Engineering 9816 Assignment 1 Due: 10 AM, Wed., Jan. 27, 2016

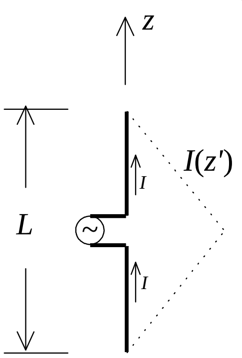
1. The electric field component of a particular e-m wave in free space is given by

$$\underset{\sim}{\vec{E}} = \hat{x}E_0 \cos(ay)\cos(\omega t - bz) \ .$$

(a) Determine the corresponding magnetic field $\underset{\sim}{\vec{H}}$. Find the relationships between the constants a, b and ω such that all of Maxwell's equations are satisfied. (a) Assuming that the given wave is the sum of two component uniform plane waves, determine the direction(s) of travel of the two waves.

2. Show from Maxwell's equations that $\vec{E}$ and $\vec{H}$ satisfy the following differential equations in a homogeneous medium containing charges and currents:

$$\begin{aligned} \vec{\nabla}^2\vec{E} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} &= \frac{1}{\epsilon}\vec{\nabla}\rho + \mu\frac{\partial\vec{J}}{\partial t} \\ \vec{\nabla}^2\vec{H} - \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2} &= -\vec{\nabla}\times\vec{J} \end{aligned}$$



3. For a short dipole antenna of length L (not a Hertzian dipole) the current distribution may be regarded to be triangular as shown. Take the current at the feed gap to be I_0 and assume that the dipole is directed along the z axis. (a) Determine the expression for the current $I(z')$ along the antenna. (b) The effective length L_{eff} of the antenna is defined as the length of a corresponding antenna which carries a uniform current I_0 over its entire length and which produces the same far-field electric field as the antenna in (a). Mathematically,

$$L_{\text{eff}} = \frac{1}{I_0}\int_{-L/2}^{L/2} I(z')dz' \ .$$

Determine the effective length of the short dipole in (a).

4. Determine the vector potential A_z for the antenna in question 3(a) assuming $|\vec{r} - \vec{r}'|$ may be approximated as r in both magnitude and phase.

5. One means of seeking solutions for the electric and magnetic fields of current sources is via the so-called Hertzian potential $\vec{\Pi}$ (it is, in fact, related to the vector potential $\vec{A}$ of the class notes). Suppose we define the magnetic field intensity in terms of $\vec{\Pi}$ as $\vec{H} = j\omega\epsilon\vec{\nabla}\times\vec{\Pi}$. (a) Show that

$$\vec{\nabla}^2\Pi + k^2\vec{\Pi} = j\frac{1}{\omega\epsilon}\vec{J}$$

where all other symbols have their usual meaning. Hint: In arriving at the result you will need to introduce a scalar potential Φ related to $\vec{E}$ in a way *similar* to that in the class notes. (b) Notice that the result above is another non-homogeneous Helmoltz-type equation. By analogy to the similar equation containing $\vec{A}$ as derived in the class notes, write down the solution for the result given in (a).

6. If magnetic current density $\vec{J}_m$ and magnetic charge density ρ_m actually existed, Maxwell's equations could be written (for time-harmonic fields) as

$$\vec{\nabla} \times \vec{E} = -j\omega\vec{B} - \vec{J}_m \qquad \vec{\nabla} \times \vec{H} = j\omega\vec{D}$$
$$\vec{\nabla} \cdot \vec{B} = \rho_m \qquad \vec{\nabla} \cdot \vec{D} = 0 \qquad \vec{\nabla} \cdot \vec{J}_m = -j\omega\rho_m$$

Let $\vec{A}_m$ and Φ_m be magnetic vector and scalar potentials, respectively, and define $\vec{D} = -\vec{\nabla} \times \vec{A}_m$. Show that the equations relating the potentials to the sources are

$$\begin{aligned}(\vec{\nabla}^2 + k_0^2)\vec{A}_m &= -\epsilon_0\vec{J}_m \\ (\vec{\nabla}^2 + k_0^2)\Phi_m &= -\frac{\rho_m}{\mu_0}\end{aligned}$$

and that

$$\vec{H} = -j\omega\vec{A}_m + \frac{\vec{\nabla}\vec{\nabla} \cdot \vec{A}_m}{j\omega\mu_0\epsilon_0}.$$

We have assumed that $\mu_R = 1$ and $\epsilon_R = 1$ so that $k = k_0 = \omega\sqrt{\mu_0\epsilon_0}$.

7. Calculate $\underset{\sim}{\vec{E}}$ and $\underset{\sim}{\vec{H}}$ if it is known that the phasor form of the vector and scalar potentials associated with the field are given by

$$\begin{aligned}\vec{A} &= A_0 e^{-jkz}\hat{x} \text{ Wb/m} \\ \Phi &= 0\end{aligned}$$

where $k = \omega\sqrt{\mu\epsilon}$. Before doing the problem, show that the potentials satisfy the Lorenz gauge condition.

8. The scalar and vector potentials for a particular e-m field are given by

$$\Phi = 0, \quad \underset{\sim}{\vec{A}} = \begin{cases} \frac{\mu_0 k}{4c}(ct - |x|)^2\,\hat{z}, & \text{for } |x| < ct \\ 0, & \text{for } |x| > ct \end{cases}$$

where k is the wavenumber and c is the free-space speed of light. (a) Use the time-dependent form of equation (1.32) of the notes to determine $\underset{\sim}{\vec{E}}$ for $|x| < ct$. (b) Use the vector potential to find the corresponding $\underset{\sim}{\vec{B}}$. (c) Determine both the divergence and curl of both these $\underset{\sim}{\vec{E}}$ and $\underset{\sim}{\vec{B}}$ fields. (d) There is no *charge* density producing the given potentials. How do we know this? (e) In fact, there is a surface current density, $\underset{\sim}{\vec{K}}$, acting as the source. Sketch the $\underset{\sim}{\vec{B}}$ field at a fixed time and, without calculation, argue from what you know about the magnetic field boundary conditions that $\underset{\sim}{\vec{K}}$ indeed exists. Find $\underset{\sim}{\vec{K}}$.