Partial Solutions to ENGI6813 Assignment #4 Fall 2009

1.

$$\vec{B}_1 = 4\hat{x} - 6\hat{y} + 8\hat{z} \quad [T]$$

and the relative permeability of iron is 5000, find B_2 in the iron?

<u>Answer</u>: $\vec{H}_1 = \frac{\vec{B}_1}{\mu_1} = \frac{1}{\mu_1}(4\hat{x} - 6\hat{y} + 8\hat{z})$

Since $\vec{B}_{N_1} = \vec{B}_{N2}$, $\vec{B}_{N2} = 8\hat{z}$

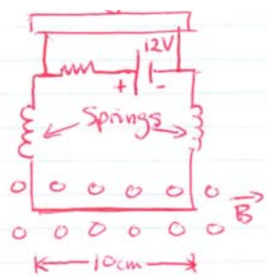
Since $\vec{H}_{t_1} = \vec{H}_{t2}$, $\vec{H}_{t2} = \frac{1}{\mu_1}(4\hat{x} - 6\hat{y})$

and $\vec{B}_{t2} = \mu_2 \vec{H}_{t2} = \frac{\mu_2}{\mu_1}(4\hat{x} - 6\hat{y})$

Since $\frac{\mu_2}{\mu_1} = \frac{\mu_{r2}\mu_0}{\mu_0} = 5000$, we have

$$\boxed{\vec{B}_2 = 20000\hat{x} - 30000\hat{y} + 8\hat{z}} \quad [Wb/m^2]$$

2.



<u>Answer</u>: Springs are characterized by a spring constant "k" where $\vec{F} = kd$ where d is the distance the spring is stretched. For these springs:

$$F = \tfrac{1}{2} mg = kd$$

$$\therefore k = \frac{mg}{2d} = \frac{(9.8 \times 5 \times 10^{-3})}{2 \times 2 \times 10^{-3}} = 12.25 \quad [N/m]$$

This means that the additional stretch means that the additional force is:

$$F = (12.25)(5 \times 10^{-3}) = 61.25 \quad [mN/Spring]$$

$$F_{TOT_{add}} = 0.123\,N$$

Electromagnetically speaking:

$$\vec{F} = I\vec{\ell} \times \vec{B}$$

Filling in the appropriate values gives $B = 410$ mT.

3.

Answer: Since $\vec{v}\times\vec{B}$ is along $\hat{x}$, voltages are induced in only the sides oriented along $\hat{x}$. The voltage induced in side ① is:

$$V_{①} = \int_{\frac{l}{2}}^{-\frac{l}{2}} [\vec{v}\times\vec{B}(y_1)]\cdot d\vec{l}$$

$$= \int_{\frac{l}{2}}^{-\frac{l}{2}} (5\hat{y}\times\hat{z}0.2e^{-0.2})\cdot\hat{x}\,dx$$

$$= -e^{-0.2}\cdot l = -2e^{-0.2} = -1.637\ [V]$$

Do an analogous calculation for the emf associated with side 2. Then, the current is found from Ohm's law to be 15.8 mA.

4.

Answer:

a) We know that: $\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho_v}{\partial t}$, and $\vec{J} = \sigma\vec{E}$

therefore $\sigma\vec{\nabla}\cdot\vec{E} = -\frac{\partial \rho_v}{\partial t}$, and $\vec{\nabla}\cdot\vec{E} = \rho_v/\epsilon$

so we get $\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon}\rho_v = 0$

given that $\rho_v = \rho_{vo}$ at $t = 0$ the solution is

$$\boxed{\rho_v(t) = \rho_{vo}e^{-(\sigma/\varepsilon)t} = \rho_{vo}e^{-t/t_r}}$$

b) (ii) Use the result in (a) and fill in the appropriate values and the t_r result from (i) to get t = 236 days (approximately).

5. (a) Get the magnetic flux density due to the current in the wire. Integrate this over the loop area to obtain the flux Φ and then get the emf from Faraday's law. **(b)** The current follows from Ohm's law.

Answer: (a) $3.45 \times 10^{-3} \sin(2\pi \times 10^4 t)$ V; (b) $0.69 \sin(2\pi \times 10^4 t)$ mA.

6.

Answer

We have: $\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho_v}{\partial t}$ (1)

and $\vec{\nabla}\cdot\vec{J} = \left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right)\cdot\left(\hat{x}z-\hat{y}3y^2+\hat{z}2x\right)\cos\omega t$

$= -3\frac{\partial}{\partial y}(y^2\cos\omega t)$

$= -6y\cos\omega t$ (2)

Now integrating both sides of eqn (1) using result (2).

$\rho_v = -\int -6y\cos\omega t\, dt$

$$\boxed{\rho_v = \frac{6y}{\omega}\sin\omega t + C_0}$$

where C_0 is a constant

7. Notice that we first convert to phasor form – this makes the math more straightforward. Then we can go back to the time domain at the end as usual.
THIS IS OFTEN A GOOD WAY TO PROCEED WITH THESE SORTS OF PROBLEMS.

Answer

In phasor form, the magnetic field is given by

$\vec{H} = \hat{x}5e^{jky}$ [A/m] (1)

we know $\vec{E} = \frac{1}{j\omega\epsilon}\vec{\nabla}\times\vec{H} = \frac{-jk}{j\omega\epsilon}\hat{z}5e^{jky}$ (2)

and also $\vec{H} = \frac{1}{-j\omega\mu}\vec{\nabla}\times\vec{E} = \frac{-jk^2}{-j\omega^2\epsilon\mu}\hat{x}5e^{jky}$ (3)

setting (1) = (3): $k = \omega\sqrt{\epsilon\mu} = \frac{\omega\sqrt{\epsilon_r}}{c} = \frac{2\pi\times10^7\sqrt{4}}{3\times10^8} = \boxed{\frac{4\pi}{30}\left[\frac{\text{rad}}{\text{m}}\right]}$

now solving (2) $\vec{E} = -\frac{4\pi/30}{2\pi\times10^7\times4\times8.854\times10^{-12}}\cdot 5e^{j4\pi y/30}\hat{z}$

$$\boxed{\vec{E} = -\hat{z}\,941e^{j4\pi y/30}\ [\text{V/m}]}$$

$$\boxed{\vec{\tilde{E}} = -\hat{z}\,941\cos\left(2\pi\times10^7 t + \frac{4\pi y}{30}\right)}$$