

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} ; \quad \vec{\nabla} \cdot \vec{D} = \rho_v ; \quad \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} ; \quad \vec{\nabla} \cdot \vec{B} = 0 ; \quad \vec{D} = \epsilon \vec{E} ; \quad \vec{B} = \mu \vec{H}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \approx \frac{10^{-9}}{36\pi} \text{ F/m} ; \quad \mu_0 = 4\pi \times 10^{-7} \text{ H/m} ; \quad \epsilon = \epsilon_0(1 + \chi_e) ; \quad \mu = \mu_0(1 + \chi_m)$$

$$\vec{D} = \epsilon_0 \vec{E} + \epsilon_0 \chi_e \vec{E} = \epsilon_0 \vec{E} + \vec{P} ; \quad \vec{B} = \mu_0 \vec{H} + \mu_0 \chi_m \vec{H} = \mu_0 (\vec{H} + \vec{M})$$

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} ; \quad \vec{J}_{cnv} = \rho_v \vec{v} ; \quad \vec{J} = \sigma \vec{E} ; \quad \vec{A}(\vec{r}, t) \equiv \vec{A} = \mathcal{R}e \{ \vec{A} e^{j\omega t} \}$$

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B}) ; \quad v_p = \frac{1}{\sqrt{\mu\epsilon}} ; \quad c = \frac{1}{\sqrt{\mu_0\epsilon_0}} ; \quad c = 3 \times 10^8 \text{ m/s} ; \quad v_p = f\lambda$$

$$k = \frac{2\pi}{\lambda} = \omega \sqrt{\mu\epsilon} ; \quad jk = \alpha + j\beta ; \quad \beta = \frac{2\pi}{\lambda} ; \quad \eta = \sqrt{\frac{\mu}{\epsilon}} ; \quad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} ; \quad \eta_0 \approx 120\pi \Omega$$

$$\vec{\nabla} \times \vec{E} = -j\omega \vec{B} ; \quad \vec{\nabla} \cdot \vec{D} = \rho_v ; \quad \vec{\nabla} \times \vec{H} = \vec{J} + j\omega \vec{D} ; \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{E}(\vec{r}) = -\vec{\nabla} \Phi(\vec{r}) ; \quad \Phi(\vec{r}) = V(\vec{r}) = \int_{v'} \frac{\rho_v(\vec{r}') dv'}{4\pi\epsilon |\vec{r} - \vec{r}'|} ; \quad \vec{\nabla}^2 \Phi = -\frac{\rho_v}{\epsilon}$$

$$\vec{A}(\vec{r}) = \int_{v'} \frac{\mu_0 \vec{J}(\vec{r}') dv'}{4\pi |\vec{r} - \vec{r}'|} = \mu_0 \vec{J}(x, y, z) \stackrel{*}{\approx} \frac{1}{4\pi |\vec{r}|}$$

$$\vec{\nabla}^2 \Phi + k^2 \Phi = -\frac{\rho_v}{\epsilon} ; \quad \vec{\nabla} \cdot \vec{A} = -j\omega \mu \epsilon \Phi ; \quad \vec{A}(\vec{r}) = \int_{v'} \frac{\mu_0 \vec{J}(\vec{r}') e^{-jk|\vec{r} - \vec{r}'|} dv'}{4\pi |\vec{r} - \vec{r}'|}$$

$$\Phi(\vec{r}) = \int_{v'} \frac{\rho_v(\vec{r}') e^{-jk|\vec{r} - \vec{r}'|} dv'}{4\pi\epsilon |\vec{r} - \vec{r}'|} ; \quad \vec{\nabla}^2 \vec{E} + k^2 \vec{E} = 0$$

$$\vec{E} = -j\omega \vec{A} - \frac{j}{\omega \mu \epsilon} \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) ; \quad \eta = \frac{E_\theta}{H_\phi} ; \quad \mathcal{F}(\omega) e^{-j\omega a} \Leftrightarrow f(t - a)$$

$$\vec{P} = \frac{1}{2} \mathcal{R}e \{ \vec{E} \times \vec{H}^* \} + \frac{1}{2} \mathcal{R}e \{ \vec{E} \times \vec{H} e^{j2\omega t} \} ; \quad \vec{P}_a = \frac{1}{2} \mathcal{R}e \{ \vec{E} \times \vec{H}^* \} ; \quad \vec{P}_a = \frac{1}{2\eta} |\vec{E}|^2 \hat{r}$$

$$U = \frac{dP_r}{d\Omega} = \frac{1}{2} r^2 \mathcal{R}e \{ \vec{E} \times \vec{H}^* \} \cdot \hat{r} ; \quad U_0 = \frac{P_r}{4\pi} ; \quad D(\theta, \phi) = \frac{U(\theta, \phi)}{U_0} = \frac{dP_r/d\Omega}{P_r/4\pi}$$

$$G(\theta, \phi) = 4\pi \frac{U(\theta, \phi)}{P_{in}} ; \quad G(\theta, \phi) = \epsilon_r D(\theta, \phi) ; \quad \text{EIRP} = P_{in} G_{max} ; \quad \epsilon_r = \frac{R_r}{R_r + R_L}$$

$$I(z') = I_0 \sin \left[k \left(\frac{\ell}{2} - |z'| \right) \right]$$

$$\vec{E}(\vec{r}) = E_\theta \hat{\theta} = \frac{j\eta I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos \left(\frac{k\ell}{2} \cos \theta \right) - \cos \left(\frac{k\ell}{2} \right)}{\sin \theta} \right] \hat{\theta}$$

$$\vec{H}(\vec{r}) = H_\phi \hat{\phi} = \frac{jI_0 e^{-jkr}}{2\pi r} \left[\frac{\cos \left(\frac{k\ell}{2} \cos \theta \right) - \cos \left(\frac{k\ell}{2} \right)}{\sin \theta} \right] \hat{\phi}$$

	$\hat{\rho}$	$\hat{\phi}$	\hat{z}		\hat{r}	$\hat{\theta}$	$\hat{\phi}$
\hat{x}	$\cos \phi$	$-\sin \phi$	0	\hat{x}	$\sin \theta \cos \phi$	$\cos \theta \cos \phi$	$-\sin \phi$
\hat{y}	$\sin \phi$	$\cos \phi$	0	\hat{y}	$\sin \theta \sin \phi$	$\cos \theta \sin \phi$	$\cos \phi$
\hat{z}	0	0	1	\hat{z}	$\cos \theta$	$-\sin \theta$	0

$$\phi = \tan^{-1} \left(\frac{y}{x} \right) ; \quad \theta = \cos^{-1} \left(\frac{z}{|\vec{r}|} \right) ; \quad d\Omega = \sin \theta d\theta d\phi$$

$$\vec{r} = \rho \hat{\rho} + z \hat{z} ; \quad d\vec{L} = d\vec{r} = d\rho \hat{\rho} + \rho d\phi \hat{\phi} + dz \hat{z} ; \quad d\vec{r} = dr \hat{r} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}$$

$$\vec{\nabla} \varphi = \frac{\partial \varphi}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial \varphi}{\partial \phi} \hat{\phi} + \frac{\partial \varphi}{\partial z} \hat{z} ; \quad \vec{\nabla} \varphi = \frac{\partial \varphi}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial \varphi}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial \varphi}{\partial \phi} \hat{\phi}$$

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{\rho} \frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} ; \quad \vec{\nabla} \cdot \vec{A} = \frac{1}{r^2} \frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(A_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\vec{\nabla} \cdot \vec{\nabla} \varphi = \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} \right) = \vec{\nabla}^2 \varphi \quad ; \quad \vec{\nabla} \cdot \vec{\nabla} \Phi = -\frac{\rho v}{\epsilon} \quad ; \quad \vec{\nabla}^2 \Phi = 0$$

$$\vec{\nabla}^2 \vec{A} = \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right] (\vec{A}) \quad ; \quad \oint_S \vec{A} \cdot d\vec{S} = \int_{\text{vol}} \vec{\nabla} \cdot \vec{A} dv \quad ; \quad \oint_C \vec{A} \cdot d\vec{L} = \int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{S}$$

$$d\vec{S}_r = r^2 \sin \theta d\theta d\phi \hat{r} \quad ; \quad d\vec{S}_\theta = r \sin \theta dr d\phi \hat{\theta} \quad ; \quad d\vec{S}_\phi = r dr d\theta \hat{\phi} \quad ; \quad d\Omega = \frac{dS}{r^2} = \sin \theta d\theta d\phi$$

$$\vec{\nabla} \times \vec{A} = \frac{1}{\rho} \begin{vmatrix} \hat{\rho} & \rho \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} \quad ; \quad \vec{\nabla} \times \vec{A} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \hat{r} & r \hat{\theta} & r \sin \theta \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix}$$

$$|\text{AF}| = \left| \frac{\sin\left(\frac{M\Psi}{2}\right)}{\sin\left(\frac{\Psi}{2}\right)} \right| \quad ; \quad \Psi = \beta + kd \cos \alpha \quad ; \quad M\Psi = \pm(2i+1)\pi, \quad i \in \mathbb{N}$$

$$\frac{M\Psi}{2} = \pm i\pi, \quad 1 \in \mathbb{N} \quad (\text{but } i \neq M, 2M, 3M, \dots) \quad ; \quad \text{BW}_{1/2} = 2\Delta\phi_{1/2} = \frac{2.65\lambda}{\pi Md}$$

$$\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \quad ; \quad \cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad ; \quad \sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b \quad ; \quad \cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$D(\phi) \approx \frac{Mkd}{\pi} \left[\frac{\sin\left(\frac{Mkd}{2} \cos \phi\right)}{\frac{Mkd}{2} \cos \phi} \right]^2 \quad ; \quad \cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$D(\phi) \approx \frac{2Mkd}{\pi} \left[\frac{\sin\left(\frac{Mkd(\cos \phi - 1)}{2}\right)}{\frac{Mkd(\cos \phi - 1)}{2}} \right]^2 \quad ; \quad \text{BW}_{1/2} = 2\Delta\phi_{1/2} = 3.26 \left[\frac{\lambda}{\pi Md} \right]^{1/2}$$

$$|\text{AF}| = 2^N \left| \cos^N \left(\frac{kd \cos \phi}{2} \right) \right| \quad ; \quad \binom{N}{n} = \frac{N!}{(N-n)!n!} \quad ; \quad \frac{\tan \beta}{\beta} = 2 - \frac{2\beta}{k\ell}$$

$$R_r = \frac{\eta}{4\pi} [0.5772 + \ln(2\pi n) - C_i(2\pi n)] \quad ; \quad X = \frac{\eta}{4\pi} S_i(2\pi n)$$

$$R_{21} = \frac{\eta}{4\pi} [2C_i(v_0) - C_i(v_1) - C_i(v_2)] \quad ; \quad X_{21} = -\frac{\eta}{4\pi} [2S_i(v_0) - S_i(v_1) - S_i(v_2)]$$

$$v_0 = kd \quad ; \quad v_1 = k(\sqrt{d^2 + \ell^2} + \ell) \quad ; \quad v_2 = k(\sqrt{d^2 + \ell^2} - \ell)$$

$$G = \frac{4\pi}{\lambda^2} A_e \quad ; \quad A_e = \varepsilon_{ap} A_p \quad ; \quad P_r = \left[\frac{G_t G_r \lambda^2}{(4\pi r)^2} \right] P_t$$

$$\vec{A}(\vec{r}) = \frac{\mu e^{-jkr}}{4\pi r} \vec{N} \quad ; \quad \vec{N} = \int \int_S \vec{J}_s(\vec{r}') e^{jkr' \cos \psi} dS'$$

$$\vec{F}(\vec{r}) = \frac{\epsilon e^{-jkr}}{4\pi r} \vec{L} \quad ; \quad \vec{L} = \int \int_S \vec{M}_s(\vec{r}') e^{jkr' \cos \psi} dS'$$

$$\vec{E}_A = -j\omega [A_\theta \hat{\theta} + A_\phi \hat{\phi}] \quad ; \quad \vec{H}_A = \frac{j\omega}{\eta} [A_\phi \hat{\theta} - A_\theta \hat{\phi}]$$

$$\vec{E}_F = j\omega\eta [-F_\phi \hat{\theta} + F_\theta \hat{\phi}] \quad ; \quad \vec{H}_F = -j\omega [F_\theta \hat{\theta} + F_\phi \hat{\phi}]$$