Engineering 7811 Assignment 2 **Due: Wed., Feb. 3, 2016**

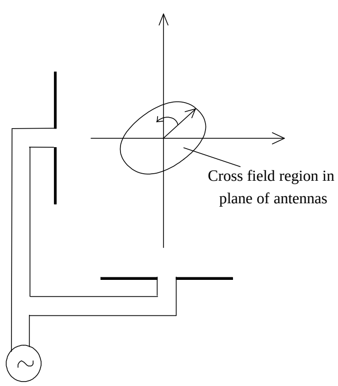
1. We have seen from Tutorial 2 that the magnetic flux density for the Hertzian dipole with $\ell \ll r, \lambda$ is given by

$$\vec{B} = \frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\hat{\phi}$$

where A_r and A_θ are the relevant components of the vector potential, which are also found in the tutorial. (a) Starting with this equation, show that the magnetic field intensity $\vec{H}$ is described by equations (2.13) and (2.14) of the class notes. (b) Using only the first terms of the E-field components given in equation (2.10) and (2.11) of the class notes along with the dominant term of H_ϕ, determine the average power density. What is the reactive component of this power density? Assuming that the angular dependencies found in the real and reactive components are similar (a big assumption), determine that the magnitude of the reactive component will exceed that of the real component when $r < \lambda/\pi$.

2. A circular "dish" antenna of radius 10 cm is used to transmit a 20 GHz signal into free space. After showing that for this situation the field distance equations associated with the various field regions described in Section 2.2.2 of the class notes apply (a) determine the reactive near-field, the Fresnel, and the Fraunhofer regions. (b) For what operating frequency would the Fresnel region begin 0.50 m from the antenna? In this case, where would the far-field begin?

3. Consider two dipole antennas oriented as shown. The "cross-field" in the plane of the antennas may be elliptical in the far-field. Suppose that this field is composed of the following linearly polarized components:



$$\utilde{E}_x = E_{x0}\sin(\omega t - kz)$$
$$\utilde{E}_y = E_{y0}\sin(\omega t - kz + \Delta\phi)$$

where $\Delta\phi$ is phase angle by which $\utilde{E}_y$ leads $\utilde{E}_x$.

(a) Show that the polarization ellipse is given by

$$a\utilde{E}_x^2 - b\utilde{E}_x\utilde{E}_y + c\utilde{E}_y^2 = 1$$

where $a = \dfrac{1}{E_{x0}^2 \sin^2\Delta\phi}$, $b = \dfrac{2\cos\Delta\phi}{E_{x0}E_{y0}\sin^2\Delta\phi}$ and $c = \dfrac{1}{E_{y0}^2\sin^2\Delta\phi}$ and (b) that the tilt angle of the ellipse is given by $\theta = \dfrac{1}{2}\tan^{-1}\left(\dfrac{2E_{x0}E_{y0}\cos\Delta\phi}{E_{x0}^2 - E_{y0}^2}\right)$. Hint: Part (b) may be accomplished by rotating the $\utilde{E}_x$-$\utilde{E}_y$ coordinate axes through angle θ to new $\utilde{E}'_x$-$\utilde{E}'_y$ coordinates and noting that the ellipse equation in these new coordinates should have no cross term – i.e. the coefficient on the $\utilde{E}'_x\utilde{E}'_y$ should be 0.

4. Determine the polarization state (i.e. linear, RHC, LHC, RHE, or LHE) and sketch the locus of $\vec{E}(0,t)$ for

$$\vec{E}(z,t) = \hat{x}a_x \cos(\omega t - kz) + \hat{y}a_y \cos(\omega t - kz + \delta)$$

where (a) $a_x = 3$ V/m, $a_y = 4$ V/m and $\delta = 0$ and (b) $a_x = 3$ V/m, $a_y = 3$ V/m and $\delta = \pi/2$.

5. Determine (a) the magnitude of the free-space electric and magnetic field intensities and the power density in the x-y plane at a distance of 100 m from a vertical dipole of length $\lambda/100$ located at the origin and carrying a current of 1 A. (b) What is the total power radiated by the antenna?