4\,\Omega$ resistor connected across the gap?



6. In a certain conducting medium, the current density is given by

$$\underset{\sim}{\vec{J}} = (\hat{x}z - \hat{y}3y^2 + \hat{z}2x)\cos\omega t\ .$$

Determine the corresponding charge distribution.

7. The magnetic field in a certain unbounded, ideal, nonmagnetic dielectric material which has $\epsilon_r = 4$ is given by

$$\underset{\sim}{\vec{H}} == \hat{x}5\cos(2\pi \times 10^7 t + ky)\ \text{A/m}\ .$$

Find k and the associated electric field $\underset{\sim}{\vec{E}}$.