An Investigation of Impropriety and Noncircularity in High Frequency Radar Data

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Abstract—Discarding the phase content of signals from single-channel high-frequency (HF) radar is commonplace among practitioners in the field. From the perspective of complex-valued statistics, this practice implicitly implies that the complex-valued HF data is second-order proper or circular. This paper presents a preliminary investigation into the validity of this assumption using HF surface wave radar (HF-SWR) field data. It is found that the HF-SWR data is indeed second-order improper or noncircular. This negates the common belief regarding the non-informativeness of the phase, and it warrants more in-depth analysis into characterizing impropriety/noncircularity for relevant HF radar applications.

Index Terms—HF radar, phase, impropriety, noncircularity, nonstationarity

I. INTRODUCTION

Discarding the phase content of signals from single-channel coherent radar, such as high-frequency (HF) radar, is commonplace in the radar community. This practice is justified by conventional radar resolution theory, strictly relevant to point targets, and based on various simplifying assumptions such as linearity and stationarity [1, 2]. In previous investigations, the insufficiency for one of these assumptions — i.e., linearity -, which leads to the use of the second-order perturbation theory, has been demonstrated [3–5].

This paper presents a preliminary investigation into the validity of another often implicit assumption that leads to the discarding of the phase. From the perspective of complex-valued statistics, the practice of discarding the phase may be justified by assuming that the underlying random variables are second-order proper or circular in nature [6–8]. Properly implies that the complex-valued HF data is uncorrelated with its complex-conjugate. Circularity means that the complex-valued HF data has a probability density function (PDF) that is invariant under rotation in the complex plane. Accordingly, discarding the phase content implicitly implies that the aforementioned assumptions are satisfied. Otherwise, valuable information about the targets in the complex-valued data is lost. This information can be of significant importance for various HF radar applications. In terms of physical properties, a necessary but insufficient condition for impropriety or noncircularity is that the random process be at least nonstationary [9]. This means that if the impropriety/noncircularity of the HF data can be estimated, one can characterize the nonstationarity of the underlying oceanic/ionospheric processes from which the HF signals are backscattered.

The remainder of this paper is organized as follows. In Section II, the formal definitions of propriety/impropriety and circularity/noncircularity are presented. In Section III, the HF datasets utilized in this paper are described. In Section IV, the analysis procedures proposed in this paper are outlined. Results and discussion are given in Section V, and concluding remarks appear in Section VI.

II. DEFINITIONS OF PROPERITY/IMPROPRIETY AND CIRCULARITY/NONCIRCULARITY

Formally, a zero-mean complex-valued random variable (i.e., $X = X_R + jX_I$) is said to be second-order proper when its pseudo-covariance is zero [6]; i.e., when

$$\Psi = E \{ X^2 \} = 0.$$  (1)

For a random vector $X$, propriety implies [6, 7]

$$\sigma_{X_R} = \sigma_{X_I},$$  (2)

and,

$$E \{ X_RX_R^T \} = E \{ X_IX_I^T \},$$  (3)

and,

$$E \{ X_RX_I^T \} = -E \{ X_IX_R^T \},$$  (4)

where $\sigma_{X_R}$ and $\sigma_{X_I}$ are the standard deviations of the real and imaginary parts of $X$, respectively; $X_R$ and $X_I$ are the real and imaginary parts of $X$, respectively; $E \{ \cdot \}$ is the expectation; and $T$ denotes the transpose.

A stronger condition for propriety is based on the PDF of the random variable. A complex-random variable $X$ is termed circular if $X$ and $X \exp(j\theta)$ have the same PDF (i.e., the PDF is rotation invariant) [8]. This means that the phase of the...
complex-valued random variable is non-informative. Hence, in this case, the PDF is a function of only the magnitude which implies that the PDF can be written as a function of $XX^*$ rather than of $X$ and $X^*$, separately ($*$ denotes the complex conjugate). Since the phase is non-informative for a proper or a circular random variable, a real-valued approach and complex-valued approach for this case are often equivalent [6, 7].

III. HF DATASETS FOR ANALYSIS

Four HF datasets are considered for analysis in this paper (see Table I). The datasets were collected using a hybrid WERA-Northern Radar HF surface wave radar installed at the site of the former Naval Station in Argentia, Newfoundland [10]. The system employs eight receive phased array antennas, and the data was collected in the SORT format\(^1\). The radar parameters for all the datasets considered are: sweep time = 0.39 s, chirp type = sawtooth, chirp bandwidth = 50 kHz, number of chirps = 1024, and center frequency = 13.385 MHz. Relevant buoy measurements (see Table I) were obtained from the Smart Bay buoy available for the site where the radar is installed [11].

Each dataset (DS) was beamformed into 61 beams covering 120° azimuthal extent. The beam angles are in the range \([-60° +\text{ boresight}, 60° +\text{ boresight}]\) with a step of 2°. Bore-sight direction is 250° from True North. This makes the size of the beamformed matrix of each DS to $50 \times 1024 \times 61$. The 3/4th beam corresponds to where the buoy is located, at range cell #3, about 9 km from the radar.

IV. PROPOSED PROCEDURES FOR NONCIRCULARITY AND IMPROPIETY ANALYSIS

Two procedures [6, 7], for analyzing noncircularity and impropriety in the complex-valued HF data, are considered. In the first method, a hypothesis test is used to examine the statistical significance of circularity/noncircularity in the data (i.e., the alternative hypothesis $H_{1,NC}$ is for noncircularity). This method employs a generalized likelihood ratio test (GLRT), based on the complex generalized Gaussian distribution (CGGD), for a specific detection threshold defined by the probability of false alarm (PFA) [7]. The CGGD utilizes the so-called augmented representation of the complex-valued random variables [8]. The shape parameter of the CGGD distribution (i.e., $\hat{c}$) can be used to characterize the nonlinearity (also known as non-Gaussianity) of the data [7]. The GGD family is general in that it encompasses a wide array of distributions with different tail characteristics from super-Gaussian ($\hat{c} < 1$) to sub-Gaussian ($\hat{c} > 1$) with specific densities such as Laplacian ($\hat{c} = 0$) and Gaussian ($\hat{c} = 1$) distributions [7, 12]. The GGD distribution allows the rate of tail decay to be varied and it is known to offer a good model for some impulsive phenomena. In the second method, the strength of the impropriety in the complex-valued HF data is quantified using the measure [6, 7]

$$|\Psi| = |E\{X^2\}|, \ 0 < |\Psi| < 1. \quad (5)$$

Note that the greater $|\Psi|$ is, the greater the impropriety effect.

V. RESULTS AND DISCUSSION

Prior to applying the noncircularity and impropriety tests, we demonstrate the goodness-of-fit for the histograms pertaining to the real and imaginary parts of the HF data with the GGD distribution. Further, a comparison with the Gaussian distribution is considered. Representative examples for qualitative goodness-of-fit are depicted in Figure 1 and Figure 2. These results are quantified in Table II using the Jensen-Shannon divergence (JSD) [13]. Note that the smallest value possible for JSD is zero, and this can only be obtained for a perfect fit.

\begin{table}[h]
\centering
\caption{HF DATASETS AND BUOY MEASUREMENTS. DS $\equiv$ DATASET, $n_S \equiv$ NUMBER OF SAMPLES PER CHIRP, $n_C \equiv$ NUMBER OF CHIRPS, $n_{Ant} \equiv$ NUMBER OF RECEIVE ELEMENTS, $h_\frac{3}{4} \equiv$ SIGNIFICANT WAVE HEIGHT.}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
DS # & Date and Time Collected & Matrix Dimension $n_S \times n_C \times n_{Ant}$ & Buoy Measurement Wind Speed | $h_\frac{3}{4}$ | Sea State | Description \\
\hline
1 & 2013-07-10 21:04 & $50 \times 1024 \times 8$ & 6.1 m/s & 0.23 m & 1 & Smooth \\
2 & 2013-07-10 21:26 & $50 \times 1024 \times 8$ & 5.5 m/s & 0.3 m & 1 & Smooth \\
3 & 2012-11-29 15:23 & $50 \times 1024 \times 8$ & 14.6 m/s & 1.34 m & 3 & Moderate \\
4 & 2013-01-31 17:01 & $50 \times 1024 \times 8$ & 13.2 m/s & 3.78 m & 6 & High \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{QUANTITATIVE RESULTS FOR GOODNESS-OF-FIT USING JSD.}
\begin{tabular}{|c|c|c|c|}
\hline
PDF & Real-part & Imaginary-part & \\
\hline
Gaussian & 0.2618 & 0.0841 & 0.6599 \\
CGGD & 10.0366 & 5.4942 & 0.9705 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{RESULTS FOR INVESTIGATION INTO NONCIRCULARITY AND IMPROPIETY IN HF RADAR DATA FOR ANT #1. PFA=0.0001.}
\begin{tabular}{|c|c|c|c|c|}
\hline
DS # & DS #2 & DS #3 & DS #4 & \\
\hline
Nonlinearity ($H_{1,NL}$) & 1 & 1 & 1 & 1 \\
Shape parameter for CGGD ($\hat{c}$) & 0.2 & 0.2 & 0.2 & 0.2 \\
Noncircularity ($H_{1,NC}$) & 1 & 1 & 1 & 1 \\
Impropriety ($|\Psi|$) & 0.0662 & 0.0722 & 0.0664 & 0.0426 \\
\hline
\end{tabular}
\end{table}

\(^1\)The data is range resolved.
These results clearly demonstrate the suitability of the GGD distribution to model the HF data. We note that the goodness-of-fit with the GGD distribution can even be increased by either considering a greater number of HF data samples or by upsampling the HF data. Also, it is noted that the HF data possesses some nonlinear characteristics as it deviates, in some cases significantly, from the Gaussian distribution. Accordingly, we adopt a CGGD-based hypothesis test for noncircularity and nonlinearity from [7]. The shape parameter $\hat{c}$ for fitting with the CGGD distribution is used to characterize the nonlinearity as described in Section IV.

Next, to examine the effect of noncircularity and impropriety along with nonlinearity, representative results are presented. Particularly, the tests are applied to all the datasets

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**Table IV**

RESULTS FOR INVESTIGATION INTO NONCIRCULARITY AND IMPROPRIETY IN HF RADAR DATA FOR ANT #4. PFA=0.0001.

<table>
<thead>
<tr>
<th></th>
<th>DS #1</th>
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<th>DS #3</th>
<th>DS #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonlinearity ($H_{1, NL}$)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Shape parameter for CGGD ($\hat{c}$)</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Noncircularity ($H_{1, NC}$)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Impropriety ($</td>
<td>\Psi</td>
<td>$)</td>
<td>0.1092</td>
<td>0.0976</td>
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</tbody>
</table>

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**Table V**

RESULTS FOR INVESTIGATION INTO NONCIRCULARITY AND IMPROPRIETY IN HF RADAR DATA FOR ANT #8. PFA=0.0001.

<table>
<thead>
<tr>
<th></th>
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<th>DS #2</th>
<th>DS #3</th>
<th>DS #4</th>
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<td>1</td>
<td>1</td>
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<tr>
<td>Shape parameter for CGGD ($\hat{c}$)</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3681</td>
<td>0.4039</td>
</tr>
<tr>
<td>Noncircularity ($H_{1, NC}$)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Impropriety ($</td>
<td>\Psi</td>
<td>$)</td>
<td>0.1113</td>
<td>0.1106</td>
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**Table VI**

RESULTS FOR INVESTIGATION INTO NONCIRCULARITY AND IMPROPRIETY IN BEAMFORMED HF RADAR DATA (BEAM #34). PFA=0.0001.

<table>
<thead>
<tr>
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<th>DS #1</th>
<th>DS #2</th>
<th>DS #3</th>
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<tr>
<td>Shape parameter for CGGD ($\hat{c}$)</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3681</td>
<td>0.4039</td>
</tr>
<tr>
<td>Noncircularity ($H_{1, NC}$)</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Impropriety ($</td>
<td>\Psi</td>
<td>$)</td>
<td>0.0464</td>
<td>0.0542</td>
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</table>
from antenna (Ant) #1 (see Table III), Ant #4 (see Table IV), and Ant #8 (see Table V). Also, representative results for the beamformed data are given in terms of beam #34 (see Table VI). Since $\mathcal{H}_{1,NC} = 1$, the results confirm the statistical significance of nonlinearity in all the complex-valued HF datasets tested. The strength of nonlinearity as characterized by $\hat{c}$ is found to be 0.2 for all the datasets pertaining to Ant #1 and Ant #4. Similar values of $\hat{c} = 0.2$ are obtained for Ant #8 as well as beam #34 from the beamformed data with the exception of DS #3 and DS #4 where $\hat{c} = 0.3861$ and $\hat{c} = 0.4039$, respectively. Note that the closer $\hat{c}$ is to 1, the less nonlinear the data. Furthermore, since $\mathcal{H}_{2,NC} = 1$, tests from all the datasets considered clearly demonstrate the statistical significance of noncircularity. The strength of impropriety for all cases is given by $|\Psi|$. This shows that, unlike the common implicit assumption of the circularity or propriety of the complex-valued HF data, the HF data is second-order noncircular or improper. Hence, the phase is indeed informative.

The results obtained in this work are promising and they warrant more in-depth investigation. In [2], the noncircularity and impropriety in complex-valued synthetic aperture radar (SAR) imagery were used to develop features found to be useful for target recognition applications. Similarly, given the relationship between noncircularity/impropriety and nonstationarity [9], it may be possible to take advantage of the noncircularity/impropriety to characterize the nonstationarity of the ocean waves, for example. Hence, future-work will address the development of a suitable method for characterizing the noncircularity/impropriety in complex-valued HF data. Also, more in-depth analysis will be conducted to study the utility of the CGGD distribution and its shape parameter (i.e., $\hat{c}$) in characterizing nonlinearity of radar data associated with the remote sensing of, for example, the ocean or the ionosphere.

VI. CONCLUSIONS

In most works on single-channel HF radar, the phase content is discarded. This is a consequence of the conventional radar resolution theory in which, among other assumptions, it is implicitly assumed that the complex-valued HF data is second-order proper or circular. Unlike that common belief, the preliminary results presented in this paper have clearly demonstrated the statistical significance of noncircularity in the complex-valued HF data. Additionally, the absolute value of the pseudo-covariance (i.e., $|\Psi|$) is used to characterize the strength of impropriety in the HF data. Moreover, the CGGD distribution is demonstrated to offer a good model for the complex-valued HF data, and its shape parameter (i.e., $\hat{c}$) is found to be suitable for characterizing nonlinearity in the complex-valued HF data. Future-work will expand upon the findings of this paper in the context of relevant HF radar applications.

ACKNOWLEDGMENTS

This work was supported by an NSERC Discovery Grant (No. RGPIN-2015-05289) and an Atlantic Innovation Fund award to Eric W. Gill (principal investigator).

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