Engineering 7811 Assignment 2 **Due: Mon., Jan. 30, 2012**

1. The vector potential for the Hertzian dipole with $\ell \ll r, \lambda$ is given in equation (2.7) of the class notes. (a) Write this expression in spherical coordinates and use it to determine that E_r and E_θ components of the E-field as given in equation (2.10) and (2.11)of the notes. (b) Using only the first terms of these E-field components along with the dominant term of H_ϕ, determine that there is a reactive power density associated with these components. Determine its value and make a brief statement based on your analytical results as to what region surrounding the antenna such a term might have to be considered. In fact, you should show that when $r < \lambda/\pi$, the reactive component of the power density will exceed the real component. (This is approximate of course.)

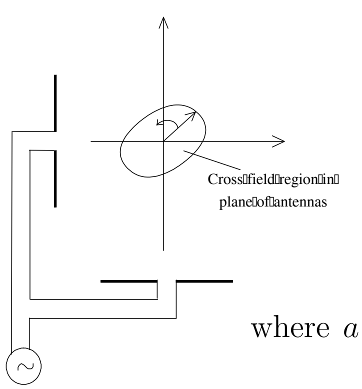
2. A circular "dish" antenna of diameter 20 cm is used to transmit a 20 GHz signal into free space. (a) Determine the reactive near-field, the Fresnel, and the Fraunhofer regions for this case. (b) For what operating frequency would the Fresnel region begin 1.0 m from the antenna? In this case, where would the far-field begin?

3. Determine the polarization state (i.e. linear, RHC, LHC, RHE, or LHE) and sketch the locus of $\vec{E}(0,t)$ for

$$\vec{E}(z,t) = \hat{x}a_x \cos(\omega t - kz) + \hat{y}a_y \cos(\omega t - kz + \delta)$$

where (a) $a_x = 3$ V/m, $a_y = 4$ V/m and $\delta = 0$ and (b) $a_x = 3$ V/m, $a_y = 3$ V/m and $\delta = \pi/2$.

4. Consider two dipole antennas oriented as shown. The "cross-field" in the plane of the antennas may be elliptical in the far-field. Suppose that this field is composed of the following linearly polarized components:



$$\begin{aligned} \underset{\sim}{E}_x &= E_{x0} \sin(\omega t - kz) \\ \underset{\sim}{E}_y &= E_{y0} \sin(\omega t - kz + \Delta\phi) \end{aligned}$$

where $\Delta\phi$ is phase angle by which $\underset{\sim}{E}_y$ leads $\underset{\sim}{E}_x$.

(a) Show that the polarization ellipse is given by

$$a\underset{\sim}{E}_x^2 - b\underset{\sim}{E}_x\underset{\sim}{E}_y + c\underset{\sim}{E}_y^2 = 1$$

where $a = \dfrac{1}{E_{x0}^2 \sin^2 \Delta\phi}$, $b = \dfrac{2\cos\Delta\phi}{E_{x0}E_{y0}\sin^2\Delta\phi}$ and $c = \dfrac{1}{E_{y0}^2 \sin^2 \Delta\phi}$ and (b) that the tilt angle of the ellipse is given by $\tau = \dfrac{1}{2}\tan^{-1}\left(\dfrac{2E_{x0}E_{y0}\cos\Delta\phi}{E_{x0}^2 - E_{y0}^2}\right)$. Hint: Part (b) may be accomplished by rotating the $\underset{\sim}{E}_x$-$\underset{\sim}{E}_y$ coordinate axes through angle τ.

5. Determine the magnitude of the free-space electric and magnetic field intensities and the power density in the x-y plane at a distance of 100 m from a vertical $\lambda/100$ dipole located at the origin and carrying a current of 1 A.