Engineering 9816 Assignment 3 **Due: Mon., March 14, 2016**

1. (a) Derive the expression for the $\vec{H}$-field far-field for a small loop as as given in equation (2.89) of the class notes and use this to derive the far-field $\vec{E}$-field as given in equation (2.90) of the same notes.

   (b) Using the results for the $\vec{E}$ and $\vec{H}$ fields, determine the time averaged Poynting vector for this small loop.

   (c) Find the radiation intensity and radiated power.

   (d) Consider a 2.5-cm diameter 12-gauge circular copper wire loop carrying a constant 0.5 A "phasor" current, the frequency of which is 10 MHz. Determine the radiation efficiency, the gain, the directivity and the EIRP for the loop (don't worry about the very poor efficiency that you get).

2. A small coil of radius $r_0 = 5$ cm and with $N = 10$ turns is used as a receiving antenna. The antenna (taken to be centred on the y-axis, say) is located 10 km away from a half-wave dipole (taken to be centred on the z-axis) and oriented for maximum magnetic flux penetration. Find the magnitude of the induced open-circuit voltage in the loop when the input power to the half-wave dipole is 5 W. The dipole is operating in the HF band at 27 MHz. Recollection of a form of Faraday's law in phasor form will help here: if the magnetic field intensity (H_ϕ, say) is everywhere perpendicular to a single loop of area A, then the induced voltage $V_{\text{ind}} = -j\omega\mu_0 H_\phi A$; note, however, that there are N turns here.

3. Let a small single-turn loop be located at $(0, r_1, 0)$ in Cartesian coordinate space. The axis of the loop is parallel to the x-axis and the loop radius is r_0. A Hertzian dipole of length $d\ell$ is centred on the z axis and carries a current I_d. (a) Find the induced open-circuit voltage in the loop and denote it by V_ℓ (this procedure follows that of Question 1). (b) Next, if current in the loop were I_ℓ find the electric field induced in the dipole. (Note that the dipole is in the far-field region and oriented to receive the maximum electric field radiated by the loop.) Also, find the open-circuit voltage V_d induced in the dipole. (c) Finally show that reciprocity holds – i.e. show that $V_d I_d = V_\ell I_\ell$.

4. The current distribution on a particular thin linear vertical antenna (not the usual dipole in this case) is given as

$$I(z') = I_0 e^{-jkz'}, 0 \leq z' \leq \ell\ .$$

   (a) Determine the far-field spherical electric and magnetic field components and (b) the radiation power density.

5. The input *reactance* of a Hertzian dipole with $\ell = \lambda/60$ and radius $a = \lambda/200$ is given by

$$X_{\text{in}} = -120 \frac{[\ln(\ell/a) - 1]}{\tan(k\ell)} \,.$$

Assuming that the dipole is copper with a conductivity of 5.7×10^7 S/m, determine at $f = 1$ GHz the

(a) loss resistance

(b) radiation resistance

(c) radiation efficiency

(d) VSWR when the antenna is connected to a 50-Ω line.