Engineering 7811 Formula and Data Sheet **Winter, 2011**

$$\vec{\nabla}\times\underset{\sim}{\vec{E}} = -\frac{\partial \underset{\sim}{\vec{B}}}{\partial t} \;;\; \vec{\nabla}\cdot\underset{\sim}{\vec{D}} = \underset{\sim}{\rho}_v \;;\; \vec{\nabla}\times\underset{\sim}{\vec{H}} = \underset{\sim}{\vec{J}} + \frac{\partial \underset{\sim}{\vec{D}}}{\partial t} \;;\; \vec{\nabla}\cdot\underset{\sim}{\vec{B}} = 0 \;;\; \underset{\sim}{\vec{D}} = \epsilon\underset{\sim}{\vec{E}} \;;\; \underset{\sim}{\vec{B}} = \mu\underset{\sim}{\vec{H}}$$

$$\epsilon_0 = 8.854\times10^{-12} \approx \frac{10^{-9}}{36\pi}\ \mathrm{F/m} \;;\; \mu_0 = 4\pi\times10^{-7}\ \mathrm{H/m} \;;\; \epsilon = \epsilon_0(1+\chi_e) \;;\; \mu = \mu_0(1+\chi_m)$$

$$\underset{\sim}{\vec{D}} = \epsilon_0\underset{\sim}{\vec{E}} + \epsilon_0\chi_e\underset{\sim}{\vec{E}} = \epsilon_0\underset{\sim}{\vec{E}} + \underset{\sim}{\vec{P}} \;;\; \underset{\sim}{\vec{B}} = \mu_0\underset{\sim}{\vec{H}} + \mu_0\chi_m\underset{\sim}{\vec{H}} = \mu_0(\underset{\sim}{\vec{H}} + \underset{\sim}{\vec{M}})$$

$$\vec{\nabla}\cdot\underset{\sim}{\vec{J}} = -\frac{\partial \underset{\sim}{\rho}_v}{\partial t} \;;\; \underset{\sim}{\vec{J}}_{cnv} = \underset{\sim}{\rho}_v\vec{v} \;;\; \underset{\sim}{\vec{J}} = \sigma\underset{\sim}{\vec{E}} \;;\; \underset{\sim}{\vec{A}}(\vec{r},t) \equiv \underset{\sim}{\vec{A}} = \mathcal{R}e\left\{\vec{A}e^{j\omega t}\right\}$$

$$\vec{F} = Q(\vec{E} + \vec{v}\times\vec{B}) \;;\; v_p = \frac{1}{\sqrt{\mu\epsilon}} \;;\; c = \frac{1}{\sqrt{\mu_0\epsilon_0}} \;;\; c = 3\times10^8\ \mathrm{m/s} \;;\; v_p = f\lambda$$

$$k = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\epsilon} \;;\; jk = \alpha + j\beta \;;\; \beta = \frac{2\pi}{\lambda} \;;\; \eta = \sqrt{\frac{\mu}{\epsilon}} \;;\; \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \;;\; \eta_0 \approx 120\pi\ \Omega$$

$$\vec{\nabla}\times\vec{E} = -j\omega\vec{B} \quad;\; \vec{\nabla}\cdot\vec{D} = \rho_v \quad;\; \vec{\nabla}\times\vec{H} = \vec{J} + j\omega\vec{D} \quad;\; \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{E}(\vec{r}) = -\vec{\nabla}\Phi(\vec{r}) \;;\; \Phi(\vec{r}) = V(\vec{r}) = \int_{v'}\frac{\rho_v(\vec{r}')dv'}{4\pi\epsilon|\vec{r}-\vec{r}'|} \;;\; \vec{\nabla}^2\Phi = -\frac{\rho_v}{\epsilon}$$

$$\vec{A}(\vec{r}) = \int_{v'}\frac{\mu_0\vec{J}(\vec{r}')dv'}{4\pi|\vec{r}-\vec{r}'|} = \mu_0\vec{J}(x,y,z) \overset{*}{_{3d}} \frac{1}{4\pi|\vec{r}|}$$

$$\vec{\nabla}^2\Phi + k^2\Phi = -\frac{\rho_v}{\epsilon} \;;\; \vec{\nabla}\cdot\vec{A} = -j\omega\mu\epsilon\Phi \;;\; \vec{A}(\vec{r}) = \int_{v'}\frac{\mu_0\vec{J}(\vec{r}')e^{-jk|\vec{r}-\vec{r}'|}dv'}{4\pi|\vec{r}-\vec{r}'|}$$

$$\Phi(\vec{r}) = \int_{v'}\frac{\rho_v(\vec{r}')e^{-jk|\vec{r}-\vec{r}'|}dv'}{4\pi\epsilon|\vec{r}-\vec{r}'|} \;;\; \vec{\nabla}^2\vec{E} + k^2\vec{E} = 0$$

$$\vec{E} = -j\omega\vec{A} - \frac{j}{\omega\mu\epsilon}\vec{\nabla}(\vec{\nabla}\cdot\vec{A}) \;;\; \eta = \frac{E_\theta}{H_\phi} \;;\; \mathcal{F}(\omega)e^{-j\omega a} \Leftrightarrow f(t-a)$$

$$\underset{\sim}{\vec{\mathcal{P}}} = \frac{1}{2}\mathcal{R}e\left\{\vec{E}\times\vec{H}^*\right\} + \frac{1}{2}\mathcal{R}e\left\{\vec{E}\times\vec{H}e^{j2\omega t}\right\} \;;\; \vec{\mathcal{P}}_a = \frac{1}{2}\mathcal{R}e\left\{\vec{E}\times\vec{H}^*\right\} \;;\; \vec{\mathcal{P}}_a = \frac{1}{2\eta}|\vec{E}|^2\hat{r}\,.$$

$$U = \frac{dP_r}{d\Omega} = \frac{1}{2}r^2\mathcal{R}e\left\{\vec{E}\times\vec{H}^*\right\}\cdot\hat{r} \;;\; U_0 = \frac{P_r}{4\pi} \;;\; D(\theta,\phi) = \frac{U(\theta,\phi)}{U_0} = \frac{dP_r/d\Omega}{P_r/4\pi}$$

$$G(\theta,\phi) = 4\pi\frac{U(\theta,\phi)}{P_{\mathrm{in}}} \;;\; G(\theta,\phi) = \varepsilon_r D(\theta,\phi) \;;\; \mathrm{EIRP} = P_{\mathrm{in}}G_{\mathrm{max}} \;;\; \varepsilon_r = \frac{R_r}{R_r + R_L}$$

$$I(z') = I_0\sin\left[k\left(\frac{\ell}{2} - |z'|\right)\right]$$

$$\vec{E}(\vec{r}) = E_\theta\hat{\theta} = \frac{j\eta I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right) - \cos\left(\frac{k\ell}{2}\right)}{\sin\theta}\right]\hat{\theta}$$

$$\vec{H}(\vec{r}) = H_\phi\hat{\phi} = \frac{jI_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right) - \cos\left(\frac{k\ell}{2}\right)}{\sin\theta}\right]\hat{\phi}\,.$$

	$\hat{\rho}$	$\hat{\phi}$	$\hat{z}$
$\hat{x}\cdot$	$\cos\phi$	$-\sin\phi$	0
$\hat{y}\cdot$	$\sin\phi$	$\cos\phi$	0
$\hat{z}\cdot$	0	0	1

	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$
$\hat{x}\cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\hat{y}\cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\hat{z}\cdot$	$\cos\theta$	$-\sin\theta$	0

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \;;\; \theta = \cos^{-1}\left(\frac{z}{|\vec{r}|}\right) \;;\; d\Omega = \sin\theta d\theta d\phi$$

$$\vec{r} = \rho\,\hat{\rho} + z\,\hat{z} \;;\; d\vec{L} = d\vec{r} = d\rho\hat{\rho} + \rho d\phi\hat{\phi} + dz\hat{z} \;;\; d\vec{r} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$$

$$\vec{\nabla}\varphi = \frac{\partial\varphi}{\partial\rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial\varphi}{\partial\phi}\hat{\phi} + \frac{\partial\varphi}{\partial z}\hat{z} \;;\; \vec{\nabla}\varphi = \frac{\partial\varphi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\varphi}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial\varphi}{\partial\phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z} \;;\; \vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(A_\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\vec{\nabla}\cdot\vec{\nabla}\varphi = \left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}\right) = \vec{\nabla}^2\varphi \quad ; \quad \vec{\nabla}\cdot\vec{\nabla}\Phi = -\frac{\rho_v}{\epsilon} \quad ; \quad \vec{\nabla}^2\Phi = 0$$

$$\vec{\nabla}^2\vec{A} = \left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]\left(\vec{A}\right) \quad ; \quad \oint_S \vec{A}\cdot d\vec{S} = \int_{\text{vol}} \vec{\nabla}\cdot\vec{A}\,dv \quad ; \quad \oint_C \vec{A}\cdot d\vec{L} = \int_S \left(\vec{\nabla}\times\vec{A}\right)\cdot d\vec{S}$$

$$d\vec{S}_r = r^2\sin\theta d\theta d\phi\hat{r} \quad ; \quad d\vec{S}_\theta = r\sin\theta dr d\phi\hat{\theta} \quad ; \quad d\vec{S}_\phi = r dr d\theta\hat{\phi} \quad ; \quad d\Omega = \frac{dS}{r^2} = \sin\theta d\theta d\phi$$

$$\vec{\nabla}\times\vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} \quad ; \quad \vec{\nabla}\times\vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

$$|\text{AF}| = \left|\frac{\sin\left(\frac{M\Psi}{2}\right)}{\sin\left(\frac{\Psi}{2}\right)}\right| \quad ; \quad \Psi = \beta + kd\cos\alpha \quad ; \quad M\Psi = \pm(2i+1)\pi\ , \quad i\in\ \text{N}$$

$$\frac{M\Psi}{2} = \pm i\pi\ , \quad 1\in\ \text{N}\ \ (\text{but } i\neq M, 2M, 3M, \ldots) \quad ; \quad \text{BW}_{1/2} = 2\Delta\phi_{1/2} = \frac{2.65\lambda}{\pi Md}$$

$$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j} \quad ; \quad \cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2} \quad ; \quad \sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\sin(a\pm b) = \sin a\cos b \pm \cos a\sin b \quad ; \quad \cos(a\pm b) = \cos a\cos b \mp \sin a\sin b$$

$$D(\phi) \approx \frac{Mkd}{\pi}\left[\frac{\sin\left(\frac{Mkd}{2}\cos\phi\right)}{\frac{Mkd}{2}\cos\phi}\right]^2 \quad ; \quad \cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$D(\phi) \approx \frac{2Mkd}{\pi}\left[\frac{\sin\left(\frac{Mkd(\cos\phi-1)}{2}\right)}{\frac{Mkd(\cos-1)}{2}}\right]^2 \quad ; \quad \text{BW}_{1/2} = 2\Delta\phi_{1/2} = 3.26\left[\frac{\lambda}{\pi Md}\right]^{1/2}$$

$$|\text{AF}| = 2^N\left|\cos^N\left(\frac{kd\cos\phi}{2}\right)\right| \quad ; \quad \begin{pmatrix} N \\ n \end{pmatrix} = \frac{N!}{(N-n)!n!} \quad ; \quad \frac{\tan\beta}{\beta} = 2 - \frac{2\beta}{k\ell}$$

$$R_r = \frac{\eta}{4\pi}\left[0.5772 + \ln(2\pi n) - C_i(2\pi n)\right] \quad ; \quad X = \frac{\eta}{4\pi}S_i(2\pi n)$$

$$R_{21} = \frac{\eta}{4\pi}\left[2C_i(v_0) - C_i(v_1) - C_i(v_2)\right] \quad ; \quad X_{21} = -\frac{\eta}{4\pi}\left[2S_i(v_0) - S_i(v_1) - S_i(v_2)\right]$$

$$v_0 = kd \quad ; \quad v_1 = k\left(\sqrt{d^2+\ell^2}+\ell\right) \quad ; \quad v_2 = k\left(\sqrt{d^2+\ell^2}-\ell\right)$$

$$G = \frac{4\pi}{\lambda^2}A_e \quad ; \quad A_e = \varepsilon_{ap}A_p \quad ; \quad P_r = \left[\frac{G_tG_r\lambda^2}{(4\pi r)^2}\right]P_t$$

$$\vec{A}(\vec{r}) = \frac{\mu e^{-jkr}}{4\pi r}\vec{N} \quad ; \quad \vec{N} = \int\int_S \vec{J}_s(\vec{r}')e^{jkr'\cos\psi}dS'$$

$$\vec{F}(\vec{r}) = \frac{\epsilon e^{-jkr}}{4\pi r}\vec{L} \quad ; \quad \vec{L} = \int\int_S \vec{M}_s(\vec{r}')e^{jkr'\cos\psi}dS'$$

$$\vec{E}_A = -j\omega\left[A_\theta\hat{\theta} + A_\phi\hat{\phi}\right] \quad ; \quad \vec{H}_A = \frac{j\omega}{\eta}\left[A_\phi\hat{\theta} - A_\theta\hat{\phi}\right]$$

$$\vec{E}_F = j\omega\eta\left[-F_\phi\hat{\theta} + F_\theta\hat{\phi}\right] \quad ; \quad \vec{H}_F = -j\omega\left[F_\theta\hat{\theta} + F_\phi\hat{\phi}\right]$$