

Your Term 6 E-M notes may be helpful for the concepts involved in some of the following problems.

1. The electric field component of a particular e-m wave in free space is given by

$$\vec{E} = \hat{z} E_0 \cos(ax) \cos(\omega t + by) \text{ V/m.}$$

- (a) Determine the corresponding magnetic field \vec{H} . Find the relationships between the constants a , b and ω such that all of Maxwell's equations are satisfied.
 (b) Assuming that the given wave is the sum of two component uniform plane waves, determine the direction(s) of travel of the two waves. Hint: Show that the general form of the phasor field is

$$\vec{E} = \hat{z} \frac{E_0}{2} \{ e^{-j\vec{k}_1 \cdot \vec{r}} + e^{-j\vec{k}_2 \cdot \vec{r}} \}$$

where $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ and \vec{k}_1 and \vec{k}_2 are the wave vectors whose hatted equivalents are the directions of travel of the component waves. Of course, \hat{k}_1 and \hat{k}_2 should be specified in terms of a and b .

2. The magnetic field of a $10 \mu\text{W}/\text{m}^2$, 200 MHz TV signal in air is given by

$$\vec{H} = \hat{z} H_0 \sin(\omega t - ax - ay + \pi/3) .$$

- (a) What are the values of H_0 and a ? (b) Find the corresponding electric field \vec{E} . (c) At $z = 0$, a straight wire detector (antenna) is used to find the electric field component along the direction in which the antenna is oriented (we'll see shortly that this concept involves the idea of "polarization"). Find the maximum value of the electric field if the antenna is oriented along (i) the x direction and (ii) the y direction.

3. At a point in space, the time-harmonic fields \vec{E} and \vec{H} , of magnitude E_0 and H_0 , respectively, are given as

$$\begin{aligned} \vec{E} &= E_0 \cos(\omega t + \phi_E) \hat{x} \\ \vec{H} &= H_0 \cos(\omega t + \phi_H) \hat{y} . \end{aligned}$$

The ϕ s are constant phase angles. Show from first principles (i.e. by doing the appropriate integration) that the time-average Poynting vector $\vec{\mathcal{P}}_a$ may be written in phasor form as

$$\vec{\mathcal{P}}_a = \frac{1}{2} \text{Re} \{ \vec{E} \times \vec{H}^* \} \hat{z}$$

where \vec{E} and \vec{H} are the phasor forms of $\underline{\vec{E}}$ and $\underline{\vec{H}}$, respectively, and * represents complex conjugation.

4. Calculate $\underline{\vec{E}}$ and $\underline{\vec{H}}$ if it is known that the phasor form of the vector and scalar potentials associated with the field are given by

$$\begin{aligned}\vec{A} &= A_0 e^{-jkz} \hat{x} \text{ Wb/m} \\ \Phi &= 0\end{aligned}$$

where $k = \omega\sqrt{\mu\epsilon}$. Before doing the problem, show that the potentials satisfy the Lorenz gauge condition.

5. The scalar and vector potentials for a particular e-m field are given by

$$\Phi = 0, \quad \vec{A} = \begin{cases} \frac{\mu_0 k}{4c} (ct - |x|)^2 \hat{z}, & \text{for } |x| < ct \\ 0, & \text{for } |x| > ct \end{cases}$$

where k is the wavenumber and c is the free-space speed of light. (a) Use the time-dependent form of equation (1.32) of the notes to determine $\underline{\vec{E}}$ for $|x| < ct$.

(b) Use the vector potential to find the corresponding $\underline{\vec{B}}$. (c) Determine both the divergence and curl of both these $\underline{\vec{E}}$ and $\underline{\vec{B}}$ fields. (d) There is no *charge* density producing the given potentials. How do we know this? (e) In fact, there is a surface current density, $\underline{\vec{K}}$ acting as the source. Sketch the $\underline{\vec{B}}$ field at a fixed time and, without calculation, argue from what you know about the magnetic field boundary conditions that $\underline{\vec{K}}$ indeed exists. You don't have to find $\underline{\vec{K}}$, although you should be able to.

6. One means of seeking solutions for the electric and magnetic fields of current sources is via the so-called Hertzian potential $\vec{\Pi}$ (it is, in fact, related to the vector potential \vec{A} of the class notes). Suppose we define the magnetic field intensity in terms of $\vec{\Pi}$ as $\vec{H} = j\omega\epsilon\vec{\nabla} \times \vec{\Pi}$. (a) Show that

$$\vec{\nabla}^2 \vec{\Pi} + k^2 \vec{\Pi} = j \frac{1}{\omega\epsilon} \vec{J}$$

where all other symbols have their usual meaning. Hint: In arriving at the result you will need to introduce a scalar potential Φ related to \vec{E} in a way *similar* to that in the class notes. You will also need an appropriate condition similar to that imposed on equation (1.36) of the class notes. (b) Notice that the result above is another non-homogeneous Helmholtz-type equation. By analogy to the similar equation containing \vec{A} as derived in the class notes, write down the solution for the result given in (a).