Engineering 6813 Formulae and Data

	$\hat{\rho}$	$\hat{\phi}$	$\hat{z}$
$\hat{x}\cdot$	$\cos\phi$	$-\sin\phi$	0
$\hat{y}\cdot$	$\sin\phi$	$\cos\phi$	0
$\hat{z}\cdot$	0	0	1

	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$
$\hat{x}\cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\hat{y}\cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\hat{z}\cdot$	$\cos\theta$	$-\sin\theta$	0

$$\vec{\nabla}^2\vec{A} = \left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]\left(\vec{A}\right) \ ; \ \oint_S \vec{A}\cdot d\vec{S} = \int_{\text{vol}} \vec{\nabla}\cdot\vec{A}\,dv \ ; \ \oint_C \vec{A}\cdot d\vec{L} = \int_S\left(\vec{\nabla}\times\vec{A}\right)\cdot d\vec{S}$$

$$\vec{\nabla}\times\vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} \ ; \ \vec{\nabla}\times\vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix} \ ; \ dS_z = \rho d\rho d\phi$$

$$\vec{\nabla}\times\vec{\nabla}V = 0 \ ; \ \vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0 \ ; \ \vec{\nabla}\times\vec{\nabla}\times\vec{A} = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \vec{\nabla}^2\vec{A} \ ; \ dS_\phi = d\rho dz$$

$$\vec{\nabla}V = \hat{\rho}\frac{\partial V}{\partial\rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial V}{\partial\phi} + \hat{z}\frac{\partial V}{\partial z} \ ; \ dS_r = r^2\sin\theta d\theta d\phi \ ; \ dv = r^2\sin\theta dr d\theta d\phi$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \ ; \ \theta = \cos^{-1}\left(\frac{z}{|\vec{r}|}\right)$$

$$\vec{r} = \rho\,\hat{\rho} + z\,\hat{z} \ ; \ d\vec{L} = d\vec{r} = d\rho\hat{\rho} + \rho d\phi\hat{\phi} + dz\hat{z} \ ; \ d\vec{r} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$$

$$(1+x)^n = 1 + nx + \tfrac{n(n-1)}{2}x^2 + \ldots \ ; \ \sin\theta = \tfrac{e^{j\theta}-e^{-j\theta}}{2j} \ ; \ \cos\theta = \tfrac{e^{j\theta}+e^{-j\theta}}{2}$$

$$\oint_S \underset{\sim}{\vec{B}}\cdot d\vec{S} = 0 \ ; \ \vec{\nabla}\times\underset{\sim}{\vec{E}} = -\frac{\partial\underset{\sim}{\vec{B}}}{\partial t} \ ; \ \vec{\nabla}\times\underset{\sim}{\vec{H}} = \underset{\sim}{\vec{J}} + \frac{\partial\underset{\sim}{\vec{D}}}{\partial t} = \underset{\sim}{\vec{J}} + \underset{\sim}{\vec{J}}_d \ ; \ \vec{\nabla}\cdot\underset{\sim}{\vec{D}} = \underset{\sim}{\rho_v} \ ; \ \vec{\nabla}\cdot\underset{\sim}{\vec{B}} = 0$$

$$\vec{\nabla}\times\vec{E}_s = -j\omega\mu\vec{H}_s \ ; \ \vec{\nabla}\times\vec{H}_s = \vec{J}_s + j\omega\vec{D}_s \ ; \ \vec{\nabla}\cdot\vec{D}_s = \rho_{v_s} \ ; \ \vec{\nabla}\cdot\vec{B}_s = 0 \ ; \ \vec{J} = \sigma\vec{E}$$

$$\mu_0 = 4\pi\times10^{-7}\text{ H/m} \ ; \ \epsilon_0 = \frac{10^{-9}}{36\pi}\text{ F/m} \ ; \ c = 3\times10^8\text{ m/s} \ ; \ \underset{\sim}{\vec{B}} = \mu\underset{\sim}{\vec{H}} \ ; \ \underset{\sim}{\vec{D}} = \epsilon\underset{\sim}{\vec{E}} \ ; \ \vec{E} = -\vec{\nabla}V$$

$$\vec{M} = \chi_m\vec{H} \ ; \ D_{1n} - D_{2n} = \rho_s \ ; \ B_{1n} - B_{2n} = 0 \ ; \ d\vec{F} = Id\vec{L}\times\vec{B} \ ; \ d\vec{T} = \vec{R}\times d\vec{F}$$

$$\hat{n}\times(\vec{E}_1-\vec{E}_2) = 0 \ ; \ \hat{n}\times(\vec{H}_1-\vec{H}_2) = \vec{K} \ ; \ \oint_C \vec{H}\cdot d\vec{L} = I_e \ ; \ \Phi = \int_S \vec{B}\cdot d\vec{S} \ ; \ \vec{H} = \int_C \frac{Id\vec{L}\times\hat{a}_R}{4\pi R^2}$$

$$\vec{\nabla}^2\vec{E}_s + \omega^2\mu\epsilon\vec{E}_s = 0 = \vec{\nabla}^2\vec{E}_s + \beta^2\vec{E}_s \text{ with one solution being } E_{x_s}(z) = E_{x0}^-e^{j\beta z} + E_{x0}^+e^{-j\beta z}$$

$$\vec{\nabla}^2\vec{E}_s - \gamma^2\vec{E}_s = 0 \text{ with one solution being } E_{x_s}(z) = E_{x0}^-e^{\gamma z} + E_{x0}^+e^{-\gamma z} \ ; \ \gamma^2 = (\sigma + j\omega\epsilon)j\omega\mu$$

$$\gamma = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}} = \alpha + j\beta \ ; \ \eta = \frac{j\omega\mu}{\gamma} \ ; \ \vec{\mathcal{P}} = \underset{\sim}{\vec{E}}\times\underset{\sim}{\vec{H}} \ ; \ P = \int_S \vec{\mathcal{P}}\cdot d\vec{S} \ ; \ <\vec{\mathcal{P}}> = \frac{1}{2}Re\left\{\vec{E}_s\times\vec{H}_s^*\right\}$$

$$<\vec{\mathcal{P}}> = \frac{E_{x0}^{+^2}}{2\eta_m}e^{-2\alpha z}\cos\theta_\eta\,\hat{z}\text{ W/m}^2 \ ; \ \delta = \frac{1}{\sqrt{\pi f\mu\sigma}} \ ; \ R = \frac{L}{\sigma S} \ ; \ 1 + \Gamma = \tau \ ; \ \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} \ ; \ v_p = \frac{\omega}{\beta} \ ; \ \tan\theta = \frac{\sigma}{\omega\epsilon} \ ; \ p_r = \left(\frac{s-1}{s+1}\right)^2 \ ; \ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \ ; \ Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta \ ; \ Z_{\text{in}} = Z_0\left[\frac{Z_L + jZ_0\tan\beta\ell}{Z_0 + jZ_L\tan\beta\ell}\right] \ ; \ \eta_0 = 120\pi\ \Omega$$