1. We know

$$\alpha = \omega\sqrt{\frac{\mu_0\epsilon_0\epsilon'_r}{2}}\left[\sqrt{1+\left(\frac{\epsilon''_r}{\epsilon'_r}\right)^2}-1\right]^{1/2}$$

and

$$\beta = \omega\sqrt{\frac{\mu_0\epsilon_0\epsilon'_r}{2}}\left[\sqrt{1+\left(\frac{\epsilon''_r}{\epsilon'_r}\right)^2}+1\right]^{1/2}$$

With the given values of ϵ'_r and ϵ''_r, it is clear that $\beta = \omega\sqrt{\mu_0\epsilon_0} = \omega/c$, and so $\lambda = 2\pi/\beta = 2\pi c/\omega = 3\times10^{10}/10^{10} = \underline{3\text{ cm}}$. It is also clear that $\alpha = \underline{0}$.

b) $\epsilon'_r = 1.04$ and $\epsilon''_r = 9.00\times10^{-4}$: In this case $\epsilon''_r/\epsilon'_r << 1$, and so $\beta \doteq \omega\sqrt{\epsilon'_r}/c = 2.13\text{ cm}^{-1}$. Thus $\lambda = 2\pi/\beta = \underline{2.95\text{ cm}}$. Then

$$\alpha \doteq \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} = \frac{\omega\epsilon''_r}{2}\frac{\sqrt{\mu_0\epsilon_0}}{\sqrt{\epsilon'_r}} = \frac{\omega}{2c}\frac{\epsilon''_r}{\sqrt{\epsilon'_r}} = \frac{2\pi\times10^{10}}{2\times3\times10^8}\frac{(9.00\times10^{-4})}{\sqrt{1.04}}$$
$$= \underline{9.24\times10^{-2}\text{ Np/m}}$$

For part (c), $\lambda = 1.33$ cm and $\alpha = 335$ Np/m.

2.

If $\mathbf{E}_s = (60/r)\sin\theta\, e^{-j2r}\,\mathbf{a}_\theta$ V/m, and $\mathbf{H}_s = (1/4\pi r)\sin\theta\, e^{-j2r}\,\mathbf{a}_\phi$ A/m in free space, find the average power passing outward through the surface $r = 10^6$, $0 < \theta < \pi/3$, and $0 < \phi < 2\pi$.

$$<\mathbf{S}> = \frac{1}{2}\text{Re}\{\mathbf{E}_s\times\mathbf{H}^*_s\} = \frac{15\sin^2\theta}{2\pi r^2}\,\mathbf{a}_r\ \text{W/m}^2$$

Then, the requested power will be

$$\Phi = \int_0^{2\pi}\int_0^{\pi/3}\frac{15\sin^2\theta}{2\pi r^2}\,\mathbf{a}_r\cdot\mathbf{a}_r\, r^2\sin\theta\, d\theta\, d\phi = 15\int_0^{\pi/3}\sin^3\theta\, d\theta$$
$$= 15\left(-\frac{1}{3}\cos\theta(\sin^2\theta+2)\right)\Big|_0^{\pi/3} = \frac{25}{8} = \underline{3.13\text{ W}}$$

Note that the radial distance at the surface, $r = 10^6$ m, makes no difference, since the power density dimishes as $1/r^2$.

3.

A hollow tubular conductor is constructed from a type of brass having a conductivity of 1.2×10^7 S/m. The inner and outer radii are 9 mm and 10 mm respectively. Calculate the resistance per meter length at a frequency of

a) dc: In this case the current density is uniform over the entire tube cross-section. We write:

$$R(\text{dc}) = \frac{L}{\sigma A} = \frac{1}{(1.2\times10^7)\pi(.01^2-.009^2)} = \underline{1.4\times10^{-3}\ \Omega/\text{m}}$$

b) 20 MHz: Now the skin effect will limit the effective cross-section. At 20 MHz, the skin depth is

$$\delta(20\text{MHz}) = [\pi f\mu_0\sigma]^{-1/2} = [\pi(20\times10^6)(4\pi\times10^{-7})(1.2\times10^7)]^{-1/2} = 3.25\times10^{-5}\text{ m}$$

This is much less than the outer radius of the tube. Therefore we can approximate the resistance using the formula:

$$R(20\text{MHz}) = \frac{L}{\sigma A} = \frac{1}{2\pi b\delta} = \frac{1}{(1.2\times10^7)(2\pi(.01))(3.25\times10^{-5})} = \underline{4.1\times10^{-2}\ \Omega/\text{m}}$$

c) 2 GHz: Using the same formula as in part b, we find the skin depth at 2 GHz to be $\delta = 3.25\times10^{-6}$ m. The resistance (using the other formula) is $R(2\text{GHz}) = \underline{4.1\times10^{-1}\ \Omega/\text{m}}$.

You should verify that to a good approximation, the cross sectional area used in (b) is indeed $2\pi b\delta$. Notice that in the solution of (b) the conductivity symbol σ is incorrectly omitted from the denominator $[1/\ 2\pi b\delta]$ **although the value correctly appears after the general formula. The formula should have** $[1/\ \sigma\, 2\pi b\delta]$.

4.

Given: $\vec{E}_s = (5\hat{x} + j10\hat{y})e^{-j2z}$ V/m .

$f = 5 \times 10^7$ Hz

$\mu_R = 1$ & $\sigma = 0$ (lossless)

(a.) To find: β, ω, v_p, λ, ϵ_R & η

(i.) From inspection (i.e. $e^{-j2z} = e^{-j\beta z}$), $\beta = 2$ rad/m

(ii.) $\omega = 2\pi f = 10\pi \times 10^7$ rad/s $\quad (= 3.14 \times 10^8$ rad/s$)$

(iii.) $v_p = \dfrac{\omega}{\beta} = \dfrac{10\pi \times 10^7 \text{ rad/s}}{2 \text{ rad/m}} = 5\pi \times 10^7$ m/s $\quad (= 1.57 \times 10^8$ m/s$)$.

(iv.) $\lambda = \dfrac{2\pi}{\beta} = \pi$ m $\quad (= 3.14$ m$)$.

(v.) $v = \sqrt{\dfrac{1}{\mu\epsilon}} = \dfrac{1}{\sqrt{\mu_R\epsilon_R}} \cdot c \quad ; \quad c = \dfrac{1}{\sqrt{\mu_0\epsilon_0}}$:

$\therefore \left(\dfrac{v}{c}\right)^2 = \dfrac{1}{\mu_R\epsilon_R} \Rightarrow \epsilon_R = \dfrac{1}{\mu_R}\left(\dfrac{c}{v}\right)^2 = 3.65$

(vi) $\eta = \sqrt{\mu/\epsilon} = \sqrt{\dfrac{\mu_R}{\epsilon_R}}\,\eta_0 = \sqrt{\dfrac{1}{3.65}} \times 377\,\Omega = 197.3\,\Omega$

(b.) To find: $\underset{\sim}{E}$ at $(0,0,0) \equiv (x,y,z)$ for $\omega t = 0, \pi/4, \pi/2, 3\pi/4, \pi$.

At this position,

$\vec{E}_s = 5\hat{x} + j10\hat{y}$

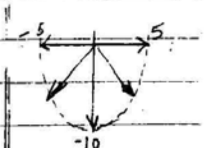
$\Rightarrow \underset{\sim}{\vec{E}} = Re\{5\hat{x} + j10\hat{y}\, e^{j\omega t}\}$

$\Rightarrow \quad \underset{\sim}{\vec{E}}\big|_{\omega t = 0} = 5\hat{x}$ V/m

$\underset{\sim}{\vec{E}}\big|_{\omega t = \pi/4} = 5\sqrt{2}/2\,\hat{x} - \dfrac{10\sqrt{2}}{2}\hat{y} = 3.5\hat{x} - 7.1\hat{y}$

$\underset{\sim}{\vec{E}}\big|_{\omega t = \frac{\pi}{2}} = -10\hat{y} \quad ; \quad \underset{\sim}{\vec{E}}\big|_{\omega t = \frac{3\pi}{4}} = -3.5\hat{x} - 7.1\hat{y}$

$\underset{\sim}{\vec{E}}\big|_{\omega t = \pi} = -5\hat{x}$



Note: $\vec{E}$ traces an ellipse. (Elliptical Polarization)

5.

Given: $f = 50\times10^6$ Hz, $\gamma = 0.25 + j2\ m^{-1}$; $\eta = 600 + j75\ \Omega$

$\hat{z} \equiv$ propagation direction.

$[\ \gamma = \alpha + j\beta \Rightarrow \alpha = 0.25,\ \beta = 2\ ]$.

a.) To find: μ_R, ϵ_R, σ.

(i.) $\eta = \sqrt{\mu/\epsilon}\left(\dfrac{1}{\sqrt{1-j\frac{\sigma}{\omega\epsilon}}}\right)$ & $\gamma = j\omega\sqrt{\mu\epsilon}\sqrt{1-j\frac{\sigma}{\omega\epsilon}}$

$\Rightarrow\ j\omega\sqrt{\mu\epsilon}\sqrt{\mu/\epsilon} = \eta\gamma$

$\Rightarrow\ j\omega\mu = (150-150) + j\cdot(1200+18.25) = j1218.25$

$\Rightarrow\ \mu_R = \dfrac{1218.25}{\omega\mu_0} = \dfrac{1218.25}{2\pi f\mu_0} = 3.086$.

(ii.) $\gamma/\eta = j\epsilon\omega\left(1-j\frac{\sigma}{\omega\epsilon}\right) = j\omega\epsilon + \sigma$

$\therefore\ j\omega\epsilon + \sigma = \dfrac{(0.25+j2)(600-j75)}{(600)^2+(75)^2}$

$\Longrightarrow\ \sigma + j\omega\epsilon = 8.21\times10^{-4} + j\,3.23\times10^{-3}$

$\therefore\ \sigma = 8.21\times10^{-4}\ \mho/m$

(iii) $\omega\epsilon = \omega\epsilon_0\epsilon_R = 3.23\times10^{-3} \Rightarrow \epsilon_R = \dfrac{3.23\times10^{-3}}{2\pi f\epsilon_0}$

$\therefore\ \epsilon_R = 1.161$

b.) $\vec{E}_s = 100\,\hat{x}$ V/m at $z=0$: In general $\vec{E}_s = 100e^{-\gamma z}\,\hat{x}$ V/m.

$\therefore$ At $z = 2m$,

$\vec{E}_s = 100e^{-\alpha z}e^{-j\beta z} = 100e^{-(0.25\times2)}e^{-j(2\times2)}\hat{x} = 100e^{-0.5}e^{-j4}\hat{x}\ \frac{V}{m}$

$\Rightarrow\ \vec{E}_s = 60.6\ \angle 130.8^\circ\ \hat{x}$ V/m where 130.8° is the angle which appears in the $\cos(\omega t - \beta z)$ factor for (βz) since $\beta z = -4_{rad} = -219.2^\circ \equiv +130.8^\circ$.

(c)

$\vec{H}_s\Big|_{z=2} = \dfrac{100e^{-\alpha z}e^{-j\beta z}}{\eta}\,\hat{y}\ \frac{A}{m}$ (recall $\hat{z}$ is propagation direction).

$= \dfrac{60.6\times(600-j75)}{(600)^2+(75)^2}\,e^{-j4}\,\hat{y}$ A/m.

$= (0.09945 - j0.01243)\,e^{-j4}\,\hat{y}$ A/m.

$= 0.1002\,e^{-j4.1243}\,\hat{y}$ A/m

OR $\vec{H}_s\Big|_{z=2} = 0.1002\ \angle 123.7\ \hat{y}$ A/m.

6.

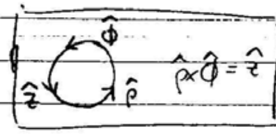
Given: $a = 0.01\,m$; $b = e \times 10^{-2}\,m$; $\vec{E} = \left(\frac{100}{\rho}\right)\cos(10^8 t - \beta z)\,\hat{\rho}\,\frac{V}{m}$.

(So direction of travel is $\hat{z}$ but $\vec{E}$ has only a radial component in $\hat{\rho}$ direction).

a.) Since wave is travelling in air,

$$\beta = \omega\sqrt{\mu_0\epsilon_0} = \frac{\omega}{c} = \frac{10^8}{3\times10^8} = 0.333\,\frac{rad}{m}$$

b.) $\vec{H} = ?$



Noting that $\hat{\rho}\times\hat{\phi} = \hat{z}$ which is direction of field "travel", $\vec{H} \equiv H\,\hat{\phi}$.

$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$; since $\vec{E}$ has only z-dependence spatially,

(Check curl expression in cylindrical coordinates) $\longrightarrow$ $\frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu_0}\frac{\partial \vec{E}}{\partial z}\hat{\phi} = \frac{-1}{\mu_0}\frac{100\beta}{\rho}\sin(10^8 t - \beta z)\,\hat{\phi}$

$$\therefore \vec{H} = \frac{100\beta}{\mu_0 \rho\, 10^8}\cos(10^8 t - \beta z)\,\hat{\phi} \quad A/m$$

$$\vec{H} = \frac{0.265}{\rho}\cos\left(10^8 t - \frac{z}{3}\right)\hat{\phi} \quad A/m$$

(c.) $\vec{P} = \vec{E}\times\vec{H} = \frac{26.5}{\rho^2}\cos^2\left(10^8 t - z/3\right)\hat{z}\ \frac{W}{m^2}$ since $\hat{\rho}\times\hat{\phi} = \hat{z}$.

(d.) Before finding P_{av} we note that $P = \int_S \vec{P}\cdot d\vec{S}$ where, here,

$$d\vec{S} = dS_z\,\hat{z} = \rho\, d\phi\, d\rho\,\hat{z}$$

$$\Rightarrow P = 26.5\cos^2\left(10^8 t - \frac{z}{3}\right)\int_0^{2\pi}\int_a^{b = e\times10^{-2}} \frac{1}{\rho^2}\rho\, d\phi\, d\rho$$

$$= 2\pi \times 26.5\cos^2(10^8 t - z/3)\ \ln\left(\frac{e\times10^{-2}}{10^{-2}}\right)$$

$\therefore$, Since $\frac{1}{T}\int_0^T \cos^2(\ldots)\,dt = \frac{1}{2}$; $\quad P_{av} = \frac{2\pi\times26.5}{2}\times 1 = 83.3\,W$

7.

Given: $f = 2.45\ GHz$; $\sigma = 1.1 \times 10^{6}\ \mho/m$

$\mu_R = 600$

Find: (a) $\delta = ?$ (skin depth)

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \times 2.45\times10^{9} \times 600 \times 4\pi\times10^{-7} \times 1.1\times10^{6}}}$$

$$\delta = 3.96 \times 10^{-7}\ m$$

(b) We know $\vec{E_s}\big|_{z=0} = 10\angle 0^\circ \hat{x} = 10 e^{j0} \hat{x}$,

For good conductor, $\gamma = \alpha + j\beta$

$\alpha = \beta = \frac{1}{\delta}$

($\hat{z}$ propagation assumed)
($\hat{x}$ direction of E_s ")

In general $\vec{E_s}\big|_z = E_s\big|_{z=0}\, e^{-\gamma z}\, \hat{x}$

$$= E_s\big|_{z=0}\, e^{-z/\delta}\, e^{-jz/\delta}$$

$$\vec{E_s}\big|_z = 10 e^{-z/\delta} e^{-jz/\delta}\ \hat{x}$$

Now, $|\vec{E_s}| = E_s = 10 e^{-z/\delta}$

and angle of E_s is $\theta = -z/\delta = \dfrac{-10^{7} z}{3.96}$

Magnitude of Phasor Electric Field vs. Its Angle