

# Lorenz, Lorentz, and the Gauge

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Soon after development of the concept of the vector potential, Ludvig Lorenz published a mathematical expression [1],

$$\nabla \cdot \mathbf{A} = -\mu_0 \epsilon_0 \frac{\partial \phi}{\partial t}, \quad (1)$$

(using modern notation for the vector potential,  $\mathbf{A}$ , and the scalar potential,  $\phi$ ) for which Hendrik A. Lorentz now gets credit [2]. This is especially curious, given the fact that the early researchers in our field of electromagnetics were few in number, and there were a limited number of journals in which to publish. How did it happen that such a mistake was made?

In the 1860s, the idea that the electromagnetic field could be expressed in terms of potentials was widely accepted. The scalar potential had been found in 1824 by Poisson, and the vector potential, by Neumann [3], in 1845, inferring what was later to be called the Coulomb gauge:

$$\nabla \cdot \mathbf{A} = 0. \quad (2)$$

The Coulomb gauge was the basis of much of James Clerk Maxwell's work on potentials.

In 1867, Lorenz, a Danish physicist, probably taking a hint from the earlier work of D'Alembert, published a paper [1] in which he proposed that the standard potentials of Neumann:

$$\phi = \frac{1}{4\pi\epsilon_0} \int_V (\rho/R) dv', \quad (3)$$

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_V (\mathbf{J}/R) dv', \quad (4)$$

in terms of the instantaneous charge,  $\rho(\mathbf{r}')$ , the current density,  $\mathbf{J}(\mathbf{r}')$ , and the position,  $R = (\mathbf{r} - \mathbf{r}')^{1/2}$ , be modified to form what became known as the retarded potentials,

$$\phi = \frac{1}{4\pi\epsilon_0} \int_V [\rho(t - R/c)/R] dv', \quad (5)$$

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_V [\mathbf{J}(t - R/c)/R] dv', \quad (6)$$

which included the time of propagation from the source. Near the end of the same paper, he showed that a mathematical consequence of his retarded potentials was the condition given in Equation (1). However, many in the electromagnetics community at the time had strong reservations about adapting retarded potentials, because retardation implies that a physical effect – propagation at the speed of light – is being imposed on what was thought to be nonphysical and non-measurable quantities,  $\mathbf{A}$  and  $\phi$ . Maxwell was less reluctant to accept the concept of retarded potentials, although in his famous treatise [4], only brief mention is made of Lorenz's proposal, concluding with the suggestion that he (Maxwell) had a prior-publication claim on the idea. To quote Maxwell: "These conclusions (of Lorenz) are similar to those of this chapter, though obtained by an entirely different method. The theory given in this chapter was first published in the *Phil. Trans.* for 1865, pp. 459-512."

One person who did take L. Lorenz seriously was H. A. Lorentz, a Dutch physicist. Although much younger than Lorenz, it is interesting that in the same year, 1880 [5], these two men published papers in the same journal and on the same topic, the relation between refractivity and density in materials [6, 7]. However, Lorentz went on to publish on many subjects in mathematical physics. Some to which his name became attached include the Lorentz force law, Lorentz contraction, Lorentz invariance, and the Lorentz transformation. But starting in 1892 [8], one year after the death of Lorenz (1829-1891), his many papers supporting the concept of the retarded potential and his clear derivation of Equation (1) strongly identified his name with the gauge. It is interesting that Lorenz's work is not referenced in Lorentz's seminal paper [8], or in his later book [9], except concerning the 1880 paper on refractivity and density.

It didn't take long after the publication of Lorentz's 1892 paper, and his several subsequent articles, for the error to creep into the literature. Witness the following excerpts [3, 10];

1904, "first by Lorentz and Poincaré," A. H. Bucherer [11]

1908, "these potentials have been employed by H. Poincaré, E. Beltrami, V. Volterra, H. A. Lorentz and others." M. Abraham [12]

1912, "the Lorentz potentials," G. A. Schott [13]

1912, "I think it was G. F. FitzGerald who brought the progressive  $\mathbf{A}$  and  $\phi$  into electromagnetics." O. Heavyside [14]

1927, "comes from Lorentz," H. Thirring [15]

H. A. Lorentz (1853-1928), almost surely knew of some of these attributions, but, if he ever disputed them, or if in fact he even knew that he didn't invent the gauge, it wasn't reported in readily available literature. It is also probably true that many of these men had a copy of Maxwell's treatise on their shelf, as do many of us today in the Dover edition, but evidently they didn't read it, either.

To the credit of many authors in the first half of the twentieth century, Equation (1) was simply referred to as "the condition" or "the continuity relation for the electromagnetic field," in analogy to the continuity condition for current and charge [16]. However, in spite of many articles in the 1800s and two books [3, 10] on the history of electromagnetic theory (one [3] revised as late as 1951), in which Lorentz was recognized as the inventor of the gauge, by the end of the 1950s, the term "Lorentz gauge" was in common use by just about everyone. That is, everyone but the Danes. In his address to the 1962 Symposium on Electromagnetic Theory and Antennas in Copenhagen [17], Mogens Pihl, of the University of Copenhagen, in the usual polite Danish way, pointed out that Ludvig Lorentz invented the gauge.

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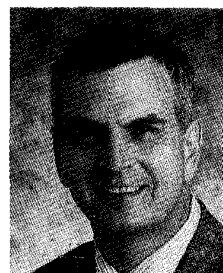
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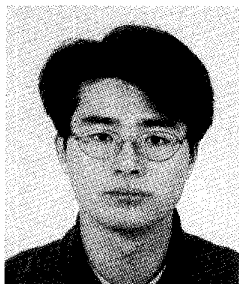
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## Introducing the Feature Article Authors



**Robert Nevels** received the BSEE degree from the University of Kentucky, the MSEE degree from Georgia Tech, and the PhD from the University of Mississippi. He joined Texas A&M University in 1978 as an Assistant Professor, became full Professor in 1993, and is currently Assistant Department Head. During the summer of 1992, he was a Visiting Professor at the Institute for Light Sources, Fudan University, Shanghai, China. Dr. Nevels served as Associate Editor of the *IEEE Transactions on Antennas and Propagation* from 1986-1989, and of the Wiley four-volume book series, *Handbook of Microwave and Optical Components*, from 1990-1992. He is currently Associate Editor of the periodical *Microwave and Optical Technology Letters*. He has received nine teaching awards, most recently, the university-level Faculty Distinguished Achievement Award in Teaching. Dr. Nevels is currently a member of the IEEE Antennas and Propagation Society ADCOM, Chair of the IEEE-AP Man and Radiation Committee, and General Chair of the 2002 IEEE International Symposium on Antennas and Propagation and USNC/URSI National Radio Science Meeting. He is a member of Eta Kappa Nu, Sigma Xi, Commission B of USNC/URSI, the Electromagnetics Academy, and the American Scientific Affiliation. Dr. Nevels' interests are in analytical and numerical techniques for electromagnetic scattering, antennas, and microwave-device design.



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## Antenna Analysis and Design Software: A Survey

A report, *Future Directions in Antenna Analysis and Design Software: A Survey*, is now available. In March of 2000, the Technical Working Group on Antennas (TWGA) of the Electromagnetic Code Consortium (EMCC) began a survey to determine the near-term (three-to-five-year) needs of the antenna community for antenna analysis and design software. The motivation for the survey was twofold: first, to bring into focus the critical issues in antenna analysis and design software, and, second, to provide US Government agencies with a basis for formulating and defending their funding plans in this area.

EMCC's focus on this area is easily justified. The number of civilian and military applications for antennas increases daily. Moreover, technological progress along several fronts is bringing within reach antenna designs that were not possible before. Examples are many: skin-embedded antenna arrays, structurally embedded arrays (where parts of the antenna array are also structural

members of the array platform), broadband arrays, reconfigurable antennas, etc. These types of antenna arrays are structurally, materially, and geometrically very complex; moreover, they cannot be designed without taking into consideration the surrounding environment, i.e., the platform they are mounted on and any other antennas with which they could interact. Exploratory studies in modern antenna systems clearly indicate that traditional antenna design methods are not sufficient to address the complexity of these systems. There is a definite need for sophisticated CEM tools: tools that are specifically tuned to antenna analysis and design.

With this in mind, the TWGA of the EMCC embarked on an effort to determine the kinds of tools that the ideal antenna computational electromagnetics (ACEM) toolbox should contain. As a first step, we compiled a wish list of our own for such a toolbox. We also compiled a list of names of antenna-design and antenna-software engineers to whom we sent our toolbox statement. Along with it, we sent a survey asking them to evaluate the toolbox and to make suggestions of their own.

The report on this survey, *Future Directions in Antenna Analysis and Design Software: A Survey*, is now available, and can be downloaded from the EMCC's Web site: <http://www.asc.hpc.mil/emcc/antenna/index.html>. We briefly describe its contents below.

Both our toolbox statement and the survey form are discussed in Section 2 of the report. In Section 3, we present some statistics about the participants in our survey and about their organizations. We analyze the responses in Sections 4 through 8. Based on these, we propose, in Section 9, a revised (and final) "antenna and antenna-platform interaction software toolbox." We also present some near-term research directions, and some action items for the EMCC. We conclude the report with Section 10, where we present some ideas about creating databases that may make the antenna designer's task easier.

Twenty-eight people participated in the survey, all of them experts, as developers or users of antenna software. We thank them all for contributing their time, and for their thoughtful and thought-provoking comments. Their input to this undertaking was invaluable.

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