Engineering 7811 Assignment 2 **Due: Mon., Feb. 8, 2010**

1. The vector potential for the infinitesimal dipole developed in class was used in Tutorial 2 to derive the magnetic field intensity $\vec{H}$. Use this $\vec{H}$ to determine the E-field components as given in equations (2.10)-(2.12) of the class notes.

2. In the phase of the vector potential for the vertical Hertzian dipole we made the far-field approximation

$$\sqrt{x^2 + y^2 + (z - z')^2} \approx r - z' \cos\theta \; .$$

Suppose that we had retained the third term of the expansion. Show that this third term has the form

$$\frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right) \; .$$

It has been shown by many investigators that a maximum phase error of 22.5° for practical antennas of $\ell > \lambda$ is not very detrimental in their analytical formulation. Show using the third term of the expansion that this maximum phase error corresponds to the required observation distance being

$$r \geq 2\left(\frac{\ell^2}{\lambda}\right) \; .$$

3. Show that the principal E-plane pattern, in the region of the y-z plane where $y > 0$, is circular for a Hertzian dipole is.

4. Determine the polarization of (a) $\vec{E} = [(1 + j)\hat{y} + (1 - j)\hat{z}]e^{-jkx}$ and (b) $\vec{E} = (j\hat{x} + j2\hat{y})e^{+jkz}$. In part (b) also give the orientation of a dipole antenna that would maximumly receive this field.

5. A dipole antenna of length ℓ which is one percent of a wavelength carries a current of $i = 100\cos(\omega t)$ mA. (a) Determine the maximun power density at a distance of 10 m from this dipole in free space. (b) What is the amplitude of the E-field at the position in (a)? (c) What is the total radiated power for the dipole?

6. The current density on a short dipole antenna of length ℓ and oriented along the z direction is given as

$$\vec{J} = \hat{z} J_0 \sin\left[k\left(\frac{\ell}{2} - |z|\right)\right] \; .$$

Find an expression for the associated charge density ρ.