Engineering 7811 Formula and Data Sheet   Winter, 2016

$$\vec{\nabla} \times \underline{\vec{E}} = -\frac{\partial \vec{B}}{\partial t} \ ; \ \vec{\nabla} \cdot \underline{\vec{D}} = \rho_v \ ; \ \vec{\nabla} \times \underline{\vec{H}} = \underline{\vec{J}} + \frac{\partial \vec{D}}{\partial t} \ ; \ \vec{\nabla} \cdot \underline{\vec{B}} = 0 \ ; \ \underline{\vec{D}} = \epsilon \underline{\vec{E}} \ ; \ \underline{\vec{B}} = \mu \underline{\vec{H}}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \approx \frac{10^{-9}}{36\pi} \text{ F/m} \ ; \ \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \ ; \ \epsilon = \epsilon_0(1 + \chi_e) \ ; \ \mu = \mu_0(1 + \chi_m)$$

$$\underline{\vec{D}} = \epsilon_0 \underline{\vec{E}} + \epsilon_0 \chi_e \underline{\vec{E}} = \epsilon_0 \underline{\vec{E}} + \underline{\vec{P}} \ ; \ \underline{\vec{B}} = \mu_0 \underline{\vec{H}} + \mu_0 \chi_m \underline{\vec{H}} = \mu_0(\underline{\vec{H}} + \underline{\vec{M}})$$

$$\vec{\nabla} \cdot \underline{\vec{J}} = -\frac{\partial \rho_v}{\partial t} \ ; \ \underline{\vec{J}}_{cnv} = \rho_v \vec{v} \ ; \ \underline{\vec{J}} = \sigma \underline{\vec{E}} \ ; \ \vec{A}(\vec{r}, t) \equiv \underline{\vec{A}} = \mathcal{R}e\left\{\vec{A}e^{j\omega t}\right\}$$

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B}) \ ; \ v_p = \frac{1}{\sqrt{\mu\epsilon}} \ ; \ c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \ ; \ c = 3 \times 10^8 \text{ m/s} \ ; \ v_p = f\lambda$$

$$k = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\epsilon} \ ; \ jk = \alpha + j\beta \ ; \ \beta = \frac{2\pi}{\lambda} \ ; \ \eta = \sqrt{\frac{\mu}{\epsilon}} \ ; \ \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \ ; \ \eta_0 \approx 120\pi \ \Omega$$

$$\vec{\nabla} \times \vec{E} = -j\omega \vec{B} \quad ; \quad \vec{\nabla} \cdot \vec{D} = \rho_v \quad ; \quad \vec{\nabla} \times \vec{H} = \vec{J} + j\omega \vec{D} \quad ; \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{E}(\vec{r}) = -\vec{\nabla}\Phi(\vec{r}) \ ; \ \Phi(\vec{r}) = V(\vec{r}) = \int_{v'} \frac{\rho_v(\vec{r}')dv'}{4\pi\epsilon|\vec{r} - \vec{r}'|} \ ; \ \vec{\nabla}^2 \Phi = -\frac{\rho_v}{\epsilon}$$

$$\vec{A}(\vec{r}) = \int_{v'} \frac{\mu_0 \vec{J}(\vec{r}')dv'}{4\pi|\vec{r} - \vec{r}'|} = \mu_0 \vec{J}(x, y, z) \overset{*}{\underset{3d}{}} \frac{1}{4\pi|\vec{r}|}$$

$$\vec{\nabla}^2 \Phi + k^2 \Phi = -\frac{\rho_v}{\epsilon} \ ; \ \vec{\nabla} \cdot \vec{A} = -j\omega\mu\epsilon\Phi \ ; \ \vec{A}(\vec{r}) = \int_{v'} \frac{\mu_0 \vec{J}(\vec{r}')e^{-jk|\vec{r} - \vec{r}'|}dv'}{4\pi|\vec{r} - \vec{r}'|}$$

$$\Phi(\vec{r}) = \int_{v'} \frac{\rho_v(\vec{r}')e^{-jk|\vec{r} - \vec{r}'|}dv'}{4\pi\epsilon|\vec{r} - \vec{r}'|} \ ; \ \vec{\nabla}^2 \vec{E} + k^2 \vec{E} = 0$$

$$\vec{E} = -j\omega\vec{A} - \frac{j}{\omega\mu\epsilon}\vec{\nabla}(\vec{\nabla} \cdot \vec{A}) \ ; \ \eta = \frac{E_\theta}{H_\phi} \ ; \ \mathcal{F}(\omega)e^{-j\omega a} \Leftrightarrow f(t - a)$$

$$\underline{\vec{\mathcal{P}}} = \frac{1}{2}\mathcal{R}e\left\{\vec{E} \times \vec{H}^*\right\} + \frac{1}{2}\mathcal{R}e\left\{\vec{E} \times \vec{H}e^{j2\omega t}\right\} \ ; \ \vec{\mathcal{P}}_a = \frac{1}{2}\mathcal{R}e\left\{\vec{E} \times \vec{H}^*\right\} \ ; \ \vec{\mathcal{P}}_a = \frac{1}{2\eta}|\vec{E}|^2 \hat{r} \ .$$

$$U = \frac{dP_r}{d\Omega} = \frac{1}{2}r^2 \mathcal{R}e\left\{\vec{E} \times \vec{H}^*\right\} \cdot \hat{r} \ ; \ U_0 = \frac{P_r}{4\pi} \ ; \ D(\theta, \phi) = \frac{U(\theta, \phi)}{U_0} = \frac{dP_r/d\Omega}{P_r/4\pi}$$

$$G(\theta, \phi) = 4\pi\frac{U(\theta, \phi)}{P_{\text{in}}} \ ; \ G(\theta, \phi) = \varepsilon_r D(\theta, \phi) \ ; \ \text{EIRP} = P_{\text{in}}G_{\text{max}} \ ; \ \varepsilon_r = \frac{R_r}{R_r + R_L}$$

$$I(z') = I_0 \sin\left[k\left(\frac{\ell}{2} - |z'|\right)\right]$$

$$\vec{E}(\vec{r}) = E_\theta \hat{\theta} = \frac{j\eta I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right) - \cos\left(\frac{k\ell}{2}\right)}{\sin\theta}\right]\hat{\theta}$$

$$\vec{H}(\vec{r}) = H_\phi \hat{\phi} = \frac{j I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right) - \cos\left(\frac{k\ell}{2}\right)}{\sin\theta}\right]\hat{\phi} \ .$$

	$\hat{\rho}$	$\hat{\phi}$	$\hat{z}$
$\hat{x}\cdot$	$\cos\phi$	$-\sin\phi$	0
$\hat{y}\cdot$	$\sin\phi$	$\cos\phi$	0
$\hat{z}\cdot$	0	0	1

	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$
$\hat{x}\cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\hat{y}\cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\hat{z}\cdot$	$\cos\theta$	$-\sin\theta$	0

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \ ; \ \theta = \cos^{-1}\left(\frac{z}{|\vec{r}|}\right) \quad ; \ d\Omega = \sin\theta d\theta d\phi$$

$$\vec{r} = \rho\,\hat{\rho} + z\,\hat{z} \ ; \ d\vec{L} = d\vec{r} = d\rho\hat{\rho} + \rho d\phi\hat{\phi} + dz\hat{z} \ ; \ d\vec{r} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$$

$$\vec{\nabla}\varphi = \frac{\partial\varphi}{\partial\rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial\varphi}{\partial\phi}\hat{\phi} + \frac{\partial\varphi}{\partial z}\hat{z} \ ; \ \vec{\nabla}\varphi = \frac{\partial\varphi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\varphi}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial\varphi}{\partial\phi}\hat{\phi}$$

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \quad ; \quad \vec{\nabla} \cdot \vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(A_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\vec{\nabla} \cdot \vec{\nabla}\varphi = \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2}\right) = \vec{\nabla}^2 \varphi \quad ; \quad \vec{\nabla} \cdot \vec{\nabla}\Phi = -\frac{\rho_v}{\epsilon} \quad ; \quad \vec{\nabla}^2 \Phi = 0$$

$$\vec{\nabla}^2 \vec{A} = \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right](\vec{A}) \quad ; \quad \oint_S \vec{A} \cdot d\vec{S} = \int_{\text{vol}} \vec{\nabla} \cdot \vec{A}\, dv \quad ; \quad \oint_C \vec{A} \cdot d\vec{L} = \int_S \left(\vec{\nabla} \times \vec{A}\right) \cdot d\vec{S}$$

$$d\vec{S}_r = r^2 \sin\theta d\theta d\phi \hat{r} \quad ; \quad d\vec{S}_\theta = r\sin\theta dr d\phi \hat{\theta} \quad ; \quad d\vec{S}_\phi = r dr d\theta \hat{\phi} \quad ; \quad d\Omega = \frac{dS}{r^2} = \sin\theta d\theta d\phi$$

$$\vec{\nabla} \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} \quad ; \quad \vec{\nabla} \times \vec{A} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

$$|\text{AF}| = \left|\frac{\sin\left(\frac{M\Psi}{2}\right)}{\sin\left(\frac{\Psi}{2}\right)}\right| \quad ; \quad \Psi = \beta + kd\cos\alpha \quad ; \quad M\Psi = \pm(2i+1)\pi \,, \ i \in \text{N}$$

$$\frac{M\Psi}{2} = \pm i\pi \,, \ 1 \in \text{N} \ (\text{but } i \neq M, 2M, 3M, \ldots) \quad ; \quad \text{BW}_{1/2} = 2\Delta\phi_{1/2} = \frac{2.65\lambda}{\pi M d}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \quad ; \quad \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad ; \quad \sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\sin(a \pm b) = \sin a\cos b \pm \cos a\sin b \quad ; \quad \cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$$

$$D(\phi) \approx \frac{Mkd}{\pi}\left[\frac{\sin\left(\frac{Mkd}{2}\cos\phi\right)}{\frac{Mkd}{2}\cos\phi}\right]^2 \quad ; \quad \cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$D(\phi) \approx \frac{2Mkd}{\pi}\left[\frac{\sin\left(\frac{Mkd(\cos\phi-1)}{2}\right)}{\frac{Mkd(\cos-1)}{2}}\right]^2 \quad ; \quad \text{BW}_{1/2} = 2\Delta\phi_{1/2} = 3.26\left[\frac{\lambda}{\pi M d}\right]^{1/2}$$

$$|\text{AF}| = 2^N\left|\cos^N\left(\frac{kd\cos\phi}{2}\right)\right| \quad ; \quad \binom{N}{n} = \frac{N!}{(N-n)!n!} \quad ; \quad \frac{\tan\beta}{\beta} = 2 - \frac{2\beta}{k\ell}$$

$$R_r = \frac{\eta}{4\pi}[0.5772 + \ln(2\pi n) - C_i(2\pi n)] \quad ; \quad X = \frac{\eta}{4\pi}S_i(2\pi n)$$

$$R_{21} = \frac{\eta}{4\pi}[2C_i(v_0) - C_i(v_1) - C_i(v_2)] \quad ; \quad X_{21} = -\frac{\eta}{4\pi}[2S_i(v_0) - S_i(v_1) - S_i(v_2)]$$

$$v_0 = kd \quad ; \quad v_1 = k\left(\sqrt{d^2 + \ell^2} + \ell\right) \quad ; \quad v_2 = k\left(\sqrt{d^2 + \ell^2} - \ell\right)$$

$$G = \frac{4\pi}{\lambda^2}A_e \quad ; \quad A_e = \varepsilon_{ap}A_p \quad ; \quad P_r = \left[\frac{G_t G_r \lambda^2}{(4\pi r)^2}\right]P_t$$

$$\vec{A}(\vec{r}) = \frac{\mu e^{-jkr}}{4\pi r}\vec{N} \quad ; \quad \vec{N} = \int\int_S \vec{J}_s(\vec{r'})e^{jkr'\cos\psi}dS'$$

$$\vec{F}(\vec{r}) = \frac{\epsilon e^{-jkr}}{4\pi r}\vec{L} \quad ; \quad \vec{L} = \int\int_S \vec{M}_s(\vec{r'})e^{jkr'\cos\psi}dS'$$

$$\vec{E}_A = -j\omega\left[A_\theta\hat{\theta} + A_\phi\hat{\phi}\right] \quad ; \quad \vec{H}_A = \frac{j\omega}{\eta}\left[A_\phi\hat{\theta} - A_\theta\hat{\phi}\right]$$

$$\vec{E}_F = j\omega\eta\left[-F_\phi\hat{\theta} + F_\theta\hat{\phi}\right] \quad ; \quad \vec{H}_F = -j\omega\left[F_\theta\hat{\theta} + F_\phi\hat{\phi}\right]$$

All the best in your career!