Engineering 7811 Assignment 4 **Due: Mon. March 21, 2016**

1. Design a two-element uniform array of vertical quarter-wave monopoles over perfect ground, positioned along the x-axis a distance of $\lambda/4$ apart, so that it has an azimuthal maximum along the $\phi = \pi/4$ direction. This means determining

   (a) the relative phase excitation of each element and

   (b) the normalized array factor. Sketch the overall normalized E-field pattern in the horizontal plane.

   (c) Repeat (a) and (b) if the array is to have an azimuthal maximum along the $\phi = 5\pi/4$ direction.

   (d) Repeat part (b) for the elevation plane.

2. Two $\lambda/2$ vertical dipoles in free space form an array along the x-axis. Determine the overall normalized E-field pattern in the horizontal plane for each of the following cases:
   (i) element spacing is $\lambda/4$ and current phasing is $0°$.
   (ii) element spacing is $\lambda/2$ and current phasing is $0°$.
   (iii) element spacing is $3\lambda/4$ and current phasing is $0°$.
   (iv) element spacing is λ and current phasing is $0°$.
   (v) element spacing is λ and current phasing is $90°$.
   In your answers, determine the maxima (including principal and secondary cases) and nulls. Sketch the patterns. Matlab or something similar will help – but be sure to establish as much as possible analytically. Notice the profound effects of varying element spacing and current phases.

3. This question is an elaboration of question 2. Show that, for a two element uniform linear array of vertical dipoles with the current on element 1 leading that on element 0 by $\pi/2$, the maxima of the array factor in the horizontal plane may be given by the solving the following equation:

$$\cos\phi = \frac{\lambda}{2d}\left[(-1)^{(n+1)}\frac{(2n+1)}{2}\right] \quad n = 0, 1, 2, \ldots$$

   Using this equation, show that there is only one such maximum for $d = \lambda/4$. How many maxima exist for $d = 3\lambda/4$ and where do they occur?

4. (a) On the same set of axes plot the total normalized horizontal field patterns for a five-element linear array of vertical dipoles aligned along the x-axis with element spacing of $\lambda/4$ when it is operating as (i) a uniform endfire array and (ii) as a binomial array. In the case of (i) be sure to give all calculations of maxima and nulls. Comment on the trade-offs for each configuration.

(b) Determine approximate values of the half-power beamwidths in (a). You may use the approximation formula in the class notes for the endfire case, but for the binomial case the procedure will be purely analytical.

5. Determine (calculate the positions of all nulls and secondary maxima) and plot the array factor (in the horizontal plane) of a 10-element uniform broadside array of vertical dipole elements spaced by $\lambda/2$ along the x-axis. Calculate the first-null and approximate half-power beamwidths. Calculate, in dB, the reduction in power of the first secondary maximum compared to the principal maximum. Calculate as an ordinary number and in dB the approximate directivity. How do these three parameter values compare with those of an identical array operating in endfire mode?